

EE 407
Microwave Engineering

Lecture 15 to 16

Designing/Simulating/Testing
Microwave filters

Dr. Sheikh Sharif Iqbal

References: Text books

- Filters are two-port networks used to control the frequency response in an RF or microwave system by allowing transmission at frequencies within the *pass-band* of the filter, and attenuation within the *stop-band* of the filter.
- Common filter responses include *low-pass*, *high-pass*, *band-pass*, *band-stop*.
- Filters are indispensable components in wireless systems, used in receivers for rejecting signals outside the operating band, attenuating undesired mixer products, and for setting the IF bandwidth of the receiver. In transmitters, filters are used to control the spurious responses of up-converting mixers, to select the desired sidebands, and to limit the bandwidth of the radiated signal.
- *Insertion loss method* is a flexible & accurate way to design & analyze filters having a specific type of frequency response. It is based on network synthesis techniques & used in the design process of low-pass filter prototype with the assumption that it is normalized in terms of impedance and cutoff frequency.
- Impedance and frequency scaling and transformations are then used to convert the normalized design to the one having the desired frequency response, cutoff frequency, and impedance level.

- Finally, Transmission line stubs (with dimensions computed using *Richard's transformation, impedance/admittance inverters, and the Kuroda identities*) OR Stepped-Impedance techniques are used to facilitate filter implementation in terms of practical microwave components.
- Three types of practical filter responses are considered here:
 - Binomial or Butterworth or Maximally flat response: *can cater the need for minimum insertion loss by providing flattest possible pass-band response.*
 - Equal-ripple or Chebyshev response: can satisfy a requirement for the sharpest cutoff region.
 - Linear Phase response: is important in multiplexing filters used in comm. systems to avoid distortion at the cost of filters sharp-cutoff characteristics.
- **Insertion loss or Power Loss Ratio (P_{LR})**: by definition P_{LR} is given by;

$$P_{LR} = \frac{\text{Power available from source}}{\text{Power delivered to load}} = \frac{P_{inc}}{P_{load}} = \frac{1}{1 - |\Gamma(\omega)|^2}$$

$\left\{ \begin{array}{l} \text{(eq. 1) where,} \\ \Gamma(\omega) \text{ is the ref. coeff.} \\ \text{looking into the filter} \end{array} \right.$

Thus, the insertion loss (IL) is : $IL = 10 \log_{10} P_{LR}$.

P_{LR} of Maximally flat Butterworth low-pass filter: By definition, power

-loss ratio is; $P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c} \right)^{2N}$ (eq. 2) $\left\{ \begin{array}{l} - N \text{ is the order of the filter,} \\ - \omega_c \text{ is the cutoff frequency} \end{array} \right.$

- Note at $\omega \gg \omega_c$; **IL** increase $20N$ dB per decade (*10) increase of frequency
- *Maximally-flat*: as first $(2N - 1)$ derivatives of above equation are zero at $\omega = 0$.

Maximally flat Low-pass filter Prototype: For a normalized two-element (N=2) low-pass filter (of fig.), the input impedance & reflection coefficient are;

$$Z_{in} = j\omega L + \frac{R(1 - j\omega RC)}{1 + \omega^2 R^2 C^2}$$

$$\Gamma = \frac{Z_{in} - 1}{Z_{in} + 1}$$

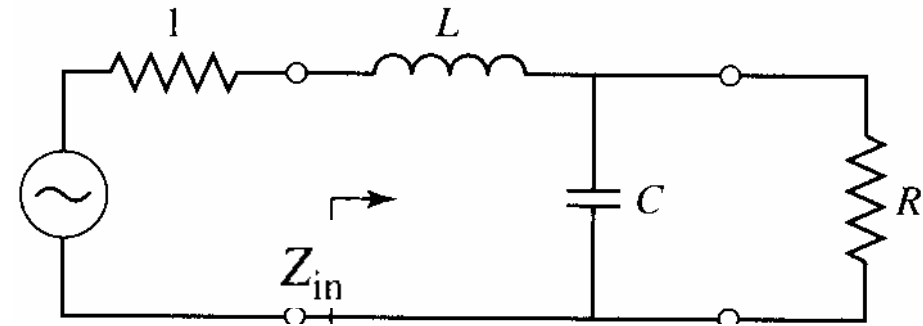


Fig.1

Thus from **eq.1**, the P_{LR} of this circuit is written as;

$$P_{LR} = \frac{1}{1 - |\Gamma|^2} = \frac{1}{1 - [(Z_{in} - 1)/(Z_{in} + 1)][(Z_{in}^* - 1)/(Z_{in}^* + 1)]} = \frac{|Z_{in} + 1|^2}{2(Z_{in} + Z_{in}^*)}$$

Since, $Z_{in} + Z_{in}^* = \frac{2R}{1 + \omega^2 R^2 C^2}$

$$|Z_{in} + 1|^2 = \left(\frac{R}{1 + \omega^2 R^2 C^2} + 1 \right)^2 + \left(\omega L - \frac{\omega C R^2}{1 + \omega^2 R^2 C^2} \right)^2$$

We have, $P_{LR} = \frac{1 + \omega^2 R^2 C^2}{4R} \left[\left(\frac{R}{1 + \omega^2 R^2 C^2} + 1 \right)^2 + \left(\omega L - \frac{\omega C R^2}{1 + \omega^2 R^2 C^2} \right)^2 \right]$

$$= \frac{1}{4R} (R^2 + 2R + 1 + R^2 C^2 \omega^2 + \omega^2 L^2 + \omega^4 L^2 C^2 R^2 - 2\omega^2 L C R^2)$$

$$= 1 + \frac{1}{4R} [(1 - R)^2 + (R^2 C^2 + L^2 - 2LCR^2)\omega^2 + L^2 C^2 R^2 \omega^4] \quad \text{(eq. 3)}$$

- If LP filter of **eq.2** has two elements (N=2) and is normalized ($Z_S=1\Omega$ & $\omega_C=1$): $P_{LR} = 1 + \omega^4$ (**eq. 2a**) (as at $\omega=\omega_C$, $\mathbf{IL}=10\log(1+k^2) \rightarrow 3\text{dB}$, Thus $\mathbf{k}=1$)
- **From eq.(2a):** $P_{LR}=1$ at $\omega=0$; substituting this in **eq.3** gives $\rightarrow \mathbf{R}=1$.
Also in **eq. 2a**, coefficient of ω^2 is 0; equating this to coeff in **eq.3** gives,
 $C^2 + L^2 - 2LC = (C - L)^2 = 0$. Thus, $\mathbf{L=C}$. Similarly, the coefficient of ω^4 in **eq. 2a** is 1; equating this to **eq.3** $\frac{1}{4}(L^2 C^2)=1$. Thus, $\mathbf{L=C}=\sqrt{2}$

- For filters with large ‘N’, this method to obtain ‘L’ & ‘C’ is impractical
- Thus, normalized LPF’s below are designed using **given** element values (see Table 1)

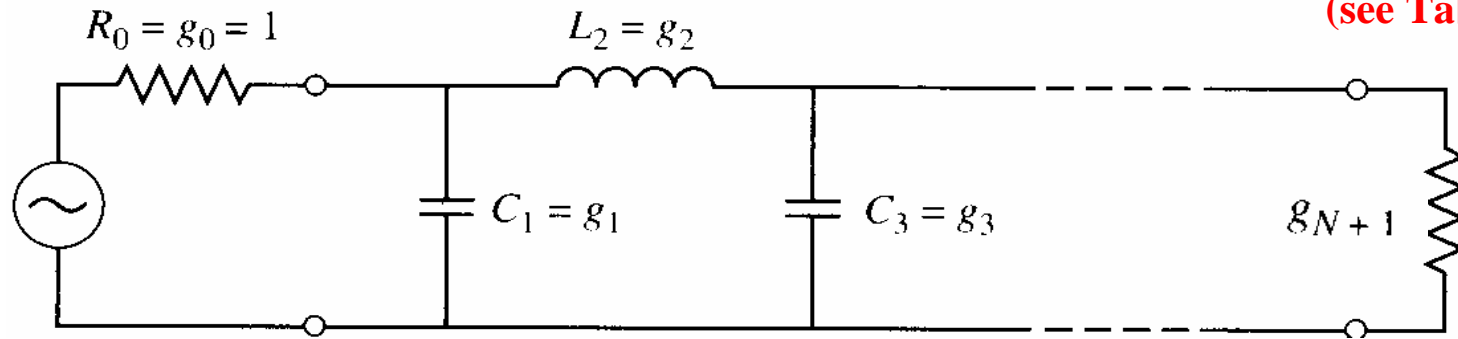


Fig 2a: Circuit begins with shunt element

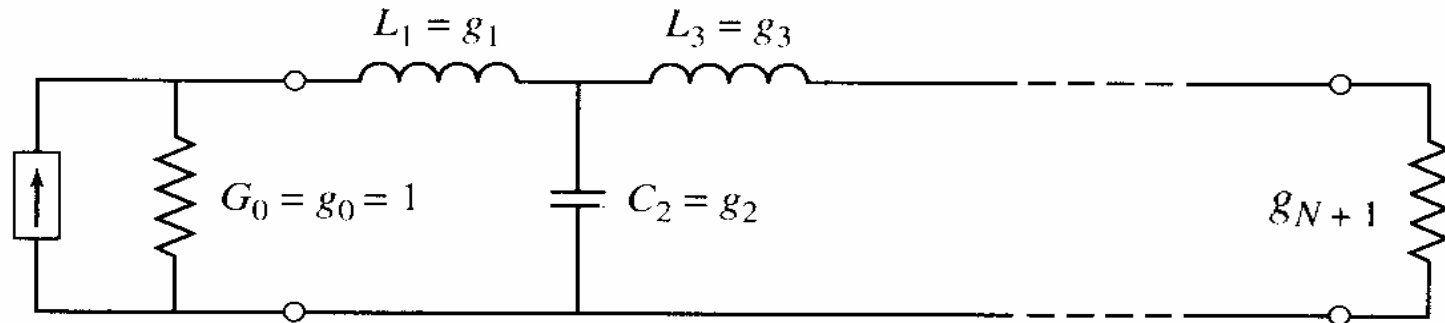


Fig 2b: Circuit begins with series element

where: $g_0 = \begin{cases} \text{generator resistance for the network of Fig 2a.} \\ \text{generator conductance for the network of Fig 2b.} \end{cases}$

$g_k = \begin{cases} \text{inductance for series inductors} \\ \text{capacitor for shunt capacitors} \end{cases}$
 $k=1 \text{ to } N$

$g_{N+1} = \begin{cases} \text{load resistance if } g_N \text{ is a shunt capacitor} \\ \text{load conductance if } g_N \text{ is a series inductor} \end{cases}$

- In previous fig's: maximally flat **LPF**'s can have 'N' reactive elements from 'g₁ to g_N'. Elements values for N=1 to 10 are tabulated below.
- In last figure, 'g₀=1=generator impedance' and 'g_{N+1}=load impedance'.

Table 1: Element ('L' & 'C') Values for Normalized (g₀=1 & ω_C=1) Maximally Flat (buterworth) Low-Pass Filter Prototype :

N	g ₁	g ₂	g ₃	g ₄	g ₅	g ₆	g ₇	g ₈	g ₉	g ₁₀
1	2.0000	1.000								
2	1.4142	1.414	1.000							
3	1.0000	2.000	1.000	1.000						
4	0.7654	1.848	1.848	0.765	1.000					
5	0.6180	1.618	2.000	1.618	0.618	1.000				
6	0.5176	1.414	1.932	1.932	1.414	0.518	1.000			
7	0.4450	1.247	1.802	2.000	1.802	1.247	0.445	1.000		
8	0.3902	1.111	1.663	1.962	1.962	1.663	1.111	0.390	1.000	
9	0.3473	1.000	1.532	1.879	2.000	1.880	1.532	1.000	0.347	1.000
10	0.3129	0.908	1.414	1.782	1.975	1.975	1.782	1.414	0.908	0.313

But if the filter circuit is not given and needed to be [designed](#):

- Order 'N' of a filter is dictated by given insertion-loss (attenuation) at some frequency and using figure below, values of 'N' can be determined (for N<10)

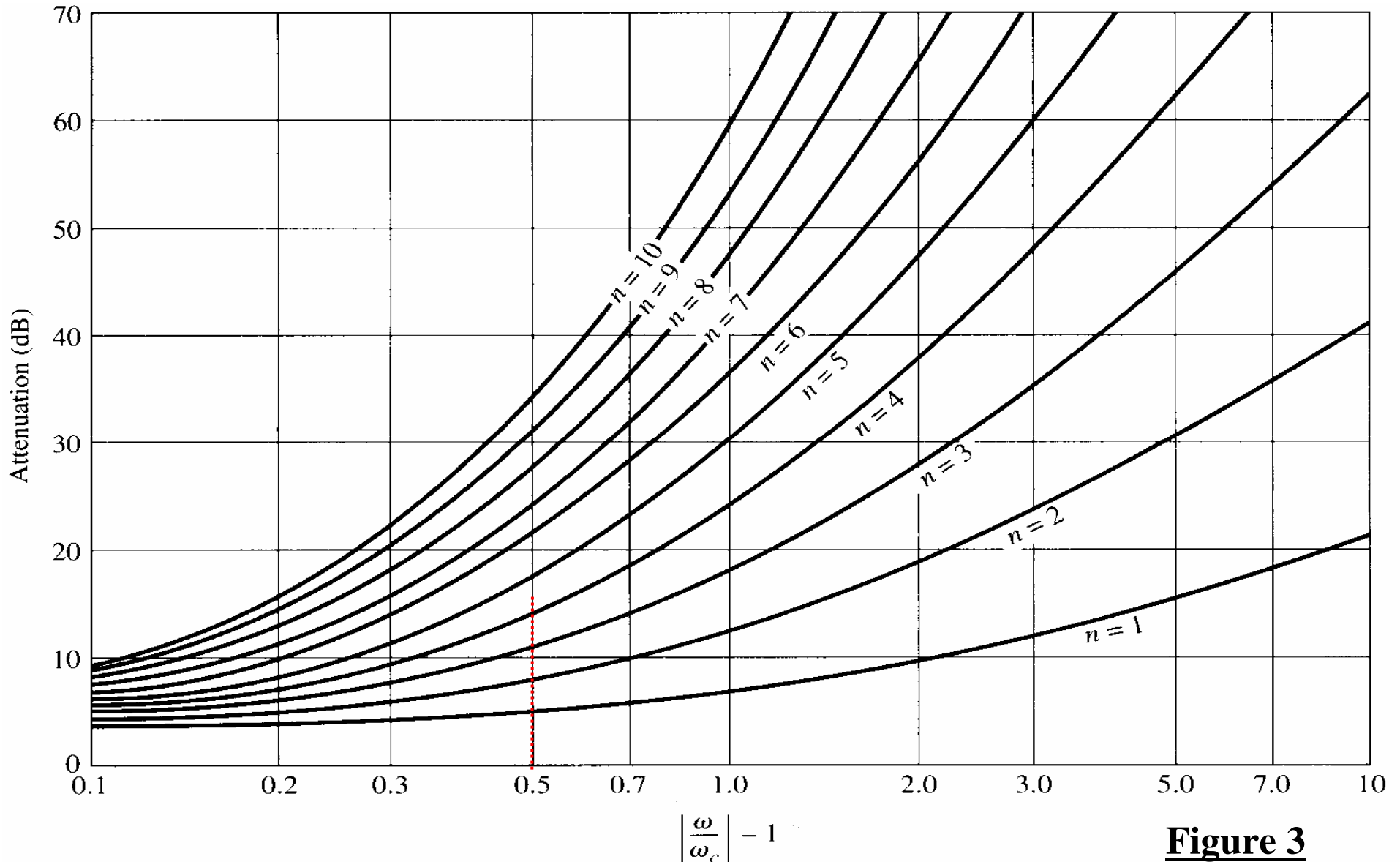


Figure 3

Example 1: A maximally-flat low-pass filter is to be designed with a cut-off frequency of 2 GHz & a minimum attenuation of 15dB at 3GHz

(a) Find the number ('N') of required filter elements.

(b) Find the un-scaled values of the filter reactive elements.

Sol.: Given, $f=3$ GHz and $f_c=2$ GHz. Thus, $|\omega/\omega_c|-1=0.5$. From fig 3 we get, $N \geq 5$ to achieve minimum attenuation of 15dB (*see red line*)

(b) From Table 1, the un-scaled values of the filter reactive elements are;

$$g_1 = 0.6180; \quad g_2 = 1.618; \quad g_3 = 2.000; \quad g_4 = 1.618; \quad g_5 = 0.618;$$

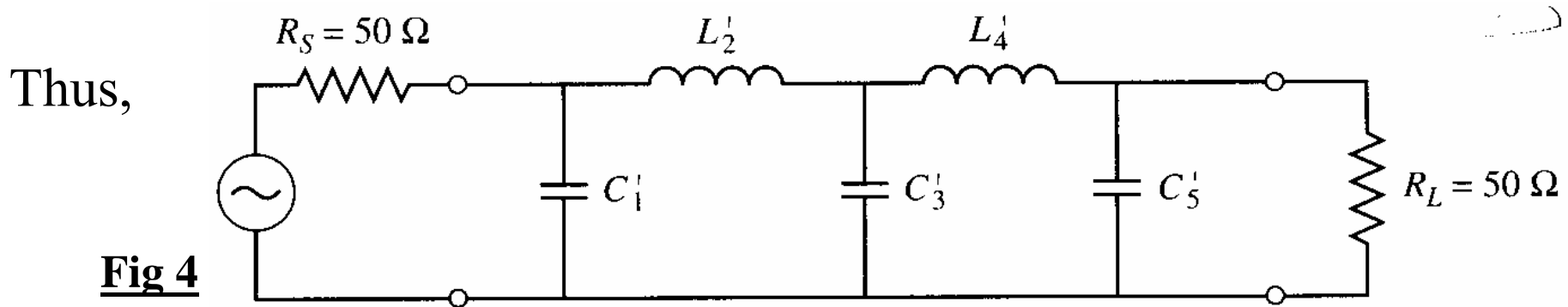
This filter circuit with five reactive elements are similar to Fig 2(a) or (b)

Scaling Designed Low pass Filter Prototypes:

- Scaling convert normalized filters to filters with prescribed impedance & ω_c
- Impedance scaling: is obtained by multiplying proto-type impedances by required source resistance (R_0). Thus, $R_S' = R_0$, $R_L' = R_L \cdot R_0$, $L' = L \cdot R_0$, $C' = C/R_0$, where R_S , R_L , L , C are normalized values and R_S' , R_L' , L' , C' are scaled values.
- Frequency scaling: convert cut-off frequency of prototype from unity to required value of ω_c . Thus, scaled element values become; $L_k' = L_k/\omega_c$ and $C_k' = C_k/\omega_c$
- For both Impedance & Frequency scaling: $L_k' = (R_0 \cdot L_k)/\omega_c$ & $C_k' = C_k / (R_0 \cdot \omega_c)$
 ($k=1,2,3\dots$) Eq. 4

Example 2: If the maximally-flat LP filter has an impedance of 50, then
 (1) find the scaled value of the reactive elements. (2) Draw the circuit.

Sol.: For the type of filter shown in **Fig 2a**; **eq.4** & example1 sol. gives,
 $C_1' = 0.984 \text{ pF}$, $L_2' = 6.438 \text{ nH}$, $C_3' = 3.183 \text{ pF}$, $L_4' = 6.438 \text{ nH}$, $C_5' = 0.984 \text{ pF}$



- Same technique is used to obtain elements for the type of filter in **fig 2b**

P_{LR} of Equal-ripple Chebyshev low-pass filter: By definition, power-

-loss ratio is; $P_{LR} = 1 + k^2 T_N^2 \left(\frac{\omega}{\omega_c} \right)$ **(eq. 5)**

- For normalized filters, $\omega_c = 1$;
 Thus for 2 element filter in fig 1; $T_2(\omega) = 2\omega^2 - 1$

Eq.5 $\rightarrow P_{LR} = 1 + k^2(4\omega^4 - 4\omega^2 + 1)$ **(eq. 6)**

- $T_N(x)$ is Chebyshev polynomial of order 'N' where,
 $T_1(x) = x$,
 $T_2(x) = 2x^2 - 1$,
 $T_3(x) = 4x^3 - 3x$,
 $T_4(x) = 8x^4 - 8x^2 + 1$,

- Again for equal-ripple filter, using **eq.6** & **eq.3** lets find ripple level (k^2)
 If (**eq. 3**) is, $P_{LR} = 1 + \frac{1}{4R} [(1 - R)^2 + (R^2 C^2 + L^2 - 2LCR^2)\omega^2 + L^2 C^2 R^2 \omega^4]$
 for even 'N' & at $\omega=0$, eq.3 & 6 gives ripple level, $k^2=(1-R)^2/(4R)$ (**eq. 7**)
- Using **eq's 3, 6 & 7**, we can find the two elements of the filter in Fig.1.
- For large 'N', **given** element tables are used to design LPF prototypes:

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1	0.699	1.000								
2	1.403	0.707	1.984							
3	1.596	1.097	1.596	1.000						
4	1.670	1.193	2.366	0.842	1.984					
5	1.706	1.230	2.541	1.230	1.706	1.000				
6	1.725	1.248	2.606	1.314	2.476	0.870	1.984			
7	1.737	1.258	2.638	1.344	2.638	1.258	1.737	1.000		
8	1.745	1.265	2.656	1.359	2.696	1.339	2.509	0.880	1.984	
9	1.751	1.269	2.668	1.367	2.724	1.367	2.668	1.269	1.750	1.000
10	1.754	1.272	2.675	1.373	2.739	1.381	2.723	1.349	2.524	0.884

Table 2:
For equal-ripple LFP
with **0.5 dB Ripple &**
 $g_0=1$ & $\omega_c=1$

Table 3: Element ('L' & 'C') Values for Normalized ($g_0=1$ & $\omega_c=1$)
equal-ripple Low-Pass Filter Prototype (with **3.0 dB** ripple)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1	1.995	1.000								
2	3.101	0.534	5.810							
3	3.349	0.712	3.349	1.000						
4	3.439	0.748	4.347	0.592	5.810					
5	3.482	0.762	4.538	0.762	3.482	1.000				
6	3.504	0.769	4.606	0.793	4.464	0.603	5.810			
7	3.518	0.772	4.639	0.84	4.639	0.772	3.518	1.000		
8	3.528	0.775	4.658	0.809	4.699	0.802	4.499	0.607	5.809	
9	3.534	0.776	4.669	0.812	4.727	0.812	4.669	0.776	3.534	1.000
10	3.538	0.777	4.677	0.814	4.743	0.816	4.726	0.805	4.514	0.609

- Again for equal-ripple filter; the order 'N', dictated by given insertion-loss (attenuation) at some frequency, is found using next two figures (for $N < 10$)

For **designing** equal-ripple LP Filter Prototype (with **0.5 dB** ripple):

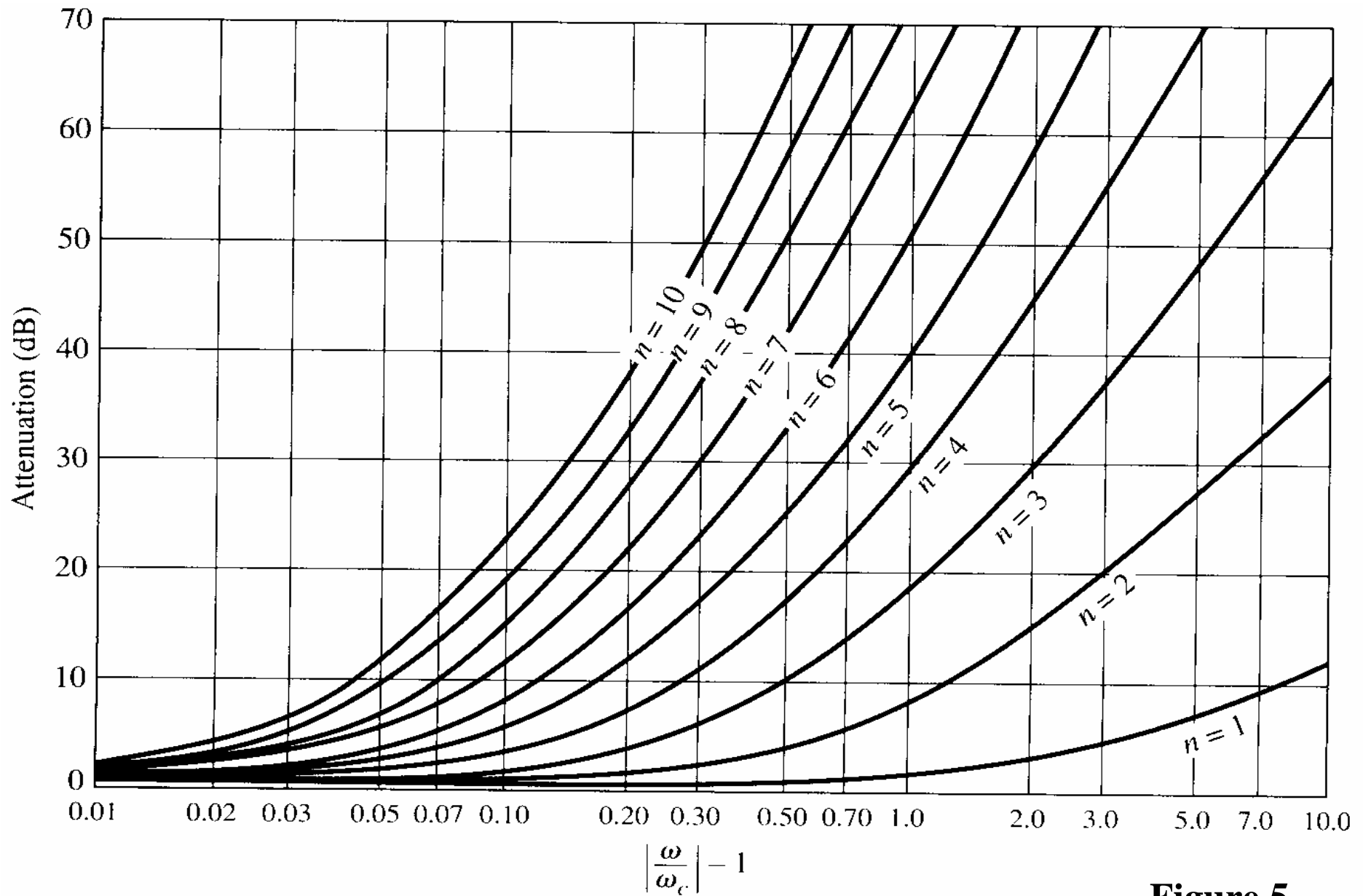


Figure 5

For **designing** equal-ripple LP Filter Prototype (with **3.0 dB** ripple):

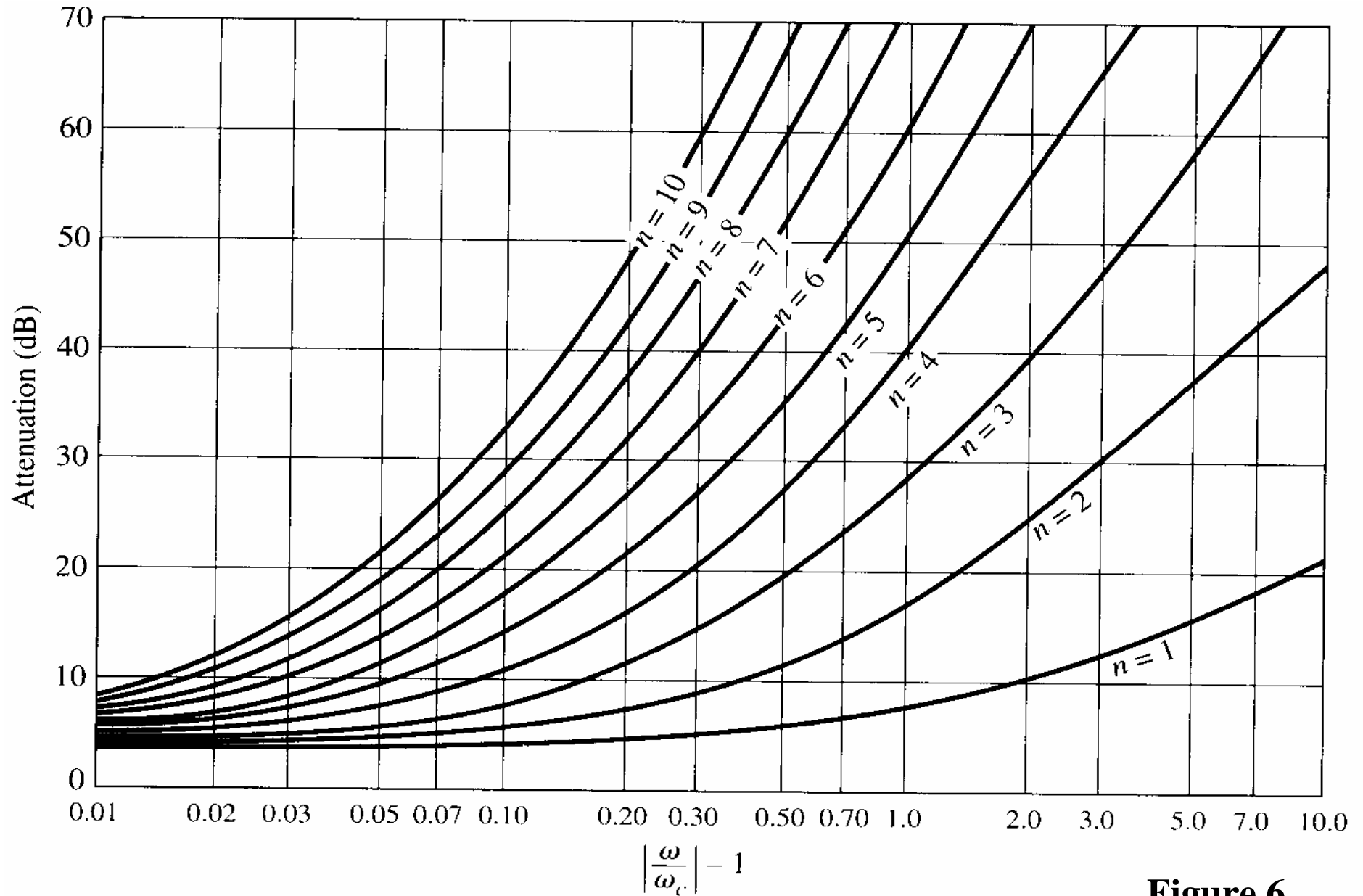


Figure 6

P_{LR} of Linear Phase low-pass filter prototype: The phase response is:

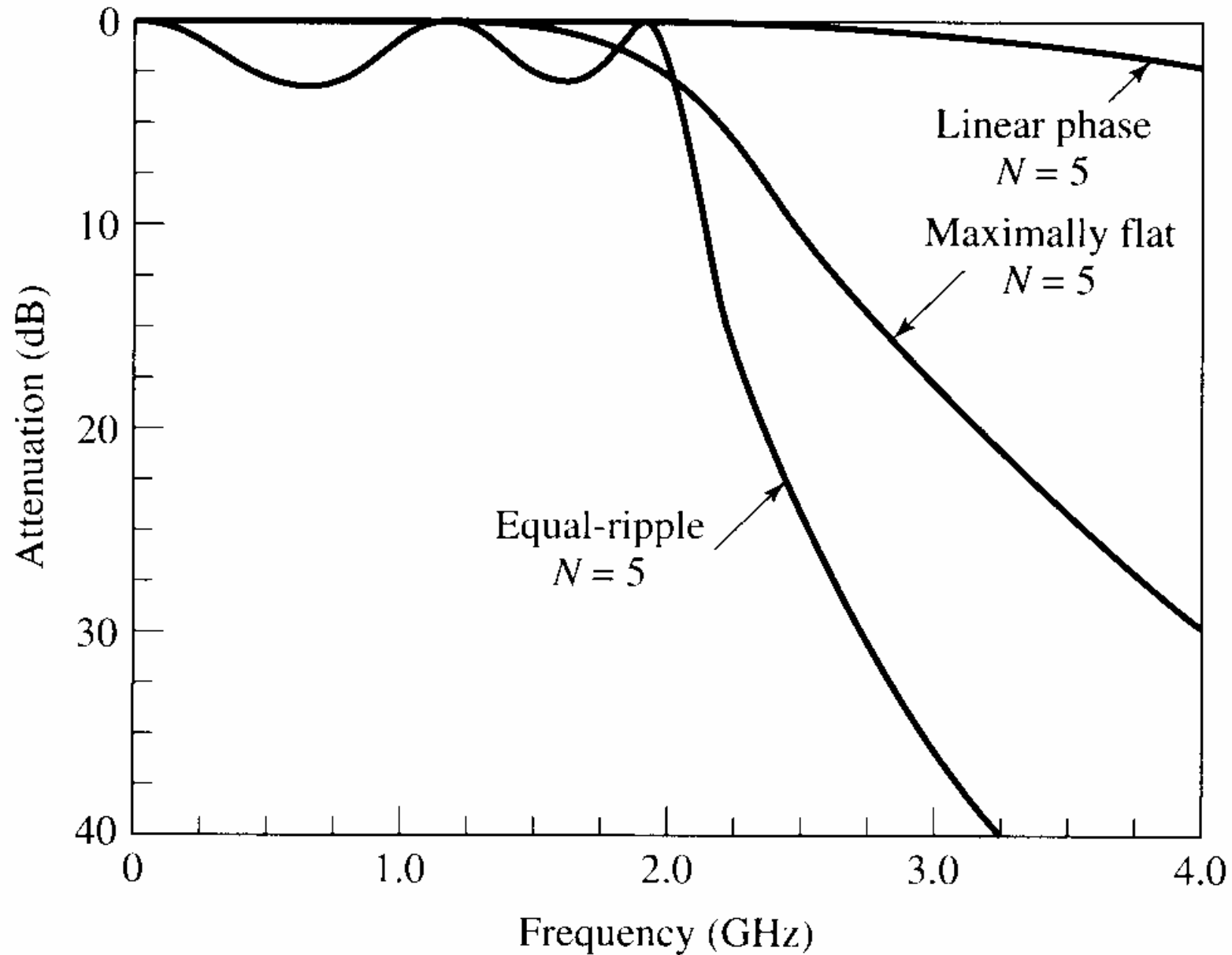
$$\phi(\omega) = A\omega \left[1 + p \left(\frac{\omega}{\omega_c} \right)^{2N} \right] \quad \text{and} \quad \tau = \frac{d\phi}{d\omega} = A \left[1 + p(2N + 1) \left(\frac{\omega}{\omega_c} \right)^{2N} \right]$$

- Using same technique, elements values for linear phase LPF is obtained
- For large ‘N’, **given** element tables are used to **design** LPF prototypes:

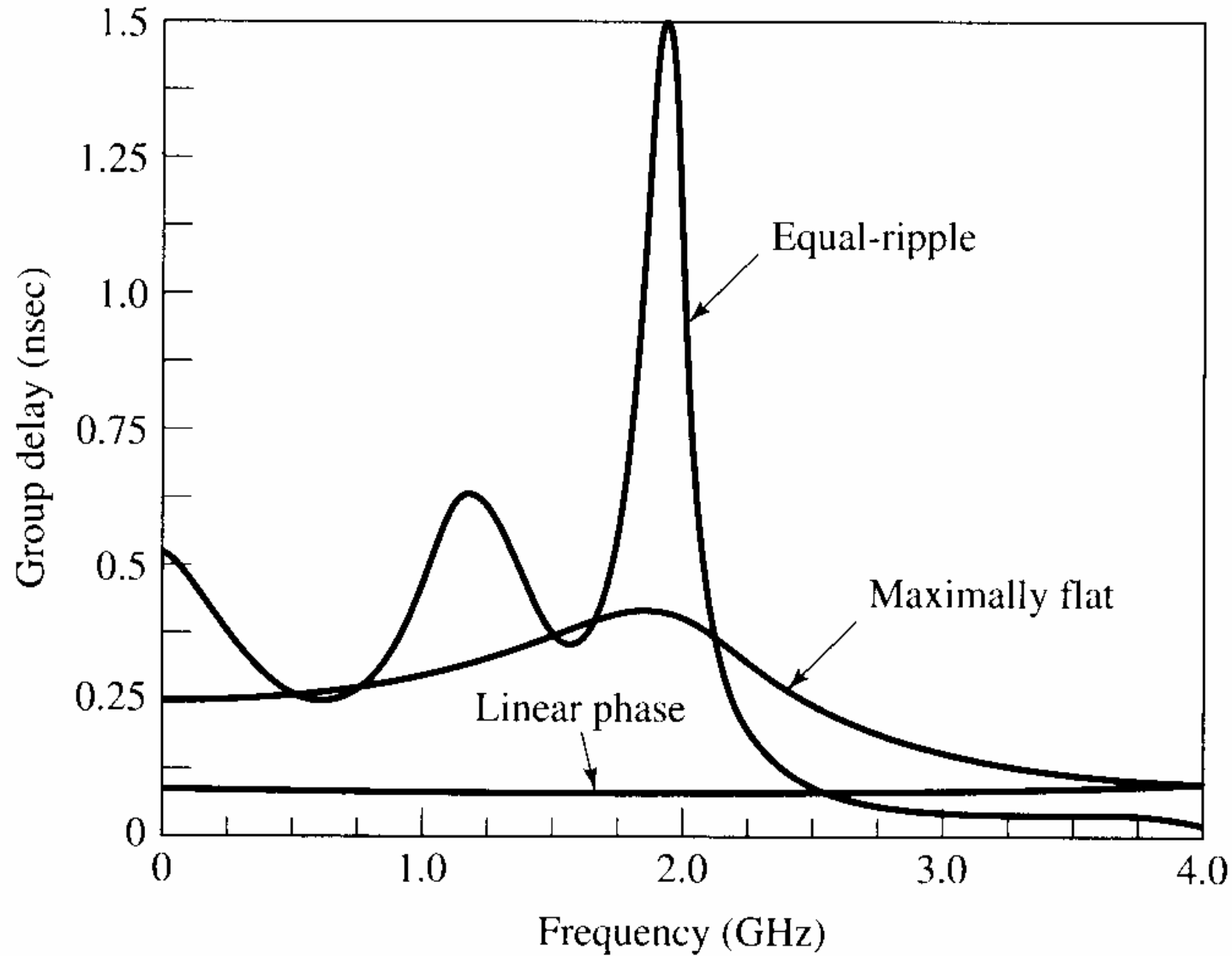
N	g ₁	g ₂	g ₃	g ₄	g ₅	g ₆	g ₇	g ₈	g ₉	g ₁₀
1	2.000	1.000								
2	1.577	0.422	1.000							
3	1.255	0.552	0.192	1.000						
4	1.059	0.511	0.318	0.110	1.000					
5	0.930	0.457	0.331	0.209	0.071	1.000				
6	0.837	0.411	0.315	0.236	0.148	0.050	1.000			
7	0.767	0.374	0.294	0.237	0.177	0.110	0.037	1.000		
8	0.712	0.344	0.273	0.229	0.186	0.138	0.085	0.028	1.000	
9	0.667	0.320	0.254	0.218	0.185	0.150	0.111	0.068	0.023	1.000
10	0.630	0.300	0.238	0.206	0.180	0.153	0.124	0.091	0.055	0.018

Table 4:
For linear phase LFP
with g₀=1 & ω_C=1

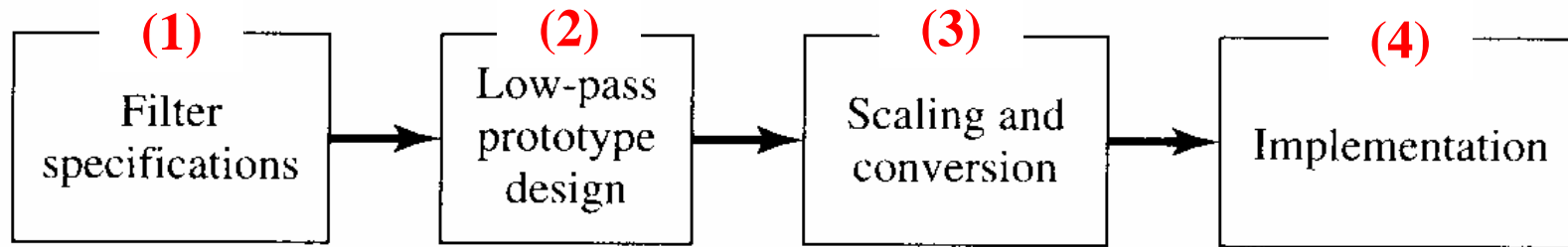
Comparison of the three responses in terms of attenuation :



Comparison of the three responses in terms of phase:

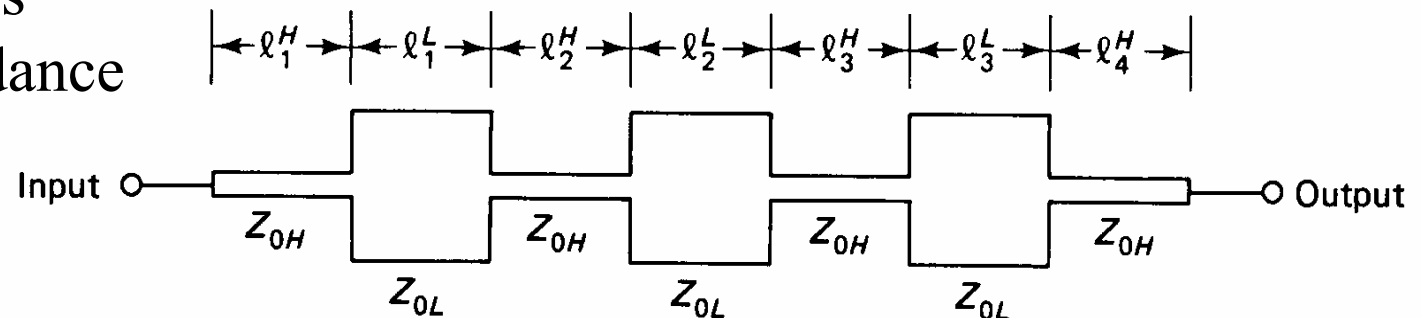
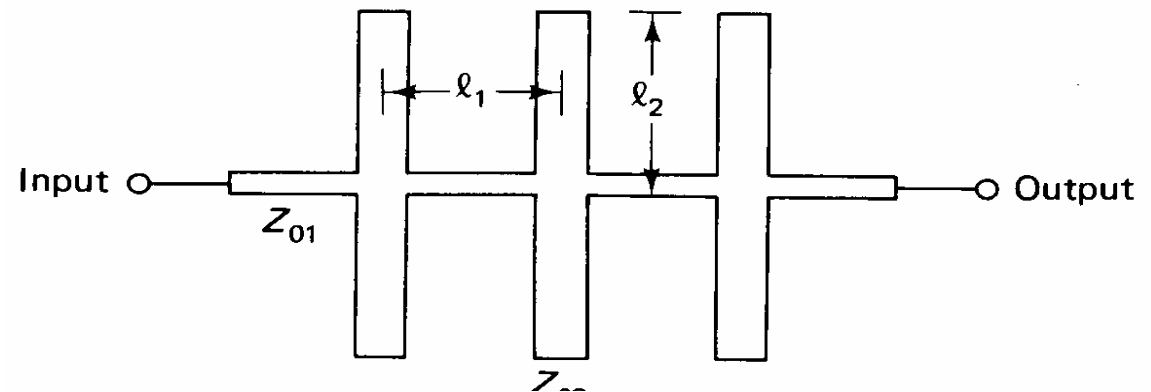
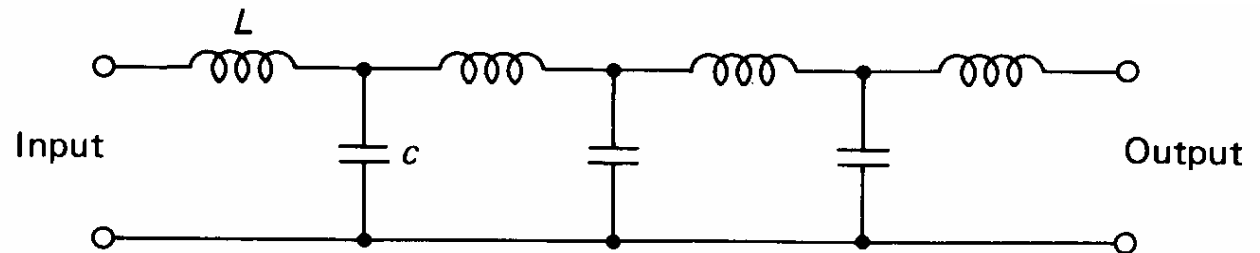


MW/RF filter implementation using Stepped-Impedance LPF's:



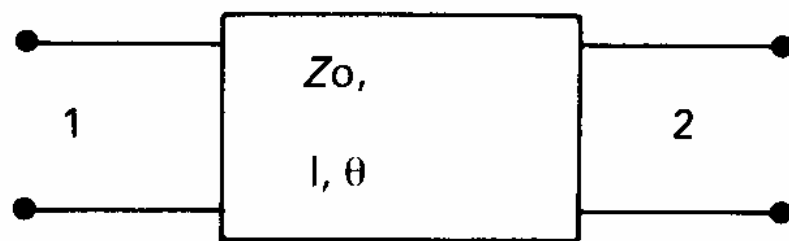
Among above steps:

- (1) is usually given
- (2) & (3) can be implemented using the method described
- (4) implemented via two methods:
 - trans-line stubs
 - stepped impedance



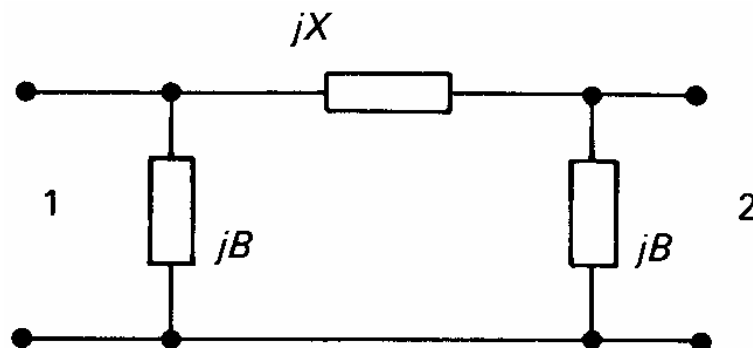
Stepped-impedance Filters:

- Relatively easy (but inefficient) way to implement MW/RF LPF/HPF's
- Mainly used in filters which do not require sharp cutoffs (mixers)
- Using [ABCD] or [T] matrix (*see examples later*), a transmission line with characteristic impedance of Z_0 and length l can be expressed as,



$$[T] = \begin{bmatrix} \cos(\theta) & jZ_0 \sin(\theta) \\ j \frac{\sin(\theta)}{Z_0} & \cos(\theta) \end{bmatrix} \quad \left\{ \begin{array}{l} \text{where,} \\ \theta = \frac{\omega l}{v} = \beta l \end{array} \right.$$

- Again for equivalent circuit of the TL, [ABCD] or [T] matrix is given by



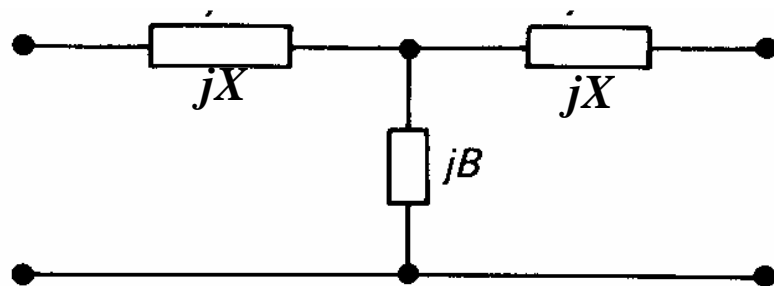
$$[T] = \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix} \times \begin{bmatrix} 1 & jX \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - XB & jX \\ jB(2 - XB) & 1 - XB \end{bmatrix}$$

- equating above expressions:

$$\cos(\theta) = 1 - XB \quad \text{and} \quad X = Z_0 \sin(\theta) \quad \text{and} \quad B = \frac{(1 - \cos(\theta))}{Z_0 \sin(\theta)} = Y_0 \tan(\theta/2)$$

- For a TL with ($l < \lambda/4$ or $\theta < \pi/2$) and ($Z_0 \gg 1$); $X \rightarrow$ ‘+’ and $B \rightarrow$ ‘small’ and the transmission line approximates a series **inductive** impedance
Thus, $X = \omega L = Z_0 \sin(\theta) = Z_0 \sin(\beta l) \cong Z_0 \beta l$ and $B \cong 0$;
- Using scaling equation (for impedance & frequency) to un-scale (L_N) gives:
 $\omega L = \omega \{ (L_N R_0) / \omega_C \} \cong Z_0 \beta l$, where $Z_0 = Z_{0h}$ or impedance of thin TL section
So at $\omega = \omega_C$, $\beta l = (L_N R_0) / Z_{0h}$, where R_0 is equivalent to filter impedance
- Similarly from, $\omega L \cong Z_0 \beta l$; $L \cong (Z_{0h} \beta l) / \omega \cong (Z_{0h} l) / (\lambda_g f)$ (shown before)
- Typically, the impedance of the inductive microstrip line ($Z_{0h} \approx Z_h$) is set to be as high as physically possible (thinnest practically fabricated line)
- **Similarly**, using the TL equivalent model below, ‘ βl ’ & ‘ C ’ values for capacitive microstrip line ($Z_{0l} = Z_l \approx$ as high impedance as possible) is obtained



$$[\mathbf{T}] = \begin{bmatrix} 1 & jX \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix} \times \begin{bmatrix} 1 & jX \\ 0 & 1 \end{bmatrix}$$

As before: $\begin{cases} B = Y_0 \sin(\theta) \\ X = Z_0 \tan(\theta/2) \end{cases} \quad \begin{cases} \text{where,} \\ \theta = \frac{\omega l}{v} = \beta l \end{cases}$

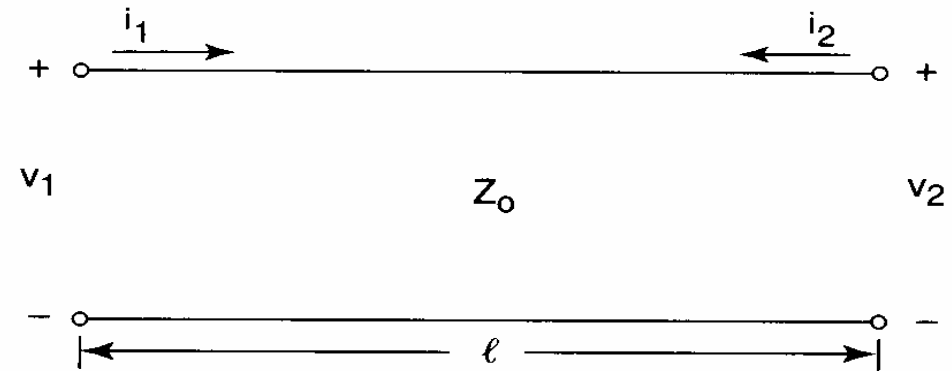
- For TL with ($l < \lambda/4$ or $\theta < \pi/2$) and ($Z_0 \ll 1$); $B \rightarrow '+'$ and $X \rightarrow \text{'small'}$ and the transmission line approximates a shunt **capacitive** impedance
Thus, $B = \omega C = Y_0 \sin(\theta) = Y_0 \sin(\beta l) \cong Y_0 \beta l$ and $X \cong 0$;
- Using scaling equation (*for impedance & frequency*) to un-scale gives:
 $\omega C = \omega \{C_N / (\omega_C R_0)\} \cong (\beta l) / Z_0$, where $Z_0 = Z_{0l}$ or impedance of thick TL section
So at $\omega = \omega_C$, $\beta l = (C_N Z_{0h}) / R_0$, where R_0 is equivalent to filter impedance
- Similarly from, $\omega C \cong (\beta l) / Z_0$; $C \cong (\beta l) / (\omega Z_{0l}) \cong l / (\lambda_g f Z_{0l})$ (*shown before*)
- Typically, the impedance of the capacitive microstrip line ($Z_{0l} \approx Z_l$) is set to be as low as possible (*thickest fabricated line allowed the ckt. size*)
- Typically, the ratio Z_h / Z_l should be as high as possible (*practically fabricated*)

Problem: Design a stepped-impedance low-pass filter with maximally flat response and a cutoff frequency of 2.5 GHz. It is necessary to have at least 25 dB attenuation at 4.0 GHz. The filter impedance is 50 ohm; the highest practical line impedances is 150ohm & the lowest is 10 ohm.

Sol: $\beta l_1 = 5.9^\circ$, $\beta l_2 = 27^\circ$, $\beta l_3 = 22.1^\circ$, $\beta l_4 = 36.9^\circ$, $\beta l_5 = 16.2^\circ$, $\beta l_6 = 9.9^\circ$)

Problem: Find the $[ABCD]$ matrix for a lossless transmission line of length (l) and characteristic impedance (Z_0) as shown in Figure below.

Sol. For a lossless TL, the output voltage or current has the same magnitude as the input but lags behind in phase by $e^{-j\beta l}$. Thus, the following can be written:



$$\begin{aligned} v_2 &= v_1 e^{-j\beta l} \Rightarrow v_1 = v_2 e^{+j\beta l} = v_2 \cos\beta l + jv_2 \sin\beta l \\ i_2 &= -i_1 e^{-j\beta l} \Rightarrow i_1 = -i_2 e^{+j\beta l} = -i_2 \cos\beta l - ji_2 \sin\beta l \end{aligned} \quad \left\{ \begin{array}{l} \text{Since } Z_L = Z_0, \\ \mathbf{v}_2 = -Z_0 \mathbf{i}_2 \end{array} \right.$$

$$\text{Then, } v_1 = (\cos\beta l)v_2 - (jZ_0 \sin\beta l)i_2 = Av_2 - Bi_2$$

$$i_1 = (jY_0 \sin\beta l)v_2 - (\cos\beta l)i_2 = Cv_2 - Di_2$$

Thus, the $[ABCD]$ matrix can be written as:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_0 \sin\beta l \\ jY_0 \sin\beta l & \cos\beta l \end{bmatrix}$$

Microwave Filter

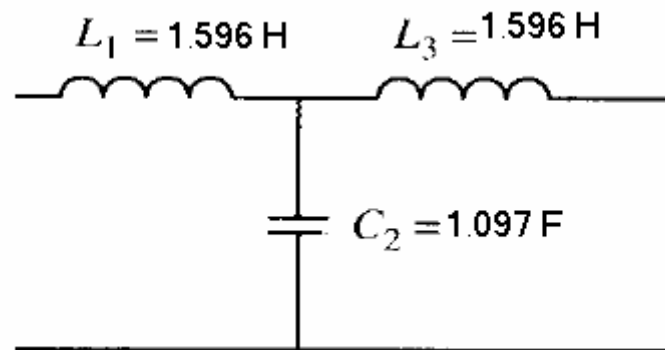
Using

Transmission line stubs

(Summery of a student project)

How to implement filter by this way?

Give this circuit as an example for 0.5 equally ripple chepysheve LPF:



The filter is designed by the following steps:

Step 1.

Lumped element low pass prototype (from tables, typically)

Lets take 0.5dB equally ripple of degree 3

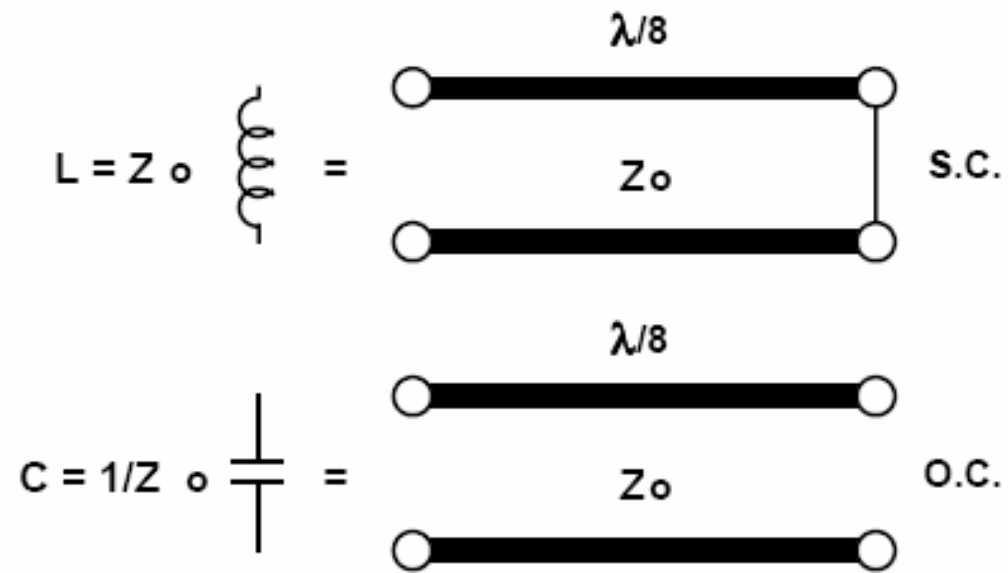
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1	0.699	1.000								
2	1.403	0.707	1.984							
3	1.596	1.097	1.596	1.000						
4	1.670	1.193	2.366	0.842	1.984					
5	1.706	1.230	2.541	1.230	1.706	1.000				
6	1.725	1.248	2.606	1.314	2.476	0.870	1.984			
7	1.737	1.258	2.638	1.344	2.638	1.258	1.737	1.000		
8	1.745	1.265	2.656	1.359	2.696	1.339	2.509	0.880	1.984	
9	1.751	1.269	2.668	1.367	2.724	1.367	2.668	1.269	1.750	1.000
10	1.754	1.272	2.675	1.373	2.739	1.381	2.723	1.349	2.524	0.884

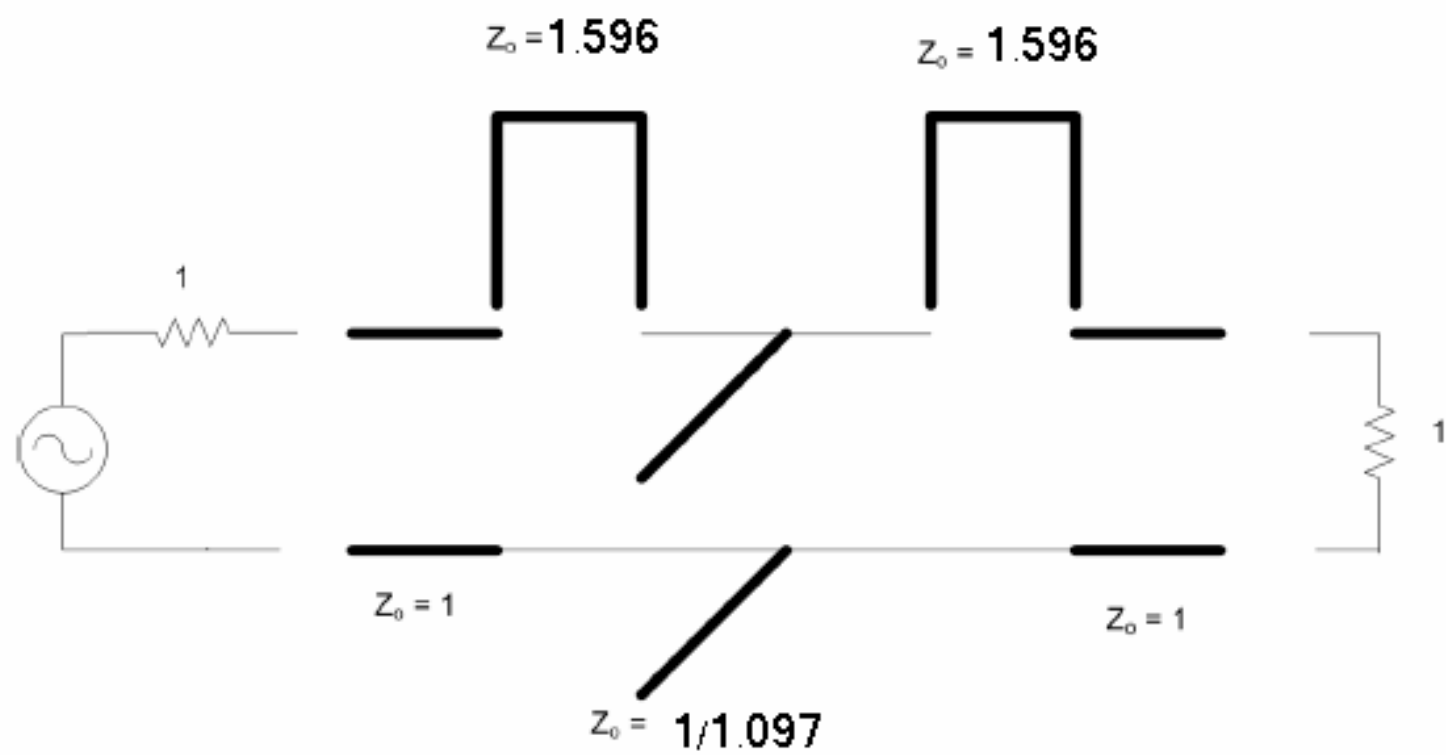
Table 2:
For equal-ripple LFP
with **0.5 dB Ripple &**
 $g_0=1$ & $\omega_c=1$

Step2.

Convert series inductors to series stubs and
shunt capacitors to shunt stubs by Richard's
rule

Richard's rule



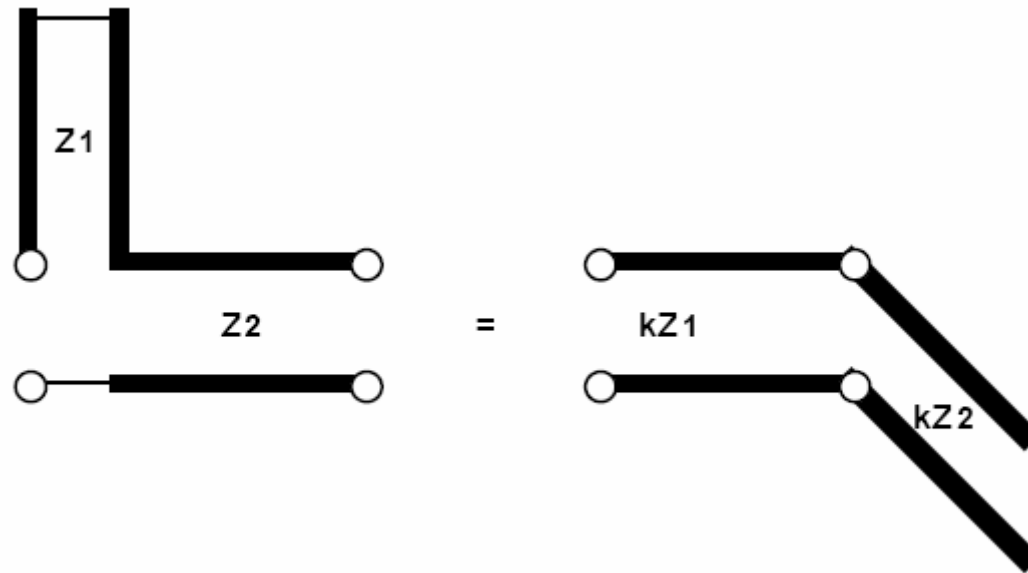


Step3.

Using Kuroda's Identities:

the series inductive stub in the diagram here
can be replaced by a shunt capacitive stub
on the other end of the $\lambda/8$ line, with
different values of characteristic impedance

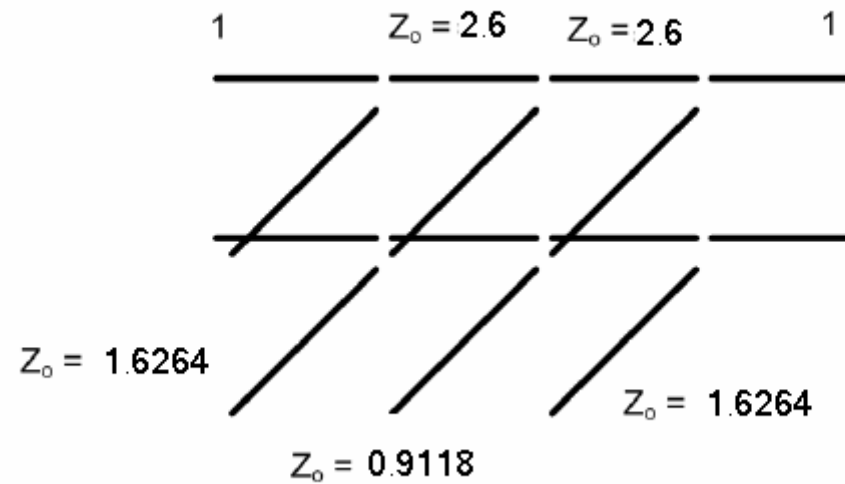
Kuroda's Identities



$$k = 1 + Z_1/Z_2$$

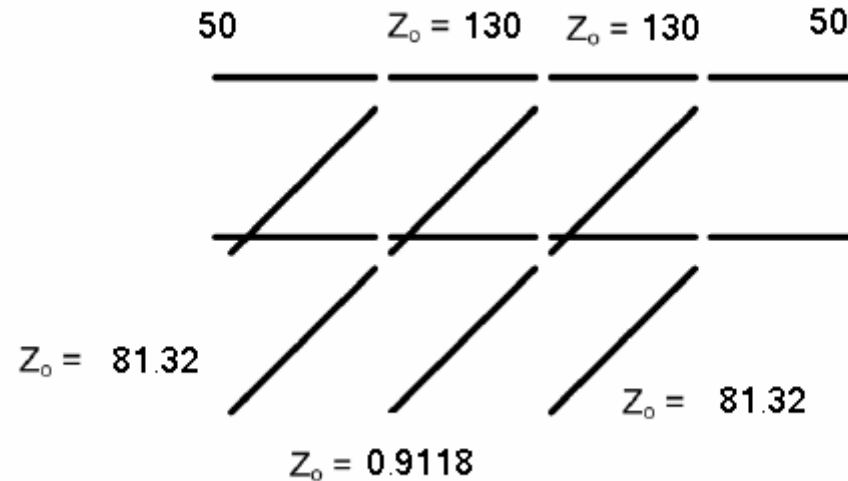
Step4.

Add $l_{um}/8$ lines of $Z_o = 1$ at input and output



Step 5.

Transform design to 50Ω and f_c to obtain physical dimensions



Step6.

Rules of width

Synthesis: Given ' Z_0 ' & ' ϵ_r ', find ' w/h ' ratio of the microstrip line:

(a) Narrow line, when $Z_0 > [44 - 2 \epsilon_r]$; $\frac{w}{h} = \left[\frac{e^H}{8} - \frac{1}{4e^H} \right]^{-1}$

$$\text{where, } H = \frac{Z_0 \sqrt{2(\epsilon_r + 1)}}{119.9} + \frac{(\epsilon_r - 1)}{2(\epsilon_r + 1)} \left[\ln \frac{\pi}{2} + \frac{1}{\epsilon_r} \ln \frac{4}{\pi} \right]$$

If $Z_0 > [63 - 2 \epsilon_r]$ it may be useful to replace ϵ_r by the effective ϵ_r , namely

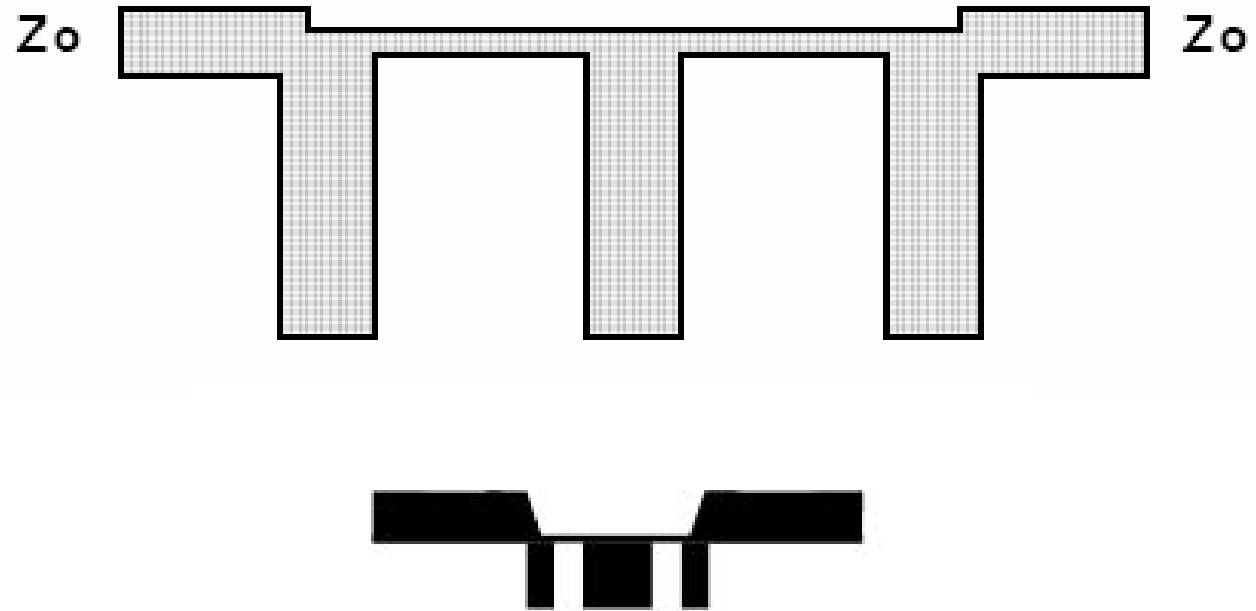
$$\epsilon_{\text{eff}} = \frac{\epsilon_r}{0.96 + \epsilon_r (0.109 - 0.004 \epsilon_r) [\log(10 + Z_0) - 1]}$$

(b) wide line, when $Z_0 < [44 - 2\epsilon_r]$

$$\frac{w}{h} = \frac{2}{\pi} \left[(d - 1) - \ln(2d - 1) + \frac{\epsilon_r - 1}{\pi \epsilon_r} \right] \times \left[\ln(d - 1) + 0.293 - \frac{0.517}{\epsilon_r} \right]$$

$$\text{where, } d = \frac{59.95 \pi^2}{Z_0 \cdot \sqrt{\epsilon_r}}$$

Final Design



Filter Transformation

(Summery of a Student Project)

- First step: find the normalized values from the table
 - i.e. $g_1, g_2, g_3, \dots, g_n$

- Second step: Impedance and frequency scaling

- Frequency Scaling for low pass filter:
 $jX_k = j\left(\frac{\omega}{\omega_c}\right)L_k = j\omega L'$ $jB_k = j\left(\frac{\omega}{\omega_c}\right)C_k = j\omega C_k'$

Where $L_k' = \frac{L_k}{\omega_c}$

where $C_k' = \frac{C_k}{\omega_c}$

Filter Transformation

Impedance Scaling: Multiply each inductor and resistor by R_0
and divide each capacitor by R_0

$$L' = \frac{R_0 L}{\omega_C}$$

$$C' = \frac{C}{R_0 \omega_C}$$

$$R_s' = R_0$$

$$R_L' = R_0 R$$

Filter Transformation

- Impedance and frequency scaling for high pass filter
- Frequency Scaling: $\omega \leftarrow \left(-\frac{\omega_c}{\omega} \right)$

$$jX_k = j\left(\frac{-\omega_c}{\omega}\right)L_k = \frac{1}{j\omega C_k'}$$

$$\text{Where } C_k' = \frac{1}{\omega_c L_k}$$

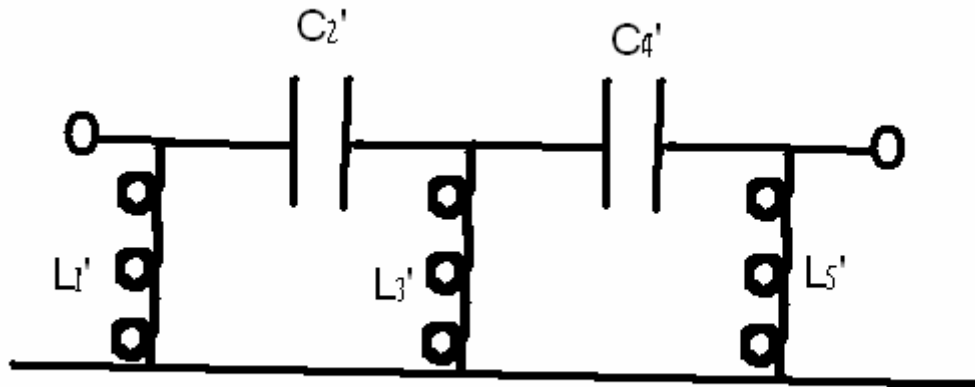
$$jB_k = j\left(\frac{-\omega_c}{\omega}\right)C_k = \frac{1}{j\omega L_k'}$$

$$\text{where } L_k' = \frac{1}{\omega_c C_k}$$

Filter Transformation

- After Impedance Scaling

$$C_k' = \frac{1}{R_o \omega_c L_k} \quad L_k' = \frac{R_o}{\omega_c C_k}$$



Filter Transformation

- frequency scaling for band pass filter:

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1} \left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right] \quad \text{Or} \quad \omega \leftarrow \frac{1}{\Delta} \left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]$$

Where $\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$ $\omega_0 = \sqrt{\omega_2 \omega_1}$

Filter Transformation

- By substitution:
$$jX_k = \frac{j}{\Delta} \left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right] L_k = j\omega L_k' + \frac{1}{j\omega C_k'}$$

where $L_k' = \frac{L_k}{\Delta\omega_0}$ And $C_k' = \frac{\Delta}{\omega_0 L_k}$

$$jB_k = \frac{j}{\Delta} \left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right] C_k = j\omega C_k' + \frac{1}{j\omega L_k'}$$

where $L_k' = \frac{\Delta}{\omega_0 C_k}$ And $C_k' = \frac{C_k}{\omega_0 \Delta}$

Filter Transformation

- Impedance Scaling:

Multiply each L_k' by R_o and divide each C_k' by R_o

Filter Transformation

- Filter Transformations:
- frequency scaling for band stop filter:

$$\omega \leftarrow \Delta \left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]^{-1}$$

Where $\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$ $\omega_0 = \sqrt{\omega_2 \omega_1}$

Filter Transformation

It can be shown that for band stop filter:

- Each series inductor is converted to a parallel LC circuit with values:

$$L_k' = \frac{\Delta L_k}{\omega_0} \text{ and } C_k' = \frac{1}{\Delta \omega_0 L_k}$$

And

- Each parallel capacitor is converted to a series LC circuit with values:

$$L_k' = \frac{1}{\Delta \omega_0 C_k} \text{ and } C_k' = \frac{\Delta C_k}{\omega_0}$$

- Impedance Scaling: Multiply each L_k' by R_o and divide each C_k' by R_o

Filter Transformation

Summary of the transformations (elements values)

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0} \quad \omega_0 = \sqrt{\omega_2 \omega_1}$$

	Low pass filter	High pass filter	Band pass filter	Band stop filter
Series elements	$L_k' = \frac{R_o L_k}{\omega_c}$	$C_k' = \frac{1}{R_o \omega_c L_k}$	$L_k' = \frac{L_k R_o}{\omega_0 \Delta}$ $C_k' = \frac{\Delta}{R_o \omega_0 L_k}$	$L_k' = \frac{L_k R_o \Delta}{\omega_0}$ $C_k' = \frac{1}{R_o \omega_0 L_k \Delta}$
Shunt Elements	$C_k' = \frac{C_k}{R_o \omega_c}$	$L_k' = \frac{R_o}{\omega_c C_k}$	$L_k' = \frac{R_o \Delta}{\omega_0 C_k}$ $C_k' = \frac{C_k}{R_o \omega_0 \Delta}$	$L_k' = \frac{R_o}{\Delta \omega_0 C_k}$ $C_k' = \frac{\Delta C_k}{R_o \omega_0}$

Filter Transformation

Why?

$$\omega \leftarrow \left(-\frac{\omega_c}{\omega} \right) \quad \text{High Pass Filter}$$

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1} \left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right] \quad \text{Band Pass Filter}$$

$$\omega \leftarrow \frac{\omega_2 - \omega_1}{\omega_0} \left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]^{-1} \quad \text{Band Stop Filter}$$

Simulating MW/RF filters

designed by Stepped-impedance method

using

‘High Frequency Structural
Simulator’ (HFSS-5.6)

Discussed Topics

- Introduce the Software HFSS
- Key features of HFSS
- Basic Steps for using HFSS
- Consider simulating the designed stepped impedance low pass filter.

Introduction to HFSS

- Typically HFSS is used for modeling electromagnetic fields in passive 3D structures
- Originally developed by HP. Currently purchased by Ansoft from Agilent Technologies.
- Can work in UNIX or Windows Operating systems
- Can cater for Isotropic/Anisotropic Materials
(ANSOFT → can simulate Tensor or ferrite material)
- HFSS is a finite element (FEM) based software
- Trade off exist between mesh size of the FEM solution and the desired accuracy in the results

Key Features of HFSS

- Uses FEM technique to solve Maxwell's Equations and can and can include dispersion.
- Can simulate unrestricted geometries that contains unlimited dielectrics and ports (Symmetries, Z-mul)
- Contains a model parts library of standard microwave devices (filters, bends, circulators etc) that can be used to model complex devices.
- Can calculate scattering parameter response for multiple modes
- Can solve for electric-field distribution, including far-field antenna radiation patterns.

Key Features of HFSS (cont.)

- Can calculate impedance and complex propagation constants at each port for an unlimited number of modes (Given Impedance/Calibration/Polarization lines, as required)
- Can display vector and contour plots
- Quantitative versus distance graphs for field solutions
- Smith Chart plots
- Shaded and Animated field plots
- Data table

Basic Steps for using HFSS

- Modeling the Structure:
 - Drawing the Geometry
 - Defining the Material
 - Defining and Calibrate the Ports
 - Defining the Surface or Boundaries
- Analyzing the EM behavior
 - Solving for S-parameters
 - Analyzing the results using the post-processor

Fabricating MW/RF filters

designed by Stepped-impedance method

using

Printed Circuit Board plotter:

‘Protomat C60’

LPKF's Protomate C60 Machine.

- Rapid PCB prototyping for SMD boards
- High precision PCB, directly from CAD data
- Support auto-contact and vacuum mechanism for multi-layer PCB's (top and bottom layers only)
- **Prototype** Microwave PCB's (FR3/Drude/Teflon..)

Software's supplied with Protomate C60

- 'CircuitCAM' software:
 - import/edit PCB data (Garber, Excellon format)
 - Draw MW PCB prototype
 - Prepare the production data for plotting (isolation channel, Rubout, Contour routing)
 - Export machining data (LMD format)

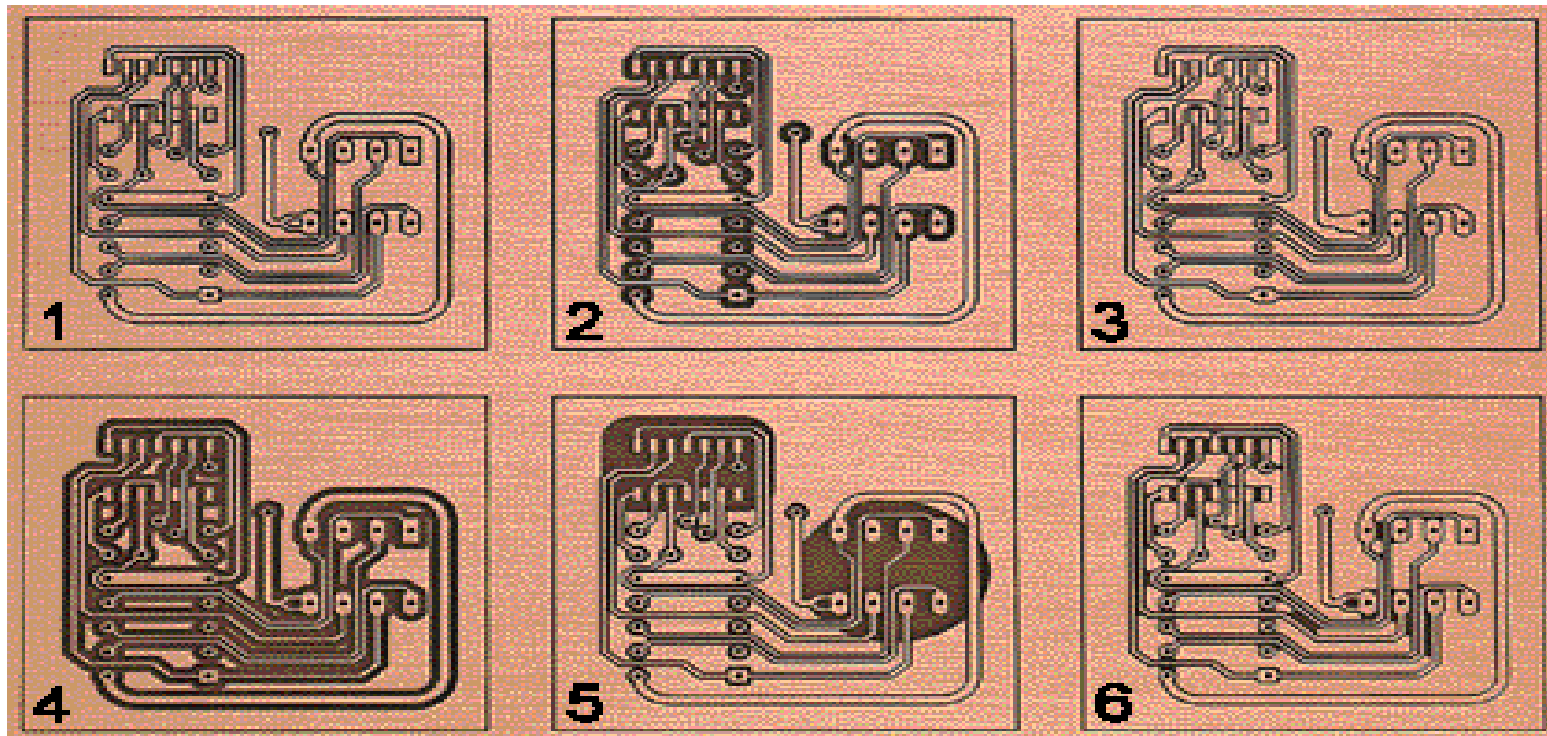
- 'BoardMaster' software:
 - Import LMD data from 'CircuitCAM'
 - Adjust board size and position of prototype
 - Select the order of production phase
 - Smart tool management (drill/mill/dispense/vacuum)
 - Enable 'auto-contact' or 'Vacuum' operation

Tools used by Protomate C60

- Universal Cutter : engrave isolation (min 0.2mm)
- RF Cutter : for RF/ μ -wave structures (RF motor)
- Spiral Drill : for drilling (0.3 mm-2.4 mm diameter)
- Micro Cutter : engrave isolation (min 0.1mm)
- Contour Router : milling contour (like guillotine)
- End Mill : wider insulation or fine milling

The Versatile Isolation Options

- **Standard Isolation** : the fastest production time for simple boards
- **Pad Clearance Isolation** : for easier soldering of SMD components
- **Micro Cut Isolation** : Micro cutter used only where necessary
- **Track Clearance Isolation** : wider spacing for high voltage or capacitance
- **Rubout** : complete or partial removal of residual copper
- **Spike removal** : as 1st item but removing unstable small areas

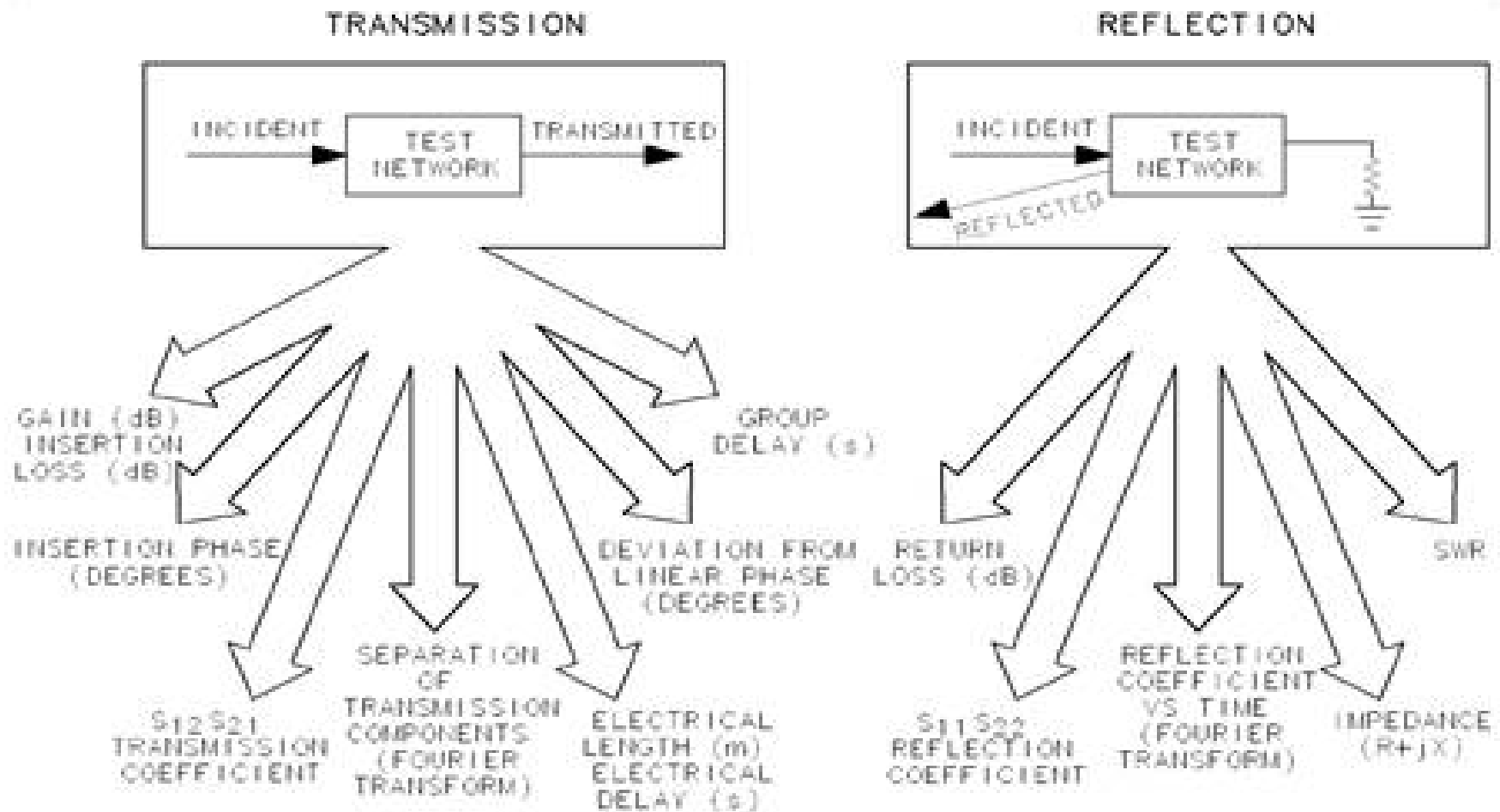


Testing MW/RF filters
using
'HP8510C Network Analyzer'

Vector network analysis is a method of accurately characterizing microwave components (amplifiers, diodes, mixers, duplexers, filters, couplers, attenuators etc.) by measuring their effect on the amplitude and phase of the swept-frequency and the swept-power test signals

- Measuring both magnitude & phase of components are important: to fully characterize a linear network and ensure distortion-free transmission; to measure complex impedance in-order to design efficient matching networks; to develop accurate models for computer-aided-engineering circuit simulators; etc.....
- HP 8510C vector network analyzer (VNA) can measure and display important network characteristics for frequencies of 45MHz - 30GHz
- VNA hardware consists of: (a) a sweeping signal source (usually internal), (b) a test set to separate forward and reverse test signals, and (c) a multi-channel, phase-coherent, highly sensitive receiver.

- **Reflection measurements:** are made by comparing reflected signal to incident signal & used for following characteristics of test network
- **Transmission measurements:** are made by comparing transmitted signal to incident signal & used to find following device characteristics



Minimum hardware configuration of 8510C:

- **Source:** The source provides the RF signal using an Agilent 8360 Series synthesized sweeper, or an 834x Series synthesized sweeper, or an 835x-Series sweep oscillator with an appropriate plug-in.
- **Test Set:** The test set separates the source signal into an incident signal (*sent to the DUT*) and a reference signal (*used to compare transmitted and reflected signals*)
- **Network:** is the Display/Processor and the IF/Detector (Receiver). It processes and display the signals.
- **Printer or plotter and disc drive.**

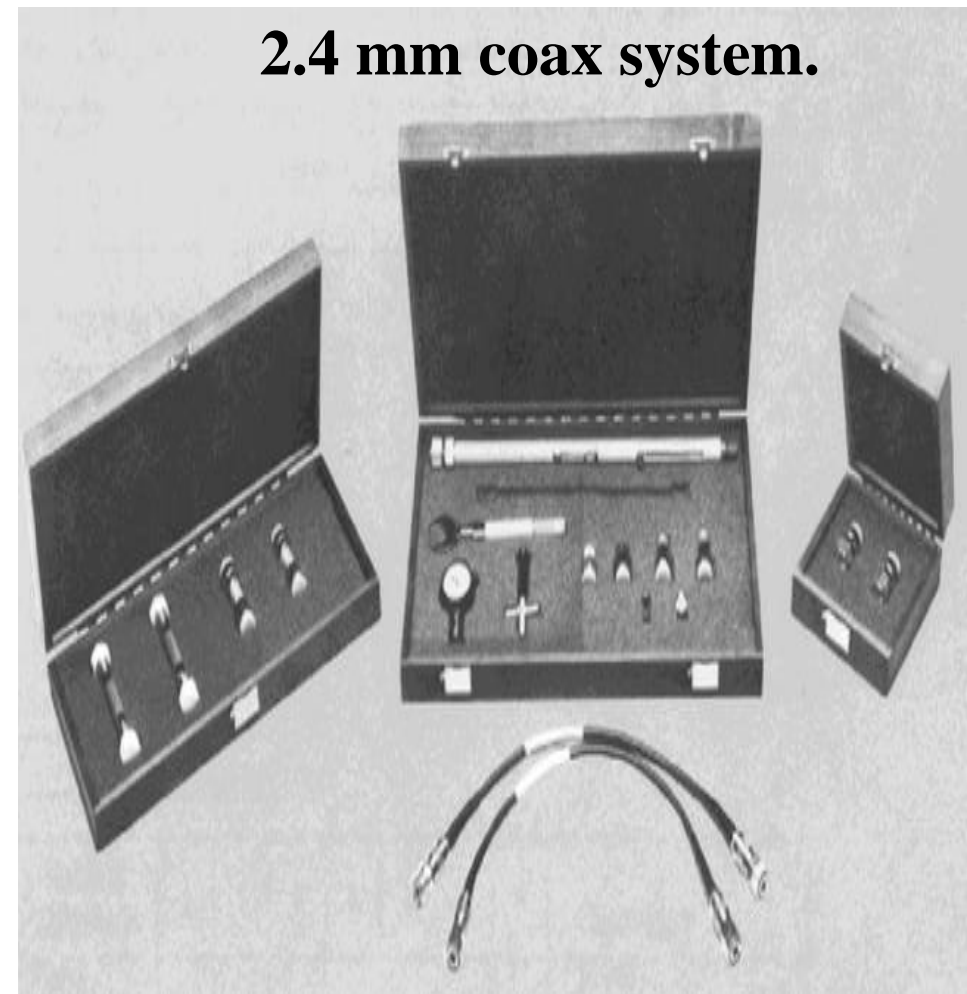


Vector Network Analyzer Calibration:

- **Why needed:** To compensate for the frequency dependent phase and amplitude characteristics of the cables and receiving system, it is necessary to calibrate using the known responses of standard loads, such as open-circuit, short-circuit & matched-load (50Ω) terminations

- **Type of available calibrations:**

- (1) RESPONSE
- (2) RESPONSE & ISOLATION
- (3) S_{11} for 1-PORT
- (4) S_{22} for 1-PORT
- (5) ONE-PATH for 2-PORT
- (6) FULL 2-PORT
- (7) TRL 2-PORT



- **Errors corrected by each type of Calibration:**
- **Response Calibration:** provides signal path frequency response error correction for the selected parameter. This model may be adequate for transmission measurements of well matched, low loss devices and for some reflection measurements where vector normalization of magnitude & phase frequency response errors provides enough measurement accuracy.
- **Response and Isolation Calibration:** This error model adds correction of the leakage term, either directivity for reflection measurements, or crosstalk for transmission measurements. This model is appropriate for transmission measurements of well matched, high loss devices and for reflection measurements where mismatch is not a large factor.
- **1-Port Calibration:** error model provides directivity, source match, and reflection signal path frequency response vector error correction for reflection measurements. This model is best applied to high accuracy

-reflection measurements of one-port devices.

- **Full 2-Port & TRL 2-Port Calibration:** provides error model for full directivity, isolation, source match, load match, and frequency response vector error correction for transmission and reflection measurements of two-port devices on S-Parameter test sets. This model provides best magnitude and phase measurement accuracy for two-port devices but requires measurement of all four S-parameters of the two-port device.

- **S_{11} Frequency Response Calibration:**

- (1) Press CAL → 2.4m coax → S_{11} → for _ CAL 1 to start calibration process.

- (2) At Port 1, connect a short, a open circuit and a load as required.

- (3) When trace is correct, press SHORT, OPEN & LOAD. (S_{11} data is measured)

- (4) Press DONE Calibration, then select a CAL SET (1 through 8).

- (5) Calibration menu is displayed with _CORRECTION ON_ .

- (6) Corrected S_{11} data is displayed

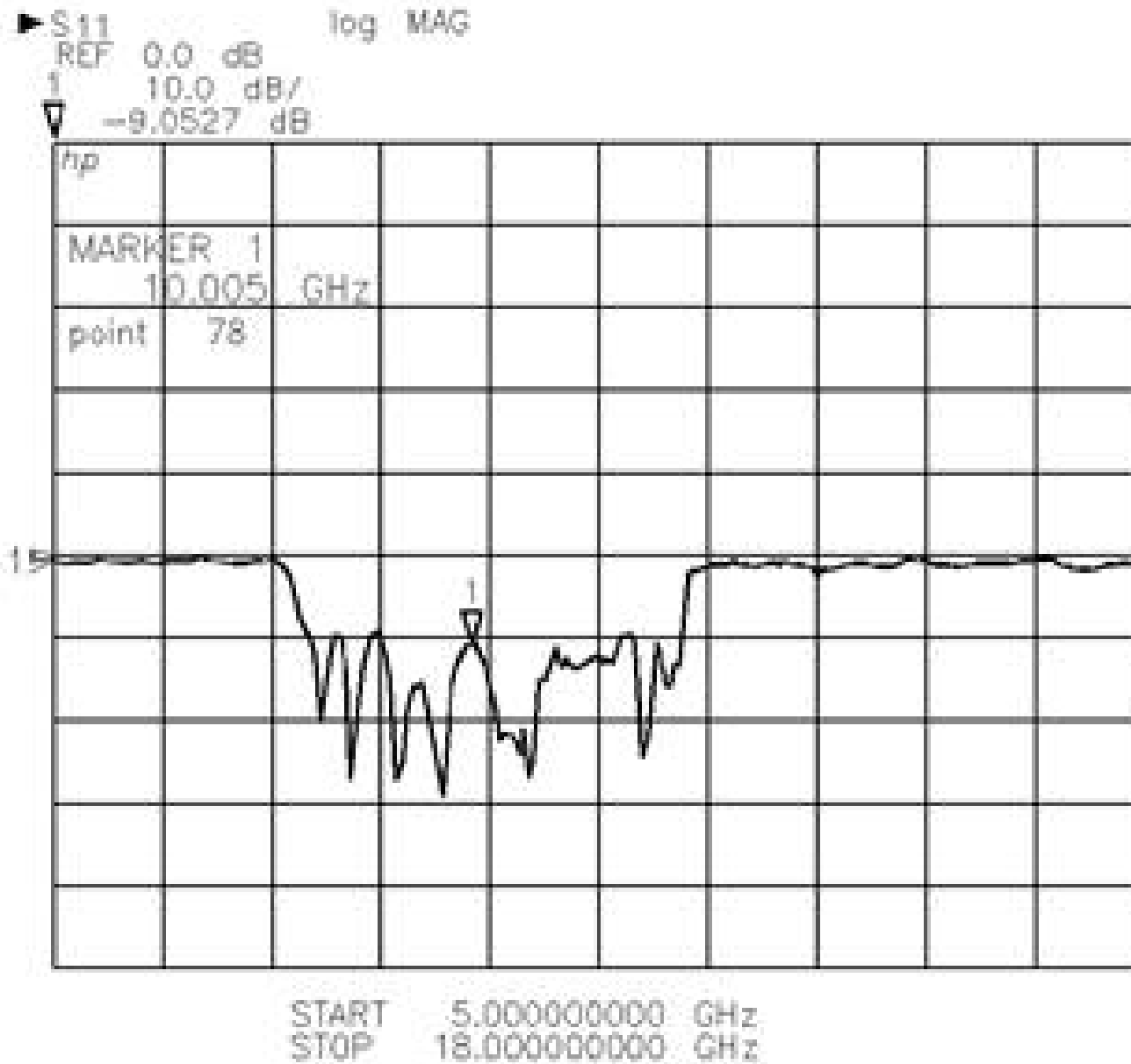
- (7) Connect the test device & **measure S_{11} (of the designed L.P. filter)**

Making a measurement using a Network Analyzer:

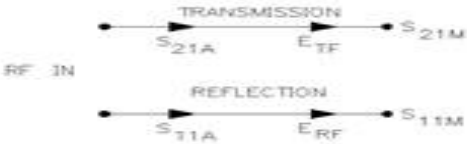
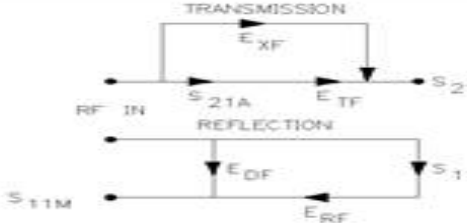
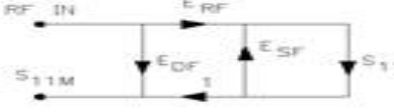
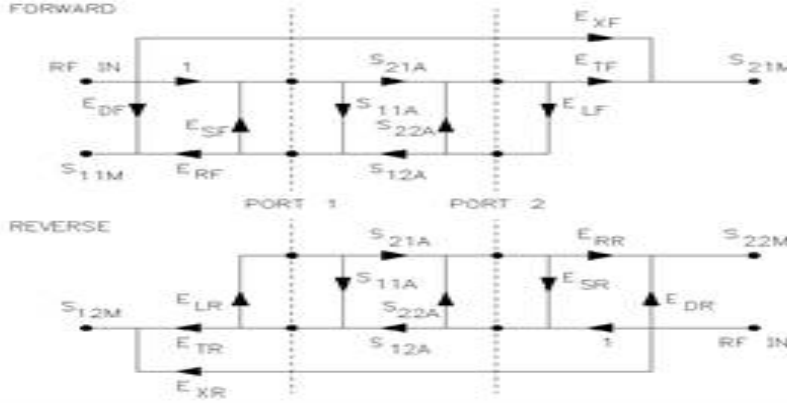
- **Two port S_{11} measurements (in log format):** in frequency-domain:
 1. Connect the DUT (LP filter) between Port 1 and Port 2.
 2. Read the Return Loss of the DUT.
 - (a) Press CHANNEL 1 ; S_{11} ; LOG MAG MARKER.
 - (b) Use the RPG knob to position the marker at any point on the trace.

The measured return loss, magnitude and value at the marker position is displayed above the graticule. The frequency value is displayed in the active function area. See the next page figure for an example of a typical display.
 4. To read the measured phase angle, press PHASE
 5. To position the trace automatically for viewing, press AUTO.

Vector Network Analyzer Display of S_{11} :



Calibration Table of Vector Network Analyzer :

RESPONSE	Selected signal path frequency response.	
RESPONSE & ISOLATION	Selected signal path frequency response and leakage.	
S11 1-PORT	S11 Directivity, Source Match, Frequency Response.	
S22 1-PORT	S22 Directivity, Source Match, Frequency Response.	All error terms with respect to PORT 2.
ONE-PATH 2-PORT (Reflection/Transmission Test Sets)	S11, S21, S12, S22 Directivity, Source Match, Frequency Response, Load Match, Isolation.	All error terms with respect to PORT 1, FORWARD 2-Port model.
FULL 2-PORT (S-Parameter Test Sets)	S11, S21, S12, S22 Directivity, Source Match, Frequency Response, Load Match, Isolation.	
TRL 2-PORT (S-Parameter Test Sets)	S11, S21, S12, S22 Directivity, Source Match, Frequency Response, Load Match, Isolation.	Same as Full 2-Port
When you select one of these cal types, the appropriate standard selection menu appears.		