Classification of Signals

Some important classifications of signals

- Analog vs. Digital signals: as stated in the previous lecture, a signal with a magnitude that may take any real value in a specific range is called an analog signal while a signal with amplitude that takes only a finite number of values is called a digital signal.
- Continuous-time vs. discrete-time signals: continuous-time signals may be analog or digital signals such that their magnitudes are defined for all values of t, while discrete-time signal are analog or digital signals with magnitudes that are defined at specific instants of time only and are undefined for other time instants.
- Periodic vs. aperiodic signals: periodic signals are those that are constructed from a specific shape that repeats regularly after a specific amount of time T_0, [i.e., a periodic signal $f(t)$ with period T_0 satisfies $f(t) = f(t+nT_0)$ for all integer values of n], while aperiodic signals do not repeat regularly.
- Deterministic vs. probabilistic signals: deterministic signals are those that can be computed beforehand at any instant of time while a probabilistic signal is one that is random and cannot be determined beforehand.
- Energy vs. Power signals: as described below.

Energy and Power Signals

The total energy contained in and average power provided by a signal $f(t)$ (which is a function of time) are defined as

$$E_f = \int_{-\infty}^{\infty} |f(t)|^2 \, dt ,$$

and

$$P_f = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 \, dt ,$$

respectively.

For periodic signals, the power P can be computed using a simpler form based on the periodicity of the signal as

$$P_{Periodic\ f} = \frac{1}{T} \int_{t_0}^{T+t_0} | f(t)|^2 \, dt ,$$

where T here is the period of the signal and t_0 is an arbitrary time instant that is chosen to simply the computation of the integration (to reduce the functions you have to integrate over one period).

Classification of Signals into Power and Energy Signals

Most signals can be classified into Energy signals or Power signals. A signal is classified into an energy or a power signal according to the following criteria

a) Energy Signals: an energy signal is a signal with finite energy and zero average power $(0 \le E < \infty,\ P = 0)$,

b) Power Signals: a power signal is a signal with infinite energy but finite average power $(0 < P < \infty,\ E \to \infty)$.

Comments:

1. The square root of the average power $\sqrt{P}$ of a power signal is what is usually defined as the RMS value of that signal.
2. Your book says that if a signal approaches zero as t approaches ∞ then the signal is an energy signal. This is in most cases true but not always as you can verify in part (d) in the following example.
3. All periodic signals are power signals (but not all non–periodic signals are energy signals).
4. Any signal f that has limited amplitude ($|f| < \infty$) and is time limited ($f = 0$ for $|t| > t_0$ for some $t_0 > 0$) is an energy signal as in part (g) in the following example.

Exercise 2: determine if the following signals are Energy signals, Power signals, or neither, and evaluate E and P for each signal (see examples 2.1 and 2.2 on pages 17 and 18 of your textbook for help).

a) $a(t) = 3\sin(2\pi t),\ -\infty < t < \infty$,

This is a periodic signal, so it must be a power signal. Let us prove it.

$$E_a = \int_{-\infty}^{\infty} |a(t)|^2\, dt = \int_{-\infty}^{\infty} |3\sin(2\pi t)|^2\, dt$$
$$= 9\int_{-\infty}^{\infty} \frac{1}{2}\left[1 - \cos(4\pi t)\right] dt$$
$$= 9\int_{-\infty}^{\infty} \frac{1}{2} dt - 9\int_{-\infty}^{\infty} \cos(4\pi t) dt$$
$$= \infty \quad \text{J}$$

Notice that the evaluation of the last line in the above equation is infinite because of the first term. The second term has a value between –2 to 2 so it has no effect in the overall value of the energy.

Since $a(t)$ is periodic with period $T = 2\pi/2\pi = 1$ second, we get

$$
\begin{aligned}
P_a &= \frac{1}{1}\int_0^1 |a(t)|^2 \, dt = \int_0^1 |3\sin(2\pi t)|^2 \, dt \\
&= 9\int_0^1 \frac{1}{2}\left[1-\cos(4\pi t)\right] dt \\
&= 9\int_0^0 \frac{1}{2} dt - 9\int_0^1 \cos(4\pi t) dt \\
&= \frac{9}{2} - \left[\frac{9}{4\pi}\sin(4\pi t)\right]_0^1 \\
&= \frac{9}{2} \quad \text{W}
\end{aligned}
$$

So, the energy of that signal is infinite and its average power is finite (9/2). This means that it is a power signal as expected. Notice that the average power of this signal is as expected (square of the amplitude divided by 2)

b) $b(t) = 5e^{-2|t|}, \ -\infty < t < \infty,$

Let us first find the total energy of the signal.

$$
\begin{aligned}
E_b &= \int_{-\infty}^{\infty} |b(t)|^2 \, dt = \int_{-\infty}^{\infty} \left|5e^{-2|t|}\right|^2 \, dt \\
&= 25\int_{-\infty}^{0} e^{4t} dt + 25\int_0^{\infty} e^{-4t} dt \\
&= \frac{25}{4}\left[e^{4t}\right]_{-\infty}^{0} + \frac{25}{4}\left[e^{-4t}\right]_0^{\infty} \\
&= \frac{25}{4} + \frac{25}{4} = \frac{50}{4} \quad \text{J}
\end{aligned}
$$

The average power of the signal is

$$
\begin{aligned}
P_b &= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} |b(t)|^2 \, dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} \left|5e^{-2|t|}\right|^2 \, dt \\
&= 25\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{0} e^{4t} dt + 25\lim_{T\to\infty}\frac{1}{T}\int_0^{T/2} e^{-4t} dt \\
&= \frac{25}{4}\lim_{T\to\infty}\frac{1}{T}\left[e^{4t}\right]_{-T/2}^{0} + \frac{25}{4}\lim_{T\to\infty}\frac{1}{T}\left[e^{-4t}\right]_0^{T/2} \\
&= \frac{25}{4}\lim_{T\to\infty}\frac{1}{T}\left[1-e^{-2T}\right] + \frac{25}{4}\lim_{T\to\infty}\frac{1}{T}\left[e^{-2T}-1\right] \\
&= 0+0=0
\end{aligned}
$$

So, the signal $b(t)$ is definitely an energy signal.
So, the energy of that signal is infinite and its average power is finite (9/2). This means that it is a power signal as expected. Notice that the average power of this signal is as expected (the square of the amplitude divided by 2)

c) $$c(t) = \begin{cases} 4e^{+3t}, & |t| \le 5 \\ 0, & |t| > 5 \end{cases},$$

d) $$d(t) = \begin{cases} \dfrac{1}{\sqrt{t}}, & t > 1 \\ 0, & t \le 1 \end{cases},$$

Let us first find the total energy of the signal.

$$\begin{aligned} E_d &= \int_{-\infty}^{\infty} |d(t)|^2 \, dt = \int_{1}^{\infty} \frac{1}{t} \, dt \\ &= \ln\left[t\right]_1^{\infty} \\ &= \infty - 0 = \infty \quad \text{J} \end{aligned}$$

So, this signal is NOT an energy signal. However, it is also NOT a power signal since its average power as shown below is zero.

The average power of the signal is

$$\begin{aligned} P_d &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |d(t)|^2 \, dt = \lim_{T\to\infty} \frac{1}{T} \int_{1}^{T/2} \frac{1}{t} dt \\ &= \lim_{T\to\infty} \left(\frac{1}{T} \ln\left[t\right]_1^{T/2} \right) = \lim_{T\to\infty} \left(\frac{1}{T} \ln\left[\frac{T}{2}\right] - \frac{1}{T} \ln\left[1\right] \right) \\ &= \lim_{T\to\infty} \left(\frac{1}{T} \ln\left[\frac{T}{2}\right] \right) = \lim_{T\to\infty} \left(\frac{\ln\left[\frac{T}{2}\right]}{T} \right) \end{aligned}$$

Using Le'hopital's rule, we see that the power of the signal is zero. That is

$$P_d = \lim_{T\to\infty} \left(\frac{\ln\left[\frac{T}{2}\right]}{T} \right) = \lim_{T\to\infty} \left(\frac{\frac{2}{T}}{1} \right) = 0$$

So, not all signals that approach zero as time approaches positive and negative infinite is an energy signal. They may not be power signals either.

e) $e(t) = -7t^2, \quad -\infty < t < \infty,$

f) $f(t) = 2\cos^2(2\pi t), \quad -\infty < t < \infty.$

g) $g(t) = \begin{cases} 12\cos^2(2\pi t), & -8 < t < 31 \\ 0, & \text{elsewhere} \end{cases}.$