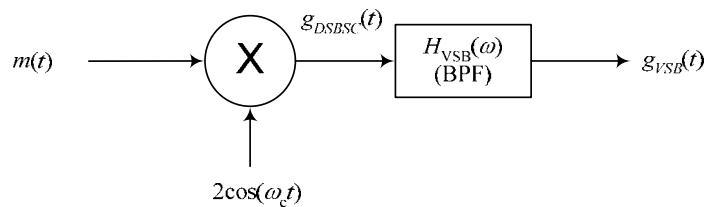
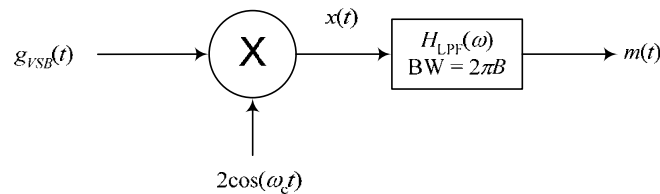


Vestigial Side Band Modulation

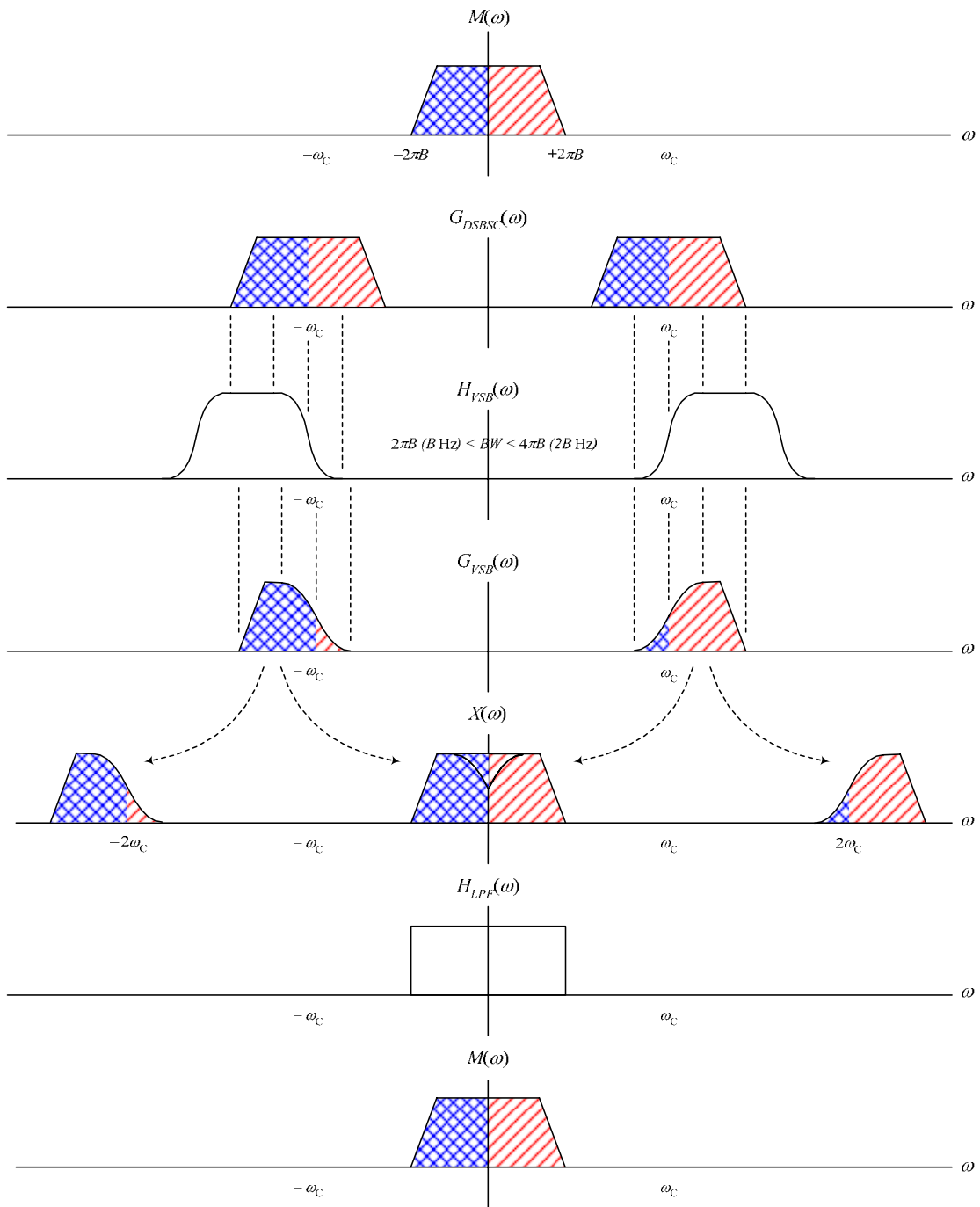
As mentioned last lecture, the two methods for generating SSB modulated signals suffer some problems. The selective-filtering method requires that the two side bands of the DSBSC modulated signal which will be filtered are separated by a guard band that allows the bandpass filters that are used to have non-zero transition band (so it allows for real filters). An ideal Hilbert transform for the phase-shifting method is impossible to build, so only an approximation of that can be used. Therefore, the SSB modulation method is hard, if not impossible, build. A compromise between the DSBSC modulation and the SSB modulation is known as Vestigial Side Band (VSB) modulation. This type of modulation is generated using a similar system as that of the selective-filtering system for SSB modulation. The following block diagram shows the VSB modulation and demodulation.



VSB Modulator (transmitter)



VSB Demodulator (receiver)



The above example for generating VSB modulated signals assumes that the VSB filter ($H_{VSB}(\omega)$) that the transition band of the VSB filter is symmetric in a way that adding the part that remains in the filtered signal from the undesired side band to the missing part of the desired side band during the process of demodulation produces an undistorted signal at baseband. In fact, this condition is not necessary if the LPF in the demodulator can take care of any distortion that happens when adding the different components of the bandpass components at baseband.

To illustrate this, consider a baseband message signal $m(t)$ that has the FT shown in the following figure.

The DSBSC modulated signal from that assuming that the carrier is $2\cos(\omega_c t)$ (the 2 in the carrier is placed there for convenience) is

$$g_{DSBSC}(t) = m(t) \cos(\omega_c t)$$

In frequency-domain, this gives

$$G_{DSBSC}(\omega) = M(\omega - \omega_c) + M(\omega + \omega_c)$$

Passing this signal into the VSB filter shown in the modulator block diagram above gives

$$G_{VSB}(\omega) = H_{VSB}(\omega) [M(\omega - \omega_c) + M(\omega + \omega_c)].$$

Note that the VSB filter is not an ideal filter with flat transfer function, so it has to appear in the equation defining the VSB signal.

Now, let us demodulate this VSB signal using the demodulator shown above but use a non-ideal filter $H_{LPF}(\omega)$ (the carrier here is also multiplied by 2 just for convenience)

$$X(\omega) = H_{VSB}(\omega - \omega_c) \left[\underbrace{M(\omega - 2\omega_c)}_{\text{at } +2\omega_c} + \underbrace{M(\omega)}_{\text{Baseband}} \right] + H_{VSB}(\omega + \omega_c) \left[\underbrace{M(\omega)}_{\text{baseband}} + \underbrace{M(\omega + 2\omega_c)}_{\text{at } -2\omega_c} \right]$$

Passing this through the non-ideal LPF in the demodulator gives an output signal that we will call $Z(\omega)$. This signal is given by

$$\begin{aligned} Z(\omega) &= H_{LPF}(\omega) [H_{VSB}(\omega - \omega_c)M(\omega) + H_{VSB}(\omega + \omega_c)M(\omega)] \\ &= H_{LPF}(\omega) [H_{VSB}(\omega - \omega_c) + H_{VSB}(\omega + \omega_c)]M(\omega) \end{aligned}$$

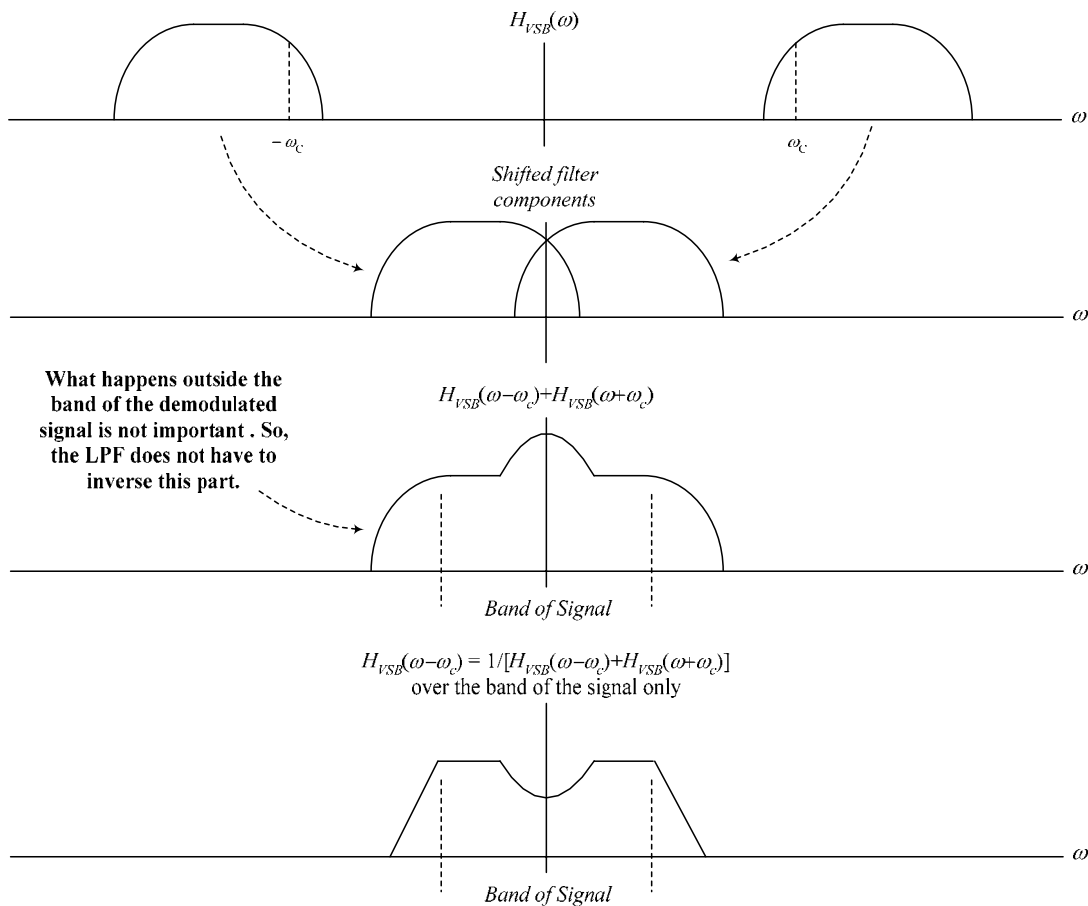
For this communication system to not distort the transmitted signal, the output signal $Z(\omega)$ must be equal to the input signal (or a scaled and shifted version of it).

$$Z(\omega) = M(\omega) = H_{LPF}(\omega)[H_{VSB}(\omega - \omega_c) + H_{VSB}(\omega + \omega_c)]M(\omega).$$

This gives us the following relationship between the LPF at the demodulator and the VSB filter at the modulator

$$H_{LPF}(\omega) = \frac{1}{H_{VSB}(\omega - \omega_c) + H_{VSB}(\omega + \omega_c)}.$$

So, this filter must be a LPF that has a transfer function around 0 frequency that is related to the VSB filter as given above. To illustrate this relationship, consider the following VSB BPF example.



Another example follows.

