Orbital Perturbations

When a satellite is to be launched into an orbit, the desired orbit is usually chosen based on many factors. The orbit of a satellite must therefore be maintained accurately in for a lot of active satellites. For example, a GEO satellite, which appears to be fixed in the sky must maintain its longitude and latitude position such that it does not leave its desired position by more than ±0.05° in the East-West direction and ±0.05° in the North-South direction. At the altitude of a GEO satellite, this corresponds to a square in space that is roughly 70 km in each side.

Keeping a satellite in that square in the sky appears to be an easy task as this square is huge. However, in practice this is not true because of many factors. Some of these factors include:

1. The weightlessness of objects in space make them subject to the slightest forces acting on them. This means that when you are weightless, even a small force that acts on you in a specific direction will result in accelerating you in that direction. Over days, months, or even years, the accumulations of these accelerations become significant that a satellite may gain relatively high speeds in an undesired direction, and hence moving its location away from the target region.

2. The Keplerian model for the orbit of satellites is only try if we assume Earth as a point mass at the center of its gravity. Clearly, this is not true because:
   - Earth is not a perfect sphere but is flatter at the two poles than at the equator. The Earth’s diameter over the equator is greater than the diameter that passing over the two poles by approximately 20 km.
   - The radius of Earth as you move over the equator changes.
   - The density of Earth is not uniform but has higher values at specific regions causing a non-uniform gravitational force as a satellite moves in its orbit.

3. The gravity of the Sun and Moon which act on the satellite in different directions as their positions with the respect to the satellite change over the time of the day, month, and year. The following figure shows the relative inclination of satellites in GEO orbit, the Moon, and the Sun.

![Diagram of Earth, Moon, Sun, Satellite, and their gravitational forces](image)

Clearly, the Sun and Moon pulls exerted on GEO satellites shown in the above figure are in these directions only at this configuration, however, these directions change as the positions of the Sun and Moon with respect to Earth and the satellites change.

4. Solar winds: The Sun has been found to have periodic flares on the order of 11 years or so. Normally, the Sun emits relatively small amounts of solar winds. However, during
these flares, huge amounts of solar winds are emitted to the point that satellite communications are often interrupted during these solar flares. These solar winds are charged particles that travel at high speeds away from the Sun. These particles act on satellites causing them to move.

**Longitude Movement**

The non-spherical shape and non-uniform density of Earth play a role on the motion of GEO satellites in the East-West direction changing their longitude position. As a result of these, it is found that Earth has 4 equilibrium points: two stable equilibrium and non-stable equilibrium points. A satellite that is placed at one of the stable equilibrium points will remain there (in terms of its longitude not latitude), while a satellite that is place in one of the non-stable points will drift until it crosses the nearest stable point and remain oscillating around that stable point for a very long time. These points have longitudes of:

- Stable points at 75° E and 252° E,
- Unstable points at 162° E and 348° E.

Therefore, to keep a GEO satellite at its specified longitude, rocket fuel must be burned periodically.

**Latitude Movement**

Although the Sun is millions of times more massive than the Moon, the distance to the Moon is much smaller than the distance to the Sun, resulting in the gravitational pull by the Moon on a satellite being about 2 times the gravitational pull of the Sun. Since the pulls caused by gravitational forces of the Moon and Sun are not equal and they act at different directions as the satellite, the Sun, and the Moon all change relative positions, the resulting forced tend to move a GEO satellite in the latitude direction (North-South). The effect of these forces if uncorrected tend to move a satellite between Latitudes of 14.67° North and 14.67° South in a period of around 26.6 years. This movement changes the inclination of the satellite orbit by about 1° per year. Again, to keep a GEO satellite in an orbit with 0° inclination, rocket fuel must be burned periodically.

**Lifetime of a Satellite**

In recent years, electronic components have become so reliable that a satellite built using high grade components will often last for 10, 15, or 20 years and possibly longer. So, in theory, a satellite launched today may still be operational 30 years from now. So, the lifetime of a satellite, usually considered to be 10 to 15 years, is not limited by the lifetime of the electronics, but by the fuel used to maneuver the satellite into its orbit whenever it moves out of it. Since building a satellite costs a lot of money and launching them also costs a lot of money that is usually proportional to the weight of the satellite, a compromise between the amount of fuel to include in the satellite and the launching cost of the satellite is made.
Usually, when the amount of fuel on board of a satellite becomes low while the electronics of the satellite are still functioning properly, the fuel is conserved as much as possible by attempting to reduce the orbit maintaining maneuvers as much as possible. For example, since all GEO satellites are located in ring over the equator, a GEO satellite

**Satellite Motion Effects**

An effect that is observed as a result of the motion of satellites with respect to an observer that is usually observed with satellites in LEO or MEO satellites that is not observed with GEO satellites is the Doppler effect. The Doppler effect (usually called Doppler shift) is the apparent difference in the carrier frequency between the transmitted and received signals. That is, the transmitted signal appears to have a specific carrier frequency while the received signal appears to have a slightly different carrier frequency. The Doppler shift occurs whenever the transmitter and/or receiver are in motion with respect to each other. The motion that concerns us here is the one that causes the transmitter and receiver to either come closer to each other or move away from each other. Motion that does not cause the distance between the transmitter and receiver to change does not cause Doppler shift. Since GEO satellites appear to be fixed in space and the distance between them and a stationary Earth station does not change with time, no Doppler shift is felt with these satellites.

The Doppler shift may either be positive or negative. The positive Doppler shift (i.e., the received carrier frequency is higher than the transmitted carrier frequency) occurs when the transmitter and receiver are coming closer to each other, while the negative Doppler shift (i.e., the received carrier frequency is lower than the transmitted carrier frequency) occurs when the transmitter and receiver are moving away from each other. This is the same phenomenon that results in the frequency of the siren of an approaching speeding ambulance, for example, to change as the ambulance moves by you and start to move away.

The Doppler shift is given by

\[
\frac{f_R - f_T}{f_T} = \frac{\Delta f}{f_T} = \frac{v_{\text{Relative}}}{c}
\]

where \(f_R\) is the received frequency, \(f_T\) is the transmitted frequency, \(v_{\text{Relative}}\) is the relative speed of the transmitter and receiver (considering positive speed to be the speed that causes the transmitter and receiver to approach each other while negative speed to be the speed that causes the transmitter and receiver to move away from each other), and \(c\) is the speed of light \((2.998 \times 10^5 \text{ km/s})\), so

\[
\Delta f = \frac{f_T \cdot v_{\text{Relative}}}{c} = \frac{v_{\text{Relative}}}{\lambda}
\]

where \(\lambda\) is transmission wavelength in (km).
Complete Satellite Earth Coverage

The question that needs to be answered is: How many satellites are needed to provide the whole surface of Earth with satellite coverage? What we mean by complete Earth coverage is that any point on Earth would at least see one of the satellites at any time instant. Let us try to answer this question.

1. Can 2 satellites do this? The answer is no. Although a satellite that is placed very far away can provide coverage to almost 1 half of the Earth’s surface, which means that two satellites would be able to provide coverage to almost the whole Earth, however, these satellites would have to be placed theoretically at an infinite distance from Earth (not practical).

2. What about 3 satellites? Three satellites also cannot cover the whole Earth’s surface. Consider for example 3 GEO satellites on the Equator. These satellites would be un-seeable to the two regions near the North and South poles (all of them would be below the horizon to people in these regions or slightly higher than the horizon but because of the low elevation angle, the signals received from them would be un-usable). In fact, a GEO satellite can only be received from regions with latitudes from around 75° N 75° S, or so. Clearly, most Earth’s inhabitants live between these latitudes, however, our aim is to provide Earth’s coverage to the whole Earth’s surface. No matter where you place the three satellites, some regions would not be able to see any of the at least part of the time.

3. So, 4 satellites would certainly be enough? No. They are not. You can cover the whole Earth’s surface using 4 satellites, but the problem is that the orbits of these satellites would make them move with respect to each other in a way that would result in some parts of Earth loosing satellite coverage for part of the satellite orbits. Take for example, the tetrahedrons configuration (similar to the structure of the Methane CH₄). Considering Earth to be in place of the C atom and a satellite at each of the H atoms would result in complete satellite coverage (given that the satellites are at far enough distance). However, placing satellites in this configuration would result in orbital plans that cross each other. Therefore, the positions of the satellites would change with respect to each other. At specific points, you may find that two or more of the satellites are at the same point results in effectively having only 3 satellites and therefore less than full Earth’s coverage.

4. I am guessing then that 5 satellites would also not be enough for the same problem faced with 4? Well, this time you also guessed wrong. Having 5 satellites is enough to provide complete Earth’s coverage given that they are high enough and placed in proper orbits (with specific orbit inclinations and have specific phases in their orbits). The configuration of these satellites would be the topic of your first project.