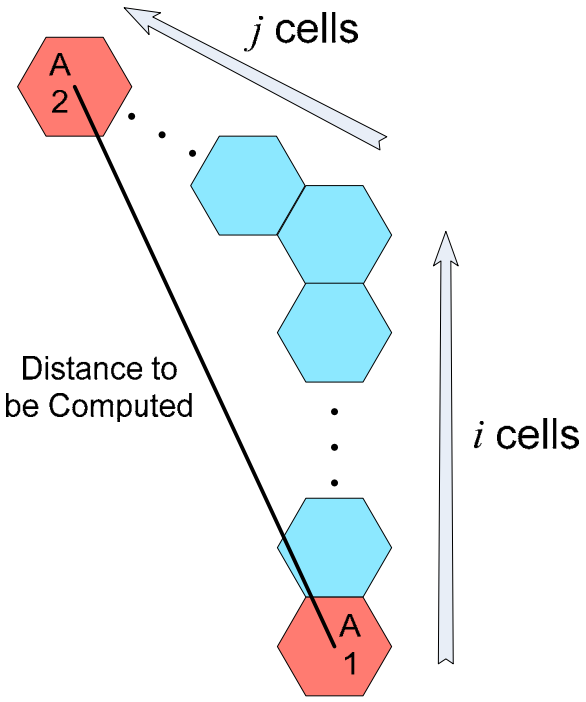
# Lecture 4: Frequency Reuse Concepts

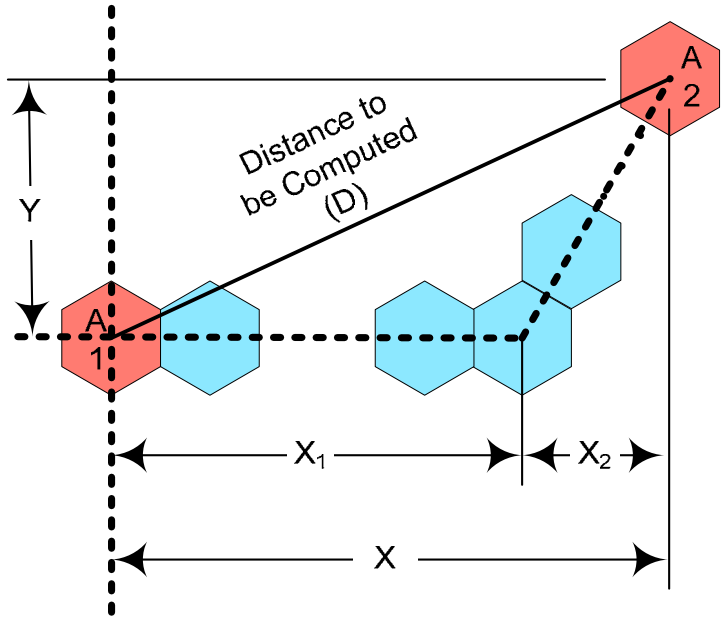
## Distance between Co-Channel Cell Centers

Knowing the relation between $i$, $j$, and $N$, we can easily find distance between the center points of two co-channel cells (cells with the same frequency band) in terms of $i$ and $j$. Consider any two co-channel cells. We will take for simplicity a cell and its co-channel cell that is located to its top-left location as shown in the below figure.



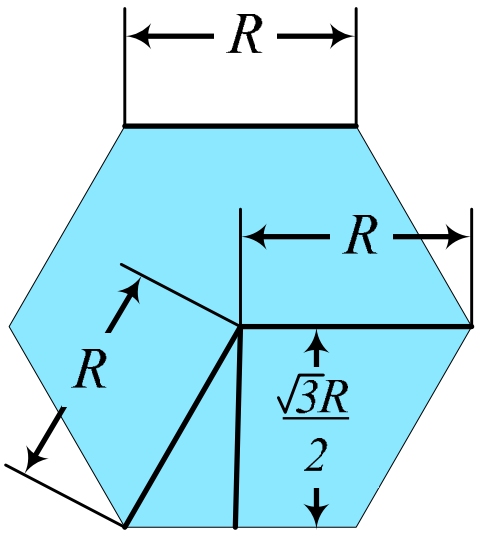

If we rotate this configuration by 90° Clockwise, and set to the origin to be the center point of Cell 1, we get the following:

# Lecture 4: Frequency Reuse Concepts




To compute the distance $D$, we need to compute $X$ and $Y$, or simply $X_1$, $X_2$, and $Y$ shown in the above figure. Once these are found, the distance is computed easily using Pythagoras Theorem.

Let us define the cell radius $R$ to be the distance from the center point of the cell to any of its corners as shown in the figure below. Note that the edge length of the cell is equal to cell radius because of the hexagonal shape, while the line from the center to mid point of any edge is equal to $\frac{\sqrt{3}R}{2}$.




Clearly,

$$X_1 = 2i\left(\frac{\sqrt{3}R}{2}\right) = iR\sqrt{3} \qquad (1)$$

$$X_2 = 2j\left(\frac{\sqrt{3}R}{2}\right)\cos(60°) = \frac{jR\sqrt{3}}{2} \tag{2}$$

Therefore,

$$\begin{aligned} X &= X_1 + X_2 \\ &= R\sqrt{3}\left(i + \frac{j}{2}\right) \end{aligned} \tag{3}$$

and *Y* is

$$Y = 2j\left(\frac{\sqrt{3}R}{2}\right)\sin(60°) = \frac{3jR}{2} \tag{4}$$

So, the distance *D* becomes

$$\begin{aligned} D &= \sqrt{3R^2\left(i + \frac{j}{2}\right)^2 + \frac{9j^2R^2}{4}} \\ &= \sqrt{3i^2R^2 + 3ijR^2 + \frac{3j^2R^2}{4} + \frac{9j^2R^2}{4}} \\ &= \sqrt{3i^2R^2 + 3ijR^2 + 3j^2R^2} \\ &= R\sqrt{3\left(i^2 + ij + j^2\right)} \end{aligned} \tag{5}$$

We notice that the part in the parentheses in the last form of *D* is nothing but *N*, which means we can rewrite it as

$$D = R\sqrt{3N} \tag{6}$$

The quantity

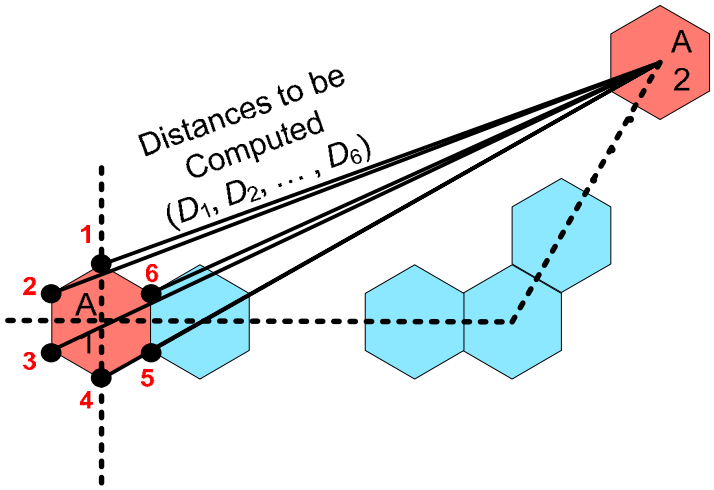
$$\frac{D}{R} = \sqrt{3N} = Q \tag{7}$$

is called the channel reuse ratio is an indication of how often frequencies are being reused.

## Distance between a Cell Corner and its Co-Channel Cell Centers

**Note that this is exactly the same as finding the distance between all corners of a cell and its co-channel cell center.** Now that we have found the distance between co-channel cells centers, we can easily extend this to find the distances between all co-channel cells centers and a specific corner of a

cell. This can be done by finding the coordinates of all corners of the original cells and taking the coordinates of these corners in consideration when computing the desired distances.




- We have already found coordinates of one co-channel cell to be $\left( R\sqrt{3}\left( i+\frac{j}{2} \right) \ , \ \frac{3jR}{2} \right)$ (8)
- Coordinates of **Corner 1** of Original Cell is $\left( 0 \ , \ R \right)$ (9)
- Coordinates of **Corner 2** of Original Cell is $\left( -\frac{\sqrt{3}R}{2} \ , \ \frac{R}{2} \right)$ (10)
- Coordinates of **Corner 3** of Original Cell is $\left( -\frac{\sqrt{3}R}{2} \ , \ -\frac{R}{2} \right)$ (11)
- Coordinates of **Corner 4** of Original Cell is $\left( 0 \ , \ -R \right)$ (12)
- Coordinates of **Corner 5** of Original Cell is $\left( \frac{\sqrt{3}R}{2} \ , \ -\frac{R}{2} \right)$ (13)
- Coordinates of **Corner 6** of Original Cell is $\left( \frac{\sqrt{3}R}{2} \ , \ \frac{R}{2} \right)$ (14)

So, now we can find all 6 distances between the center of the co-channel cell and the 6 corners of the original cell.

## Interference and System Capacity

There are different types of interference that cellular calls suffer from. Among these types are **Co-Channel Interference** (interference that results from the transmission of signals at the same frequency of a particular channel transmitted from a nearby co-channel cell). Also, there is **Adjacent-Channel Interference** (interference that results from transmitting a signal with a nearby frequency but from the

same tower or another mobile unit in the same cell. Because of the limited capabilities of analog and digital filters, it may be very difficult to remove adjacent channels completely).

## Co-Channel Interference and System Capacity

Interference is measured in terms of received Signal (Power) to received Interference (Power) Ratio or SIR, which is sometimes expressed as $\frac{S}{I}$. First note that the power $P_R$ received at some distance from antenna $d_R$ over the power $P_0$ received at some other distance $d_0$ is given by

$$\frac{P_R}{P_0} = \left(\frac{d_0}{d_R}\right)^n \tag{15}$$

which can be written in the form

$$P_R = P_0\left(\frac{d_0}{d_R}\right)^n \tag{16}$$

where $P_0$ is power received at some distance $d_0$, $P_R$ is power received at some distance $d_R$, and $n$ is called path loss exponent.

## Computation of Co-Channel Interference in Different Configurations

To compute the Forward channel SIR, we use equation (16) to compute the Signal power and again use equation (16) to compute the Interference power as follows

$$\frac{S}{I} = \frac{P_R \text{ of Signal}}{P_R \text{ of All Intefering Signals}}$$

$$= \frac{P_0\left(\frac{d_0}{\text{Dist. of Mobile to Original Tower}}\right)^n}{P_0\left(\frac{d_0}{\text{Dist. of Mobile to Tower 1}}\right)^n + \ldots + P_0\left(\frac{d_0}{\text{Dist. of Mobile to Tower 6}}\right)^n} \tag{17}$$

Clearly, $P_0$ and $d_0$ have no effect on the SIR and they cancel out to give

$$\frac{S}{I} = \frac{\left(\frac{1}{\text{Dist. of Mobile to Original Tower}}\right)^n}{\left(\frac{1}{\text{Dist. of Mobile to Tower 1}}\right)^n + \ldots + \left(\frac{1}{\text{Dist. of Mobile to Tower 6}}\right)^n} \tag{18}$$

The performance of a system is usually measured in terms of the minimum SIR, which occurs at the corners of the cell providing service to the mobile station. The distance in this worst case of serving tower to the mobile station is $R$ while the distance from the co-channel cell towers to the mobile station are obtained by finding the Cartesian distances of the co-channel cell tower coordinates given in (8) and the corners of the original tower given in (9) – (14).

## Numerical Evaluation of Worst SIR for Several Configurations

To find numerical results of worst SIR for several configurations, an MS Excel sheet is developed for this reason. Simple parameters such as $R$ (which has no real effect on the worst SIR), $i$, $j$, and $n$.