EE 407
Microwave Engineering

Lecture 3 & 4

Transmission line characteristics

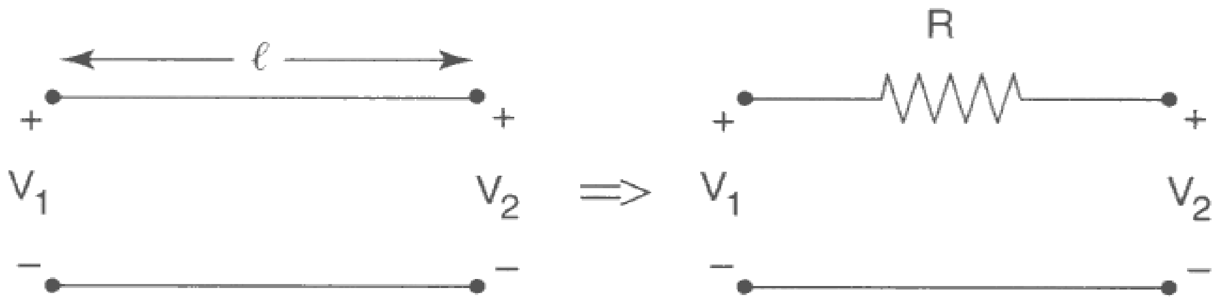
Dr. Sheikh Sharif Iqbal

References: Text books and Agilent notes

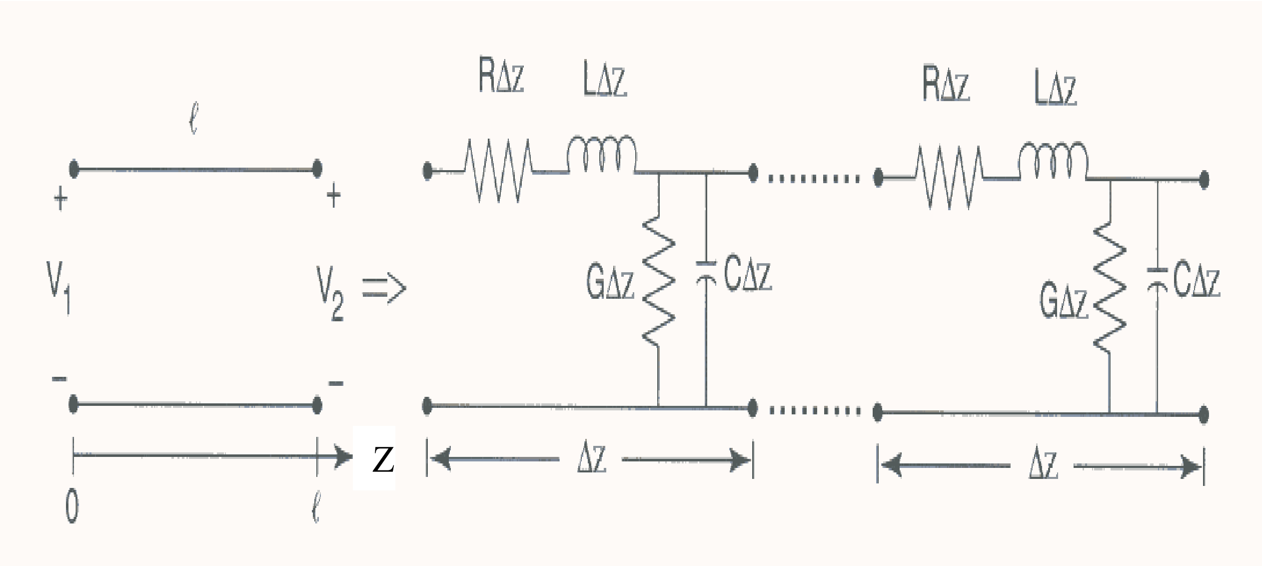
Behavior of basic Transmission media:

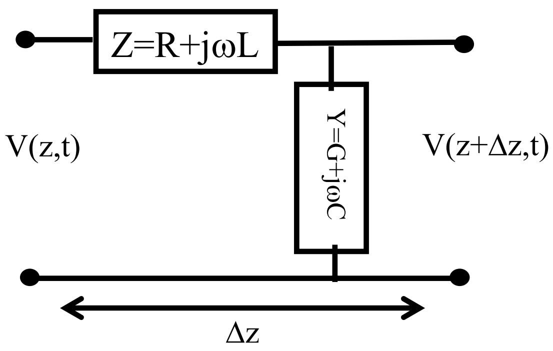
•At low frequencies: TL is considered to be a short wire with a negligible distributed resistance, represented as **lumped** for the purpose of analysis.

$V_1 = V_2 + IR$



• At high frequencies: analyzed by **M.E's** or **distributed** circuit model;





• **$dV/dz = -ZI$** *and* **$dI/dz = -YV$** ; *where* 'Z' & 'Y' *are function of frequency*

- *Here, 'Z' & 'Y' are function of frequency only and* $\Delta Z \rightarrow 0$;.

$$dV/dz = -Z\, I \qquad \text{(eq. 1)}$$

$$dI/dz = -Y\, V \;;$$

- *Differentiating again gives*: {as Z➔ Z(f)}

$$d^2V/dz^2 = -Z.\, dI/dz = ZYV \qquad \text{(eq. 2)}$$

$$d^2I/dz^2 = -Y.dV/dz = YZI$$

- *If no reflected wave is present;*

$$V = V(z,t) = V_0^+ e^{j\omega t-\gamma z} \quad \textit{and} \quad V^+ = V(z) = V_0^+ e^{-\gamma z} \qquad \text{(eq. 3)}$$

$$I = I(z,t) = I_0^+ e^{j\omega t-\gamma z} \quad \textit{and} \quad I^+ = I(z) = I_0^+ e^{-\gamma z}$$

- ***Substitute*** **V(z,t)** *and* **I(z,t)** *into eq.2 and differentiating yields*:

$$\gamma^2 V_0^+ e^{j\omega t-\gamma z} = Z\, Y\, V_0^+ e^{j\omega t-\gamma z} \qquad \text{(eq. 4)}$$

$$\gamma = \pm\sqrt{ZY}$$

$\gamma = \pm\sqrt{(R+j\omega L)(G+j\omega C)}$ ≡> ***called Propagation Constant.***

Again, $\gamma \equiv \alpha + j\beta$, *where* α=**attenuation cons** & β=**phase cons**=$2\pi/\lambda$.

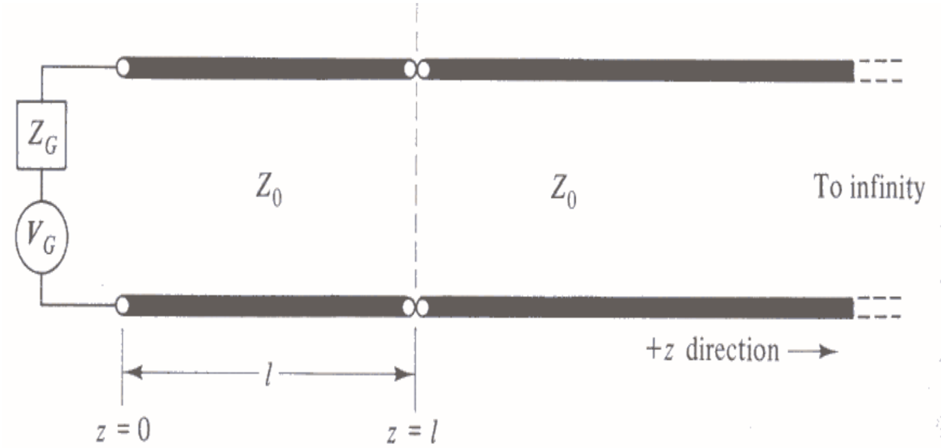
•***Substituting* V(z,t) *and* I(z,t) *into eq.1 gives*:**

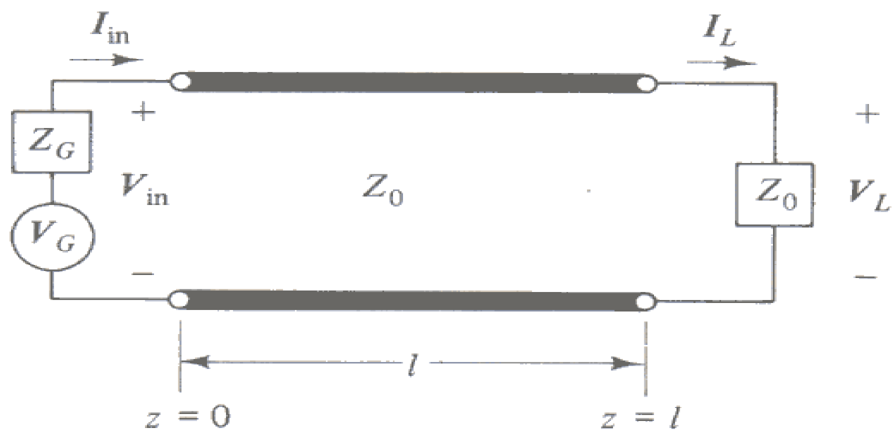
$$-\gamma V_0^+ e^{j\omega t-\gamma z} = -Z I_0^+ e^{j\omega t-\gamma z}$$

$$\frac{V_0^+}{I_0^+} = \frac{Z}{\gamma} = \pm\sqrt{\frac{Z}{Y}} = \pm\sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad \text{Again,} \quad \frac{V_0^+}{I_0^+} = Z_0 = \pm\sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

•**Note:** '+' & '-' is used for observer looking into the 'load' & 'generator'

Infinitely long Transmission Line (or $Z_L=Z_0$):



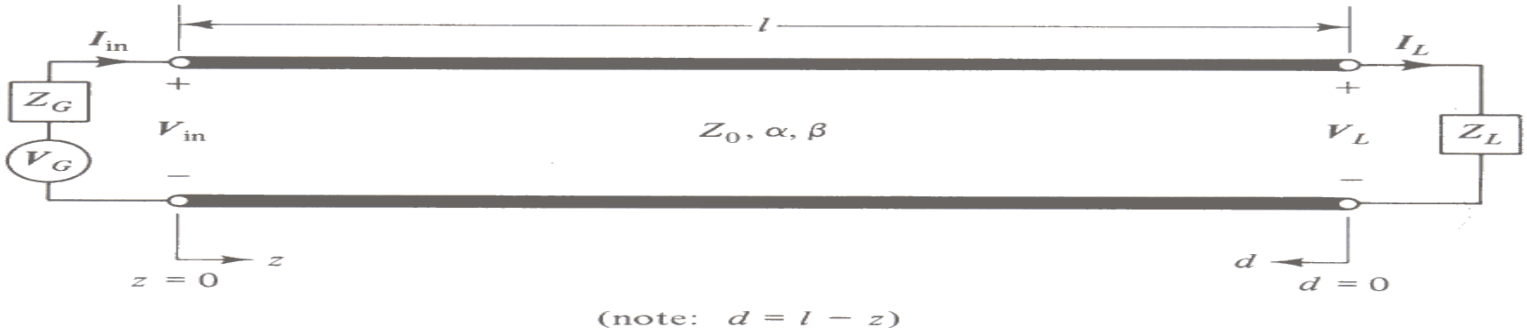


• If '$Z_L = Z_0$' of TL ⇒ Matched ⇒ No reflection occurs ⇒ ≈ infinite line

• Time-average *incident power* **&** *loss/length* alone the line (*if 'Z_0' is real*)

$$P^+(z) = \frac{1}{2}\left[\mathrm{Re}V^+(z)V^+(z)^*\right] = \frac{V_0^{+2}}{2Z_0}e^{-2\alpha z} \qquad \alpha = \frac{-\Delta P/\Delta Z}{2P} = \frac{-(P_2-P_1)/d}{2P_1}$$

<u>Mismatched Transmission Line</u> ($Z_L \neq Z_0$):



- If $Z_L = \infty$; when incident wave arrive at the O/C end, it must satisfy ;
  - (1) For the traveling wave: $\mathbf{V^+/I^+ = V^-/I^- = Z_0}$ of the transmission line
  - (2) Ohm's law at O/C requires an infinite impedance as current is zero
- Creation of reflected waves ($\mathbf{V^-, I^-}$) satisfies both requirements.
- Thus at any pt.of TL, $\mathbf{V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} = V^+ + V^-}$ *and* $\mathbf{I(z) = I^+ - I^-}$
- Load <u>**reflection** coefficient</u>, $\Gamma_L = \mathbf{V^-/V^+ = (V_0^- e^{\gamma l}) / (V_0^+ e^{-\gamma l}) = (V_0^-/V_0^+) . e^{2\gamma l}}$
- Thus, ref. coeff. Γ, at any point $\mathbf{d=(l-z)}$ from the load end, is given by;

$$\Gamma = \mathbf{(V_0^- \cdot e^{\gamma(l-d)}) / (V_0^+ \cdot e^{-\gamma(l-d)}) = (V_0^- \cdot e^{+\gamma l} \cdot e^{-\gamma d}) / (V_0^+ \cdot e^{-\gamma l} \cdot e^{+\gamma d})}$$

$$\mathbf{= [\{(V_0^- / V_0^+) . e^{2\gamma l}\} e^{-\gamma l} . e^{-\gamma d}] / (e^{-\gamma l} . e^{+\gamma d}) = \Gamma_L . e^{-2\gamma d}} \qquad \text{(eq. 5)}$$

- Thus at z =0 (*when* ***d=l***), $\Gamma_{\mathbf{in}} = \Gamma_{\mathbf{L}} \mathbf{. e^{-2\gamma l} = \Gamma_L . e^{-2(\alpha+j\beta)l} = |\Gamma_L|. e^{-2\alpha l} \angle \varphi_L - 2\beta l}$

- Load ref. coeff. Γ_L can also be determined from Z_0 and Z_L values;

  With d=0 (*when z=l*), $\mathbf{V_L = V^+ + V^- = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} = V_0^+ e^{-\gamma l}[1+ \Gamma_L]}$

  Similarly, $\mathbf{I_L = I_0^+ e^{-\gamma l} [1- \Gamma_L]}$

- Since $\mathbf{Z_L = V_L/I_L = Z_0 \{(1+ \Gamma_L) / (1- \Gamma_L)\}}$ (eq. 6a)
- or $\mathbf{\Gamma_L = (Z_L - Z_0) / (Z_L + Z_0)}$ (eq. 6b)
- ***Similarly***, $\mathbf{\Gamma_G = (Z_G - Z_0) / (Z_G + Z_0)}$ '*where* $\mathbf{Z_G}$ *is the source impedance*'

Transmission coefficient:

- Ratio of the transmitted voltage current over incident voltage or current
- Similar to "Γ_L", using equation '$\mathbf{V(z)=V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} = V_{tr} e^{-\gamma z}}$' yields;

  Transmission coefficient, $\mathbf{T = (2Z_L) / (Z_L + Z_0)}$ (eq. 7)

Power Flow: For the circuit in previous figure, if $\mathbf{Z_G}$ is real;

- Input power : $P_{in} = \dfrac{V_G^2}{4Z_G} \dfrac{(1-|\Gamma_G|^2)(1-|\Gamma_{in}|^2)}{|1-\Gamma_G \Gamma_{in}|^2}$
- Power delivered to load is: $P_L = \dfrac{V_G^2}{4Z_G} \dfrac{(1-|\Gamma_G|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_G \Gamma_L e^{-2\gamma l}|^2} e^{-2\alpha l}$

Standing wave ration (SWR):

- If both incident (**V**$^+$) and reflected (**V**$^-$) wave are present, the voltage at any point in line is the phasor sum of **V**$^+$ and **V**$^-$.
- Thus $|\mathbf{V}_{max}| = |\mathbf{V}^+|+|\mathbf{V}^-| = |\mathbf{V}^+| + |\Gamma_L|.|\mathbf{V}^+| = |\mathbf{V}^+|\{1+|\Gamma_L|\}$
- and $\mathbf{V}_{min} = |\mathbf{V}^+|-|\mathbf{V}^-| = |\mathbf{V}^+|\{1-|\Gamma_L|\}$
- By definition, **VSWR** $= |\mathbf{V}_{max}| / |\mathbf{V}_{min}| = (1+|\Gamma_L|) / (1-|\Gamma_L|)$

  or $|\Gamma_L| = (\mathrm{VSWR}-1) / (\mathrm{VSWR}+1)$
- In a lossless line, $1<\mathrm{VSWR}<\infty$ and is same everywhere along the line

Impedance Transformation:

Impedance at pt.'d' due to Z_L is;

$$Z_d = Z_0 \frac{1+\Gamma_L e^{-2\gamma d}}{1-\Gamma_L e^{-2\gamma d}} = Z_0 \frac{(Z_L+Z_0)e^{\gamma d} + (Z_L-Z_0)e^{-\gamma d}}{(Z_L+Z_0)e^{\gamma d} - (Z_L-Z_0)e^{-\gamma d}}$$

(Using eq's 5, 6 & Fig)

$$Z_d = Z_0 \frac{Z_L + Z_0 \tanh \gamma d}{Z_0 + Z_L \tanh \gamma d}$$

(*where,* $\cosh \gamma z = (e^{\gamma z} + e^{-\gamma z})/2$; $\sinh \gamma z = (e^{\gamma z} - e^{-\gamma z})/2$)

- In previous figure, if d=*l*, input impedance is; $$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$
- For lossless case ($\alpha=0$), $\tanh \gamma l \Rightarrow j \tan \beta l$

- *Special cases:*

(1.a) *Open Circuited TL:* $\mathbf{Z_L = \infty}$; $\mathbf{Z_{in(o/c)} = Z_0 \coth \gamma \mathit{l}}$

(1.b) *Short Circuited TL:* $\mathbf{Z_L = 0}$; $\mathbf{Z_{in(s/c)} = Z_0 \tanh \gamma \mathit{l}}$

From these eq's: $\mathbf{\tanh \gamma l = \sqrt{(Z_{in(s/c)} / Z_{in(o/c)})}}$ *and* $\mathbf{Z_0 = \sqrt{(Z_{in(o/c)} Z_{in(s/c)})}}$

(2) *Loss-free line:* $\alpha=0$ *or* $\mathbf{R=G=0}$; $\mathbf{\gamma = j\omega\sqrt{(LC)}}$ *and* $\mathbf{Z_0 = \sqrt{(L/C)}}$

(2) *Low-loss line:* $\mathbf{G \approx 0}$ *and* $\mathbf{R \ll \omega L}$; $\mathbf{\gamma = \{R\sqrt{(C/L)}\}/2 + j\omega\sqrt{(LC)}}$

$$\mathbf{Z_0 = \sqrt{(L/C)} - \{jR\sqrt{(1/LC)}\}/(2\omega)}$$

- *Example Problems (also solve the assignments given in the class):*

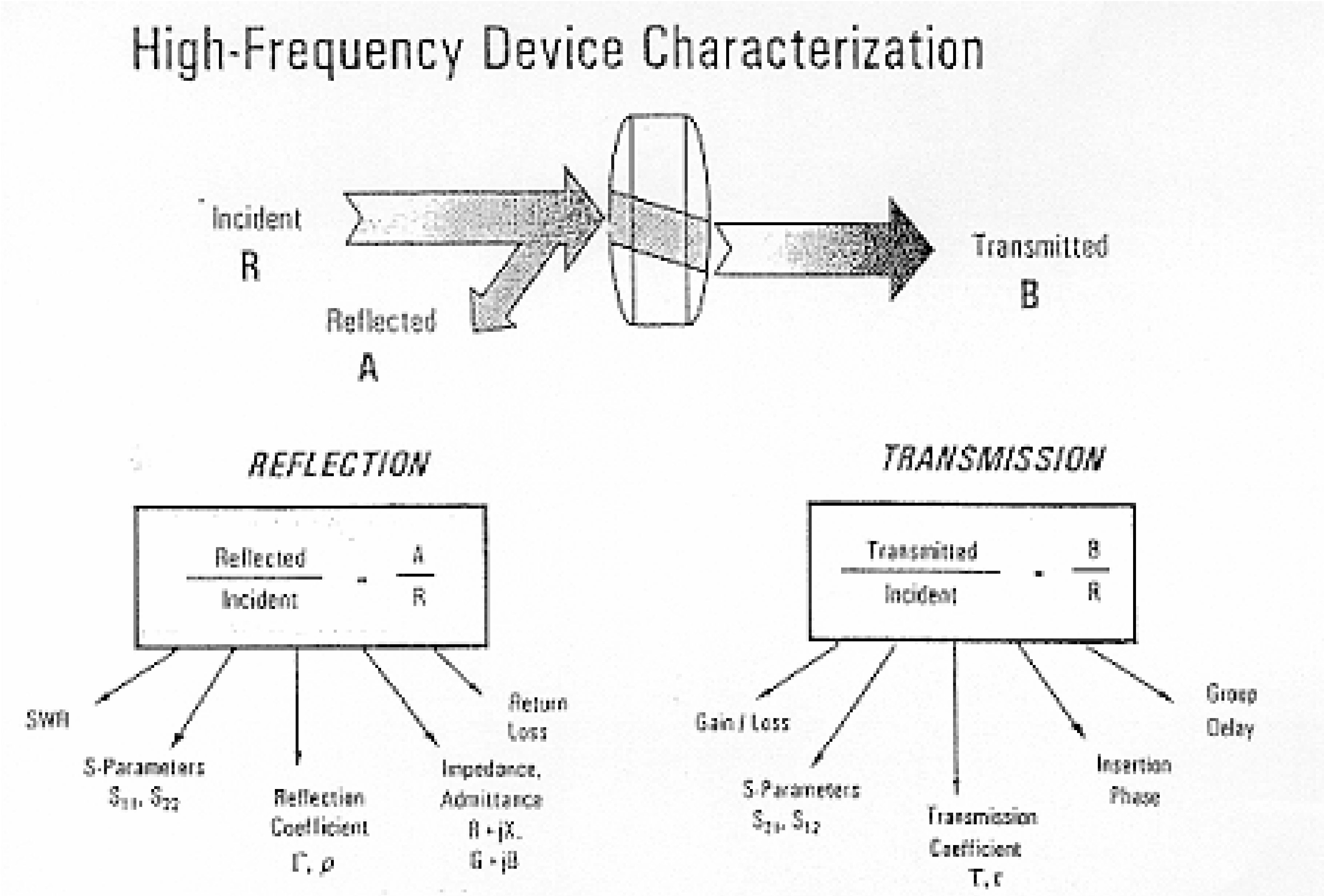
(1) *At 1 GHz, an air filled coaxial line has;* R=4 Ω/m, L=450 nH/m, $G=7\times10^{-4}$ mho/m, C=50 pF/m. Find the related $\mathbf{Z_0}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{v_p}$ *and* λ .

(2) *For a loss-less TL,* L=0.60 μH/m, C=240 pF/m and $\omega=2\pi\times10^8$ rad/m.

(a) Find the related β *and* λ in the line. (*assume air filled line*)

(b) If the line length $\mathbf{l=\lambda/4}$ and the line is terminated by $\mathbf{Z_L=-j100}$ Ω, find the input impedance ($\mathbf{Z_{in}}$) of the line. *(Hint: $\beta=2\pi/\lambda$)*

Review



High-Frequency Device Characterization
Incident
R
Reflected
A
Transmitted
B
REFLECTION
Reflected / Incident = A / R
SWR
S-Parameters
S11, S22
Reflection
Coefficient
Γ, ρ
Impedance,
Admittance
R + jX,
G + jB
Return
Loss
TRANSMISSION
Transmitted / Incident = B / R
Gain / Loss
S-Parameters
S21, S12
Transmission
Coefficient
T, τ
Insertion
Phase
Group
Delay

Review

Reflection Parameters

Reflection Coefficient

$$\Gamma = \frac{V_{reflected}}{V_{incident}} = \rho \angle \Phi = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Return loss $\triangleq -20 \log(\rho)$, $\rho = |\Gamma|$

(Sign)

Voltage Standing Wave Ratio

$$VSWR = \frac{E_{max}}{E_{min}} = \frac{1+\rho}{1-\rho}$$

No reflection *($Z_L = Z_0$)*

Full reflection *(Z_L = open, short)*