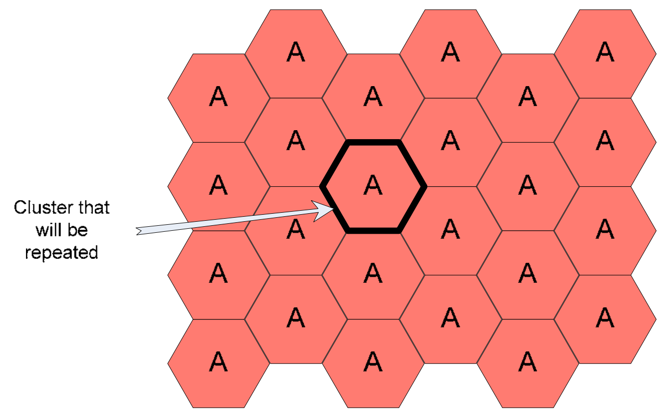
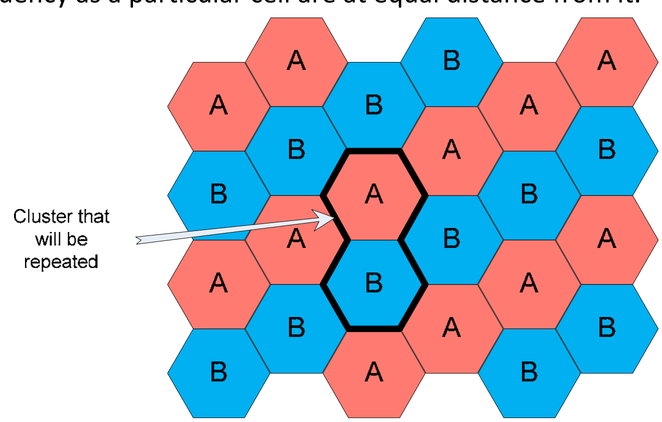
How Often Are Frequencies Reused (Frequency Reuse Factor)?

The band of frequency allocated for cellular system use can be reused with different CLUSTERS. We mean by cluster here the configuration of cells over which the complete frequency band is divided and this configuration of cells is repeater over and over. The frequency reuse factor is defined as 1 over the number of cells in the cluster of the system. Valid clusters are those that result in 6 cells with the same frequency of a particular cell located at equal distance from it. Considering several cluster shapes with different frequency reuse factors:

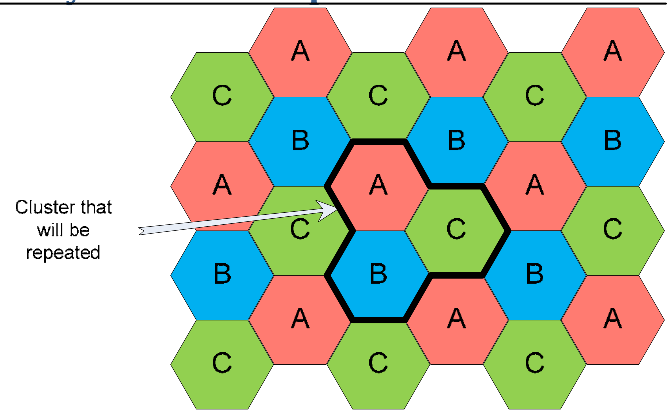
1) **1-Cell Frequency Reuse Cluster (Frequency Reuse Factor = 1) [Valid]:** This means that the whole band of frequency of used in a cell and reused in each of the adjacent cells. Because the same frequency is used in all cells, high interference occurs in this system making it impractical. Note that there are 6 cells with the same frequency band around each cell.



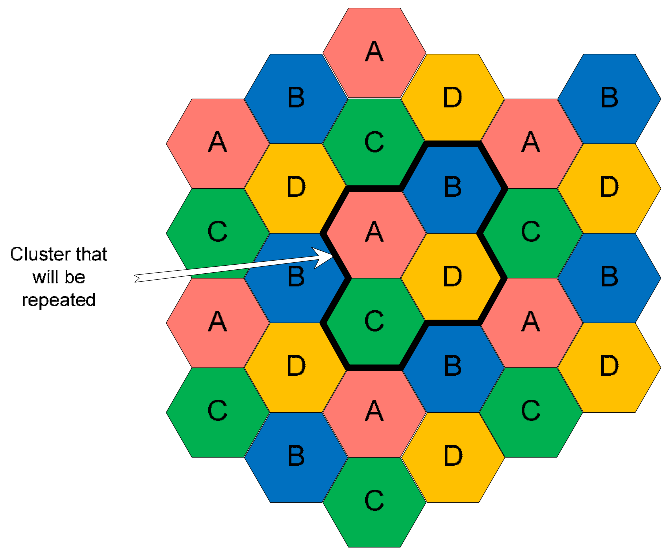
2) **2-Cell Frequency Reuse Cluster [Invalid]:** In this cluster, the allocated band is divided into 2 bands and the two sub-bands are reused in an alternating fashion somehow. Clearly, only two cells with the same frequency as a particular cell are at equal distance from it.



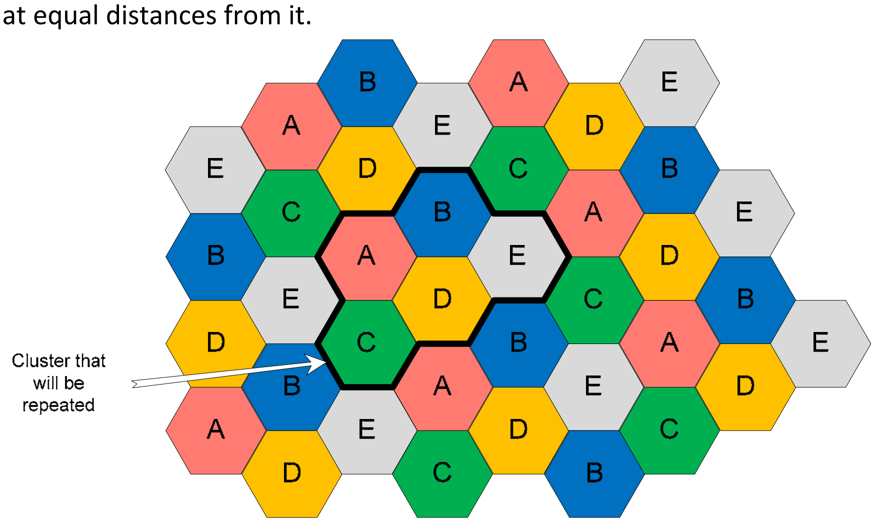
3) **3-Cell Frequency Reuse Cluster (Frequency Reuse Factor = 1/3) [Valid]:** Here, the allocated band is divided into 3 bands (possibly with equal bandwidth) and the three sub-bands are reused in an alternating fashion. No neighboring cells have the same frequency in this configuration resulting in it being the cluster with the least number of cells that is provides practical frequency reuse.



4) **4-Cell Frequency Reuse Cluster (Frequency Reuse Factor = 1/4) [Valid]:** This means that the allocated band is divided into 4 bands and the four sub-bands are reused in an alternating fashion. No neighboring cells have the same frequency in this configuration. Note that if the a slightly different configuration can be achieved with 4 cells in the cluster that performs worse than this configuration.



5) **5-Cell Frequency Reuse Cluster (Frequency Reuse Factor = 1/5) [Invalid]:** This configuration is invalid because as you see in the figure below, a cell does not have 6 co-channel cells at equal distances from it.



Lecture 3: Frequency Reuse Concepts

What Makes a Cell Frequency Pattern "Valid" or "Invalid"?

It is not wither you can stack clusters near each other to cover the whole desired coverage area or not. For example, 2-Cell and 5-cell frequency reuse clusters can cover the whole area without gaps. However, if you look at either the 2 or 5, you note that to each cell there are some close co-channel cells (not equal to 6) and there are some co-channel cells at a farther away distance (also not equal to 6). This makes the interference be dominated by the close co-channel cells. So, we are splitting the frequency band into smaller regions in the hope of reducing the interference but we are not necessarily getting the benefits of this.

What do We Gain – What do We Loose with Frequency Reuse?

Reusing frequencies by dividing the allocated band by a specific integer number of cells and assigning each cell one division and then repeating the assignment over and over produces a tradeoff between network capacity and reception quality as follows:

- The higher the number of divisions of the spectrum over cells (higher cell-reuse factor), the lower the capacity of the network but the further away cells with similar frequency allocations are located resulting in lower interference.
- The lower the number of divisions of the spectrum over cells (Lower cell-reuse factor), the higher the capacity of the network but the closer cells with similar frequency allocations are located resulting in higher interference.

Frequency Allocation Concepts

Assume that the **total frequency band allocated for a cellular system is B Hz**, and that each **half-duplex channel requires W Hz**, **the number of full-duplex channels S that the total band** supports (one channel for transmission and one for reception) is

$$S = \frac{B}{2W} \tag{1}$$

Let the total number of full-duplex channels be divided **equally** among N cells (in an N-Cell Frequency reuse system). The **total number of channels k assigned to each cell** becomes

$$k = \frac{S}{N} \tag{2}$$

with a frequency reuse factor FRF given by

$$FRF = \frac{1}{N} \qquad (3)$$

Note that it is possible that S may not be divisible by N. In this case, some cells will have $\left\lfloor \frac{S}{N} \right\rfloor$ full-duplex channels while the others will have $\left\lceil \frac{S}{N} \right\rceil$ full-duplex channels, where $\lfloor x \rfloor$ is the integer smaller than or equal to x and $\lceil x \rceil$ is the integer grater than or equal to x.

The N cells over which the complete frequency band has been divided is called a CLUSTER. If **this cluster is repeated M times over the coverage area**, this gives a total number of full-duplex channels in the coverage area C equal to

$$C = MS = MkN \qquad (4)$$

The value C is strongly related to the capacity of the cellular system. Generally, a larger C means higher capacity and higher number of calls that can take place at the same time while lower C means lower capacity and lower number of calls that can take place at the same time.

Studying Equation (3) in more detail (assuming a fixed system bandwidth B and a fixed channel bandwidth W):

- If we increase the cluster size N, the number of channels per cell k will drop by the same factor (because their produce must be S). At the same time, because N is larger, we will need a smaller number of clusters M to completely cover the desired area but cells with similar frequencies will be located further away from each other ➔ C will drop resulting in lower capacity but the interference between frequencies will be lower given better performance.
- If we decrease the cluster size N, the number of channels per cell k will increase by the same factor (because their produce must be S). At the same time, because N is smaller, we will need a larger number of clusters M to completely cover the desired area but cells with similar frequencies will be located closer to each other ➔ C will increase resulting in higher capacity but the interference between frequencies will be lower giving lower performance.

Possible Cluster Sizes

Is any cluster size possible? The answer is NO. There are some cluster sizes that if repeated they will be able to cover the complete region (try for example 8). In fact, there following formula give the cluster sizes N that are possible:

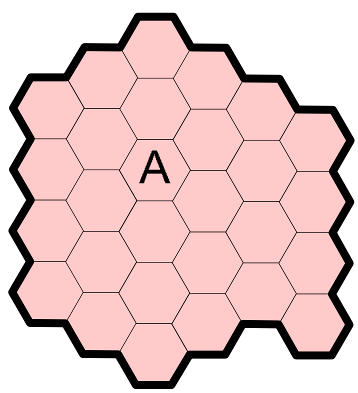
$$N = i^2 + ij + j^2 \qquad (5)$$

where $i \geq 0$ and $j \geq i$. Applying this equation for all possible values of $0 \leq i \leq 12$ and all possible values of $i \leq j \leq 12$ gives the table below

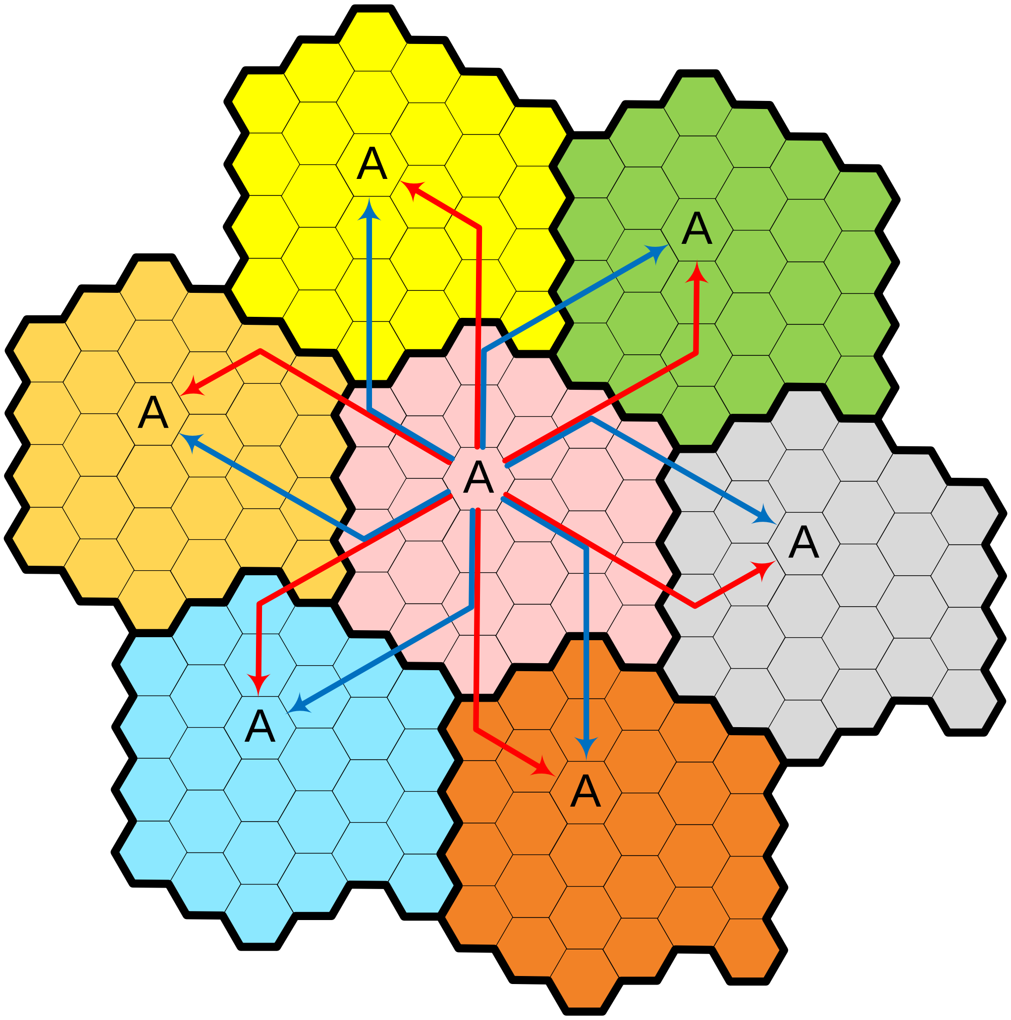
N		***j***												
		0	1	2	3	4	5	6	7	8	9	10	11	12
i	0	0	1	4	9	16	25	36	49	64	81	100	121	144
	1		3	7	13	21	31	43	57	73	91	111	133	157
	2			12	19	28	39	52	67	84	103	124	147	172
	3				27	37	49	63	79	97	117	139	163	189
	4					48	61	76	93	112	133	156	181	208
	5						75	91	109	129	151	175	201	229
	6							108	127	148	171	196	223	252
	7								147	169	193	219	247	277
	8									192	217	244	273	304
	9										243	271	301	333
	10											300	331	364
	11												363	397
	12													432

Relation Between *i*, *j*, and *N*

To illustrate the relation between i, j, and N, let us consider the cluster size $N = 28$. This cluster size is obtained by having $i = 2$ and $j = 4$. Also, assume that in this cluster, one of the cells has been allocated a channel group A.



If we repeat this cluster 6 times around the current cluster, we get the following structure.



We see that the nearest cells with the same channel group A (Called Co-Channel Cells) can be located by:

1. Moving perpendicular to any of the 6 surfaces of the original cell and passing over $i = 4$ cells then rotating 60° counter-clock wise and moving along $j = 2$ cells to reach all adjacent co-channel cells (**RED Arrows** in the above figure),
2. Moving perpendicular to one of the surfaces of the original cell and passing over $j = 2$ cells then rotating 60° clock wise and moving along $i = 4$ cells to reach all adjacent co-channel cells (**BLUE Arrows** in the above figure).

Comments about Different Cluster Sizes

The practical cluster sizes are generally 4, 7, and 12. Other cluster sizes either result in too much interference (such as $N = 1$) or waste system resources by insuring a very low interference level that is much lower than the maximum acceptable value (such as $N = 27$).