ECONOMIC LOAD DISPATCH MULTIOBJECTIVE OPTIMIZATION PROCEDURES USING LINEAR PROGRAMMING TECHNIQUES

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ABSTRACT

This paper outlines the optimization problem of real and reactive power, and presents the new algorithm for studying the load shedding and generation reallocation problem in emergencies where a portion of the transmission system is disabled and an a.c. power solution cannot be found for the overloaded system.

The paper describes a novel and efficient method and algorithm to obtain the optimal shift in power dispatch related to contingency states or overload situations in power system operation and planning phases under various objectives such as economy, reliability and environmental conditions.

The optimization procedures basically utilize linear programming with bounded variables and it incorporates the techniques of the Section Reduction Method and the Third Simplex Method.

The validity and effectiveness of the algorithm is verified by means of two examples: a 10-bus system and the IEEE 30-Bus, six generators System.

INTRODUCTION

The main purpose of the economic dispatch of electric energy systems have so far been confined to determine generation schedule that minimizes the total generation and operation cost and does not violate any of the system operating constraints such as line overloading, bus voltage profiles and deviations. In general, power system possesses multiple objectives to be achieved such as economic operations, reliability, security and minimal impact on environment, which inherently have different characteristics, and hence conflicting relations hold among these objectives.

A new technique which can be almost generally used in extremizing the quadratic objective function with a linear programming method has been developed. It is considered to be more effective than such a technique as the "Multi-Segment Curve Method" or the Linear Objective Function Method". In addition to the algorithm for calculating real power generation which minimizes the total fuel cost, the algorithm for optimizing bus voltage and reactive power is presented.

As a measure to counter the line overload, the algorithm is developed for settling the Economic Load Dispatch (ELD) condition and the overload elimination condition simultaneously. As a Linear Programming technique, the Third simplex Method has been contrived for reducing execution time and memory size (1-3).

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This paper presents a new method to determine the optimal shift in power dispatch related to contingency states or overload situations in the system. The approach incorporates the condition that the additional cost incurred in shifting generation should be minimized (4-7).

The method used for solving the load flow problem is the Newton-Raphson method. The Sensitivity $K_{\ell i}$ between line flow of line ℓ and power injection of bus i has also been calculated.

In order to demonstrate the effectiveness of the proposed algorithm, two examples: 10 bus system and the IEEE 30 bus, six generators system are considered. Objectives selected vary from economy, security to minimal environmental impact. Numerical results have clearly shown that the optimal solution by means of the proposed algorithm is successfully and favourably compared to the existing techniques and algorithms (8-11).

PROBLEM FORMULATION

The multi-objective power flow optimization problem can be formulated as in the following steps:

1. OPTIMUM SCHEDULING OF REAL GENERATION

The objective function of the optimum scheduling of the real generation is to:

$$\text{Minimize } J = a + b^T P_{gs} + P_{gs}^T [c] P_{gs} \quad (1)$$

$$= \sum_{i=1}^{n_{gs}} (a_i + b_i P_{gi} + c_i P_{gi}^2)$$

where

$$a = \sum_{i=1}^{n_{gs}} a_i \quad ,$$

a_i is the Basic Cost Coefficient for generator i,

n_{gs} is the number of generators inclusive of the slack bus generator.

$$b = \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_i \\ . \\ . \\ b_{n_{gs}} \end{bmatrix}$$, b_i is the Linear Cost Coefficient for generator i

$$c = \begin{bmatrix} c_1 & & & & 0 \\ & c_2 & & & \\ & & \ddots & & \\ & & & c_i & \\ 0 & & & & \ddots \\ & & & & & c_n \end{bmatrix}_{gs}$$ c_i is the Quadratic Cost Coefficient for generator i

$$P_{gs} = \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gi} \\ \vdots \\ P_{gn} \end{bmatrix}_{gs} = \begin{bmatrix} P_g \\ \\ P_{gs} \end{bmatrix}$$

Taking the perturbation of the both sides of Eqn. 1, we get

$$J+\Delta J = a + b^T(P_{gs}+\Delta P_{gs}) + (P_{gs}+\Delta P_{gs})^T[c](P_{gs}+\Delta P_{gs}) \qquad (2)$$

then

$$\Delta J = b^T \Delta P_{gs} + P^T_{gs}[c]\Delta P_{gs} + \Delta P^T_{gs}[c]P_{gs} + \Delta P^T_{gs}[c]\Delta P_{gs}$$

$$= (b^T + 2P^T_{gs}[c])\Delta P_{gs} + \Delta P^T_{gs}[c]\Delta P_{gs} \qquad (3)$$

As the value of P_{gs} approach to the optimal point, ΔP_{gs} becomes so small that we can neglect the term of $(\Delta P_{gs})^2$. Then we have the new objective function

$$\Delta J = b'^T \Delta P_{gs} \qquad (4)$$

where

$$b'^T = b^T + 2P^T_{gs}[c]$$
$$= (b'_1 \; b'_2 \dots b'_k \dots b'_n)_{gs}$$

with $b'_k = b_k + 2c_k P_{gk}$

The control variables ΔP_{gi} must obey:

$$P^m_{gi} \le P^o_{gi} + \Delta P_{gi} \le P^M_{gi}$$
$$P^m_{gi} - P^o_{gi} \le \Delta P_{gi} \le P^M_{gi} - P^o_{gi} \qquad (5)$$

where

- P^o_{gi} is the initial value of real power generated at generating bus i
- P^M_{gi} is the upper limit of real power generated at generating bus i
- P^m_{gi} is the lower limit of real power generated at generating bus i

Since some of the ΔP_{gi} may be negative and linear programming can only operate on nonnegative variables, new variables should be introduced through :

$$X_{gi} \triangleq \Delta P_{gi} + P^o_{gi} - P^m_{gi} \qquad (6)$$

so that the conditions of Eqn. 5 reduce to:

$$0 \le X_{gi} \le P^M_{gi} - P^m_{gi} \qquad (7)$$

Then we can express the original variables ΔP_{gi} with respect to the new variables X_{gi} as follows:

$$\Delta P_{gi} = X_{gi} - P^o_{gi} + P^m_{gi} \qquad (8)$$

Substituting of Eqn. 8 into Eqn. 4, we have

$$\Delta J = b'(X_{gs} - P^o_{gs} + P^m_{gs})$$
$$= b' X_{gs} + b'(-P^o_{gs} + P^m_{gs}) \qquad (9)$$

Since the constant terms of the objective function to be extremized are insignificant, we can use the following objective function

$$\Delta J = b' X_{gs} = \sum_{k=1}^{n_{gs}} b'_k X_{gk} \qquad (10)$$

which is a linear function with respect to the new variables X_{gi}.

CONSTRAINTS

The Newton-Raphson method formulates a set of linear equations expressing the real and reactive power injection errors at buses as follows:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \qquad (11)$$

where, H, N, J, L are Jacobian matrices defined as $\frac{\partial P}{\partial \theta}$, $\frac{\partial P}{\partial V}$, $\frac{\partial Q}{\partial \theta}$ and $\frac{\partial Q}{\partial V}$ respectively (4-7).

In the real power optimization procedure, setting $\Delta V = 0$ firstly. The relationships of ΔP, ΔQ with $\Delta\theta$ can be expressed by the following equations (3-5):

$$\Delta P = [H]\Delta\theta \qquad (a)$$
$$\Delta Q = [J]\Delta\theta \qquad (b) \qquad (12)$$

(i) Real Power Balance Condition

The real power balance condition is defined as the Equality Constraints. The slack bus real power balance condition can be written similar to Eqn. (12(a)) as follows:

$$\Delta P_s = [H_s]\Delta\theta \qquad (13)$$

From Eqn. (12(a)), we have

$$\Delta\theta = [H]^{-1}\Delta P = [\mathcal{H}_g \mid \mathcal{H}_c \mid \mathcal{H}_\ell] \begin{bmatrix} \Delta P_g \\ \Delta P_c \\ \Delta P_\ell \end{bmatrix} \qquad (14)$$

where $\mathcal{H}$ is the inverse of the Jacobian matrix H (3-7). $\mathcal{H}_g$, $\mathcal{H}_c$, $\mathcal{H}_\ell$ are matrices consisting of the column vectors of the matrix H pertaining to generation buses, control buses, and load buses, respectively.

Since the bus power of the non-generator buses cannot be changed, i.e. $\Delta P_c = 0$ and $\Delta P_\ell = 0$. Equation (14) can be written as follows:

$$\Delta\theta = [\mathcal{H}_g]\Delta P_g \qquad (15)$$

Substituting Eqn. (15) into Eqn. (13) leads to:

$$\Delta P_s = [H_s][\mathcal{H}_g]\Delta P_g \qquad (16)$$

Using equation (8)

$$\Delta P_{gi} = X_{gi} - P^o_{gi} + P^m_{gi}$$

and substituting in Eqn. (16) in view of Eqn. (8), then

$$X_{gs} - P^{o}_{gs} + P^{m}_{gs} = [H_s][JH_g](X_g - P^{o}_{g} + P^{m}_{g})$$

or expressed in the form

$$[H_s][JH_g]X_g - X_{gs} = [H_s][JH_g](P^{o}_{g} - P^{m}_{g}) - P^{o}_{gs} + P^{m}_{gs} \qquad (17)$$

(ii) Line Overload Prevention Condition

The line overload prevetion condition is defined as the Inequality Constraint (4,5). The overloading in a transmission line can lead to system collapse in an extreme case. The following condition must be satisfied for preventing the line overload; as a security constraint or security index:

$$Z_{\ell} + \sum_{i=1}^{n_{gs}} k_{\ell i}\,\Delta P_{gi} \le U_{\ell} \qquad (18)$$

where

Z_{ℓ} is the line flow through line ℓ

U_{ℓ} is the upper limit of transmission power of line ℓ

and $K_{\ell i}$ is the sensitivity between line flow of line ℓ and power injection of bus i (i.e. $\Delta Z_{\ell}/\Delta P_i$)

Substituting from Eqn. (8) for ΔP_{gi} in Eqn. (18), we get:

$$Z_{\ell} + \sum_{i=1}^{n_{gs}} K_{\ell i}(X_{gi} - P^{o}_{gi} + P^{m}_{gi}) \le U_{\ell}$$

or

$$\sum_{i=1}^{n_{gs}} K_{\ell i} X_{gi} \le U_{\ell} - Z_{\ell} + \sum_{i=1}^{n_{gs}} K_{\ell i}(P^{o}_{gi} - P^{m}_{gi}) \qquad (19)$$

The problem size can be reduced by taking into consideration only the constraints of the lines whose loading is approaching their maximum rating U_{ℓ}. It can be taken as 90% of normal loading (6,7).

(iii) Upper/Lower Bounds on Each Generator Output Power:

Rewriting Eqn. (17) in a compact form leads to:

$$0 \le X_g \le P_g - P^{m}_{g} \qquad (20)$$

The Simplex Method with Bounded variables handles these constraints on the control variables implicitly without increasing the problem size.

(iv) Environmental Impact (9):

Generation emission can be taken as an index for environment conservation. The following condition must be satisfied:

$$\alpha + \beta P_G + \gamma P_G^{2} + \xi \exp^{\epsilon P_G} \le V$$

where α, β, γ, ξ, ϵ are coefficient of generator emission characteristic and V is the allowable upper limit.

2. OPTIMUM SCHEDULING OF REACTIVE GENERATION

The objective function of the optimum scheduling of the reactive generation is obtained as follows:

Setting $\Delta\theta = 0$ in Eqn. (11), the changes in the active power and reactive power caused by the bus voltage change can be expressed as follows:

$$\Delta P = [N_{gs}]\ \Delta V$$
$$\Delta Q = [L]\ \Delta V \qquad (21)$$

where:

$[N_{gs}]$ is the $(n_{gs} \times n_b)$ matrix consisting of the rows of the Jacobian matrix [N] pertaining to the generator buses.

Substituting Eqn. (21) for ΔP_{gs} into Eqn. (4) leads to

$$\Delta J = b'^{T}\ [N_{gs}]\,\Delta V$$

$$= b''^{T}\,\Delta V = \sum_{k=1}^{n_b} b''_k\,\Delta V_k \qquad (22)$$

where: $b''^{T} \triangleq b'^{T}[N_{gs}] = (b''_1, b''_2, \text{---------}, b''_{nb})$.

The control variable ΔV_i must satisfy the following conditions:

$$V^{m}_{i} \le V^{o}_{i} + \Delta V_i \le V^{M}_{i} \qquad (23)$$

Thus, by introducing the new nonnegative variables X_{vi}, Eqn. (23) is reduced to:

$$0 \le X_{vi} \le V^{M}_{i} - V^{m}_{i} \qquad (24)$$

where:

$$X_{vi} \triangleq \Delta V_i + V^{o}_{i} - V^{m}_{i}, \qquad (25)$$

and $\Delta V_i = X_{vi} - V^{o}_{i} + V^{m}_{i}$

Substituting Eqn. (25) into Eqn. (22) and eliminating the constant terms, the linear objective function , with respect to, the new variable is obtained as:

$$\Delta J = b'' X_v \qquad (26)$$

CONSTRAINTS

(i) Conditions on Reactive Power

Separating the Eqn. (21) into two parts, one pertaining to the voltage-controlled buses and the other pertaining to the load buses, we get:

$$\begin{bmatrix} \Delta Q_{gc} \\ \Delta Q_{\ell} \end{bmatrix} = \begin{bmatrix} L_{gc} \\ L_{\ell} \end{bmatrix} .\Delta V \qquad (27)$$

where

$[L_{gc}]$ and $[L_{\ell}]$ are the matrices consisting of the rows of the matrix [L] pertaining to the voltage-controlled buses and the load buses respectively.

For the voltage-controlled buses, the reactive power must be inside the permissible range given by:

$$Q^{m}_{gc} \le Q^{o}_{gc} + \Delta Q_{gc} \le Q^{M}_{gc} \qquad (28)$$

For the load buses, the reactive power cannot be changed; i.e.

$$\Delta Q_{\ell} = 0 \qquad (29)$$

Substituting Eqn. (27) into Eqns. (28), (29) leads to

$$\begin{aligned} Q^{o}_{gc} + [L_{gc}]\ \Delta V &\le Q^{M}_{gc} \\ Q^{o}_{gc} + [L_{gc}]\ \Delta V &\le Q^{m}_{gc} \\ [L_{\ell}]\ \Delta V &= 0 \end{aligned} \qquad (30)$$

or in the form of Eqn. (25) as follows:

$$\begin{aligned} [L_{gc}]X_v &\le Q^{M}_{gc} - Q^{o}_{gc} + [L_{gc}](V^{o} - V^{m}) \\ [L_{gc}]X_v &\le Q^{m}_{gc} - Q^{o}_{gc} + [L_{gc}](V^{o} - V^{m}) \\ [L_{\ell}]X_v &= [L_{\ell}](V^{o} - V^{m}) \end{aligned} \qquad (31)$$

(ii) Upper and Lower Bounds on Each Bus Voltage:

Rewriting Eqn. (24) in a compact form such as:

$$0 \le X_v \le V^M - V^m \tag{32}$$

which can be implicitly treated without increasing the problem size.

3. CONTINGENCY ANALYSIS

When it is impossible to dispatch the load without overloading the lines, the solution of the linear programming problem in the real power optimization procedure appears to be infeasible. In such cases, the solution can be feasible by assuming the possibility of load shedding. In this case, the objective function is written as the sum of the following two terms: the first is the objective function in the real power optimization procedure, i.e. Eqn. (10) and the second is a weighted sum of load shedding quantities given as:

$$J = \sum_{k=1}^{n_{gs}} b'_k X_{gk} + \sum_{j=1}^{n_l} (WF)_j X_{\ell j} \tag{33}$$

where:

$X_{\ell j} = \Delta P_{\ell j}$ is the variable representing the load shedding quantity $\Delta P_{\ell j}$ of the load bus j, which is nonnegative and needs not be replaced, and

$(WF)_j$ is the weighting factor assigned to reflect the load priority of load bus j, which must be made greater than the largest values of b' for k = 1, 2, ... n_{gs}.

CONSTRAINTS

(i) Real Power Balance Condition:

Since the power change quantity of a bus equals to the sum of the power generation change and the load change, we can write

$$\Delta P = \Delta P_G + \Delta P_L \tag{34}$$

$$\Delta P_S = \Delta P_{GS} + \Delta P_{LS} \tag{35}$$

In analogy with Eqns. (16), (17), it is possible to rewrite:

$$\Delta P_{GS} + \Delta P_{LS} = [H_S][\mathcal{H}](\Delta P_G + \Delta P_L) \tag{36}$$

or in view of Eqn. (8) and the definition of $X_\ell \overset{\Delta}{=} \Delta P_L$,

$$X_{GS} - P^o_{GS} + P^m_{GS} + X_{\ell S} = [H_S][\mathcal{H}](X_G - P^o_G + P^m_G + X_\ell); \text{ or}$$

$$[H_S][\mathcal{H}](X_G + X_\ell) - X_{GS} - X_{\ell S} = [H_S][\mathcal{H}](P^o_G - P^m_G) + P^o_{GS} - P^m_{GS} \tag{37}$$

(ii) Line Overload Prevention Condition

In the same manner as in the case of real power optimization, we get:

$$Z_\ell + \sum_{i=1}^{n_b} K_{\ell i}(X^o_{Gi} - P^m_{Gi}) + \sum_{j=1}^{n_b} K_{\ell j} X_{\ell j} \le U_\ell \text{ , or}$$

$$\sum_{i=1}^{n_b} K_{\ell i} X_{Gi} + \sum_{j=1}^{n_b} K_{\ell j} X_{\ell j} \le U_\ell - Z_\ell + \sum_{i=1}^{n_b} K_{\ell i}(P^o_{Gi} - P^m_{Gi}) \tag{38}$$

(iii) Bounds on X_{Gi} and X_{Li}

The bounds on X_{Gi} and $X_{\ell i}$ can be presented as:

$$0 \le X_{Gi} \le P^M_{Gi} - P^m_{Gi}$$

$$0 \le X_{\ell i} \le X^M_{\ell i}$$

where

$X^M_{\ell i}$ is the upper limit of load shedding quantity ΔP_{Li} at bus i.

SECTION REDUCTION METHOD

At the first iteration, the temporary section of each control variable is set to be the part of the original range within the distance of λD from the initial point of the control variable, where D denotes the width of the original range between upper and lower limits of the control variable. The value λ is determined according to the estimated maximum distance between the initial points and the optimal points ($0.1 \le / \lambda \le 1.0$).

As the iteration is repeated, the temporary section of the next iteration is reduced to k times as wide as that of the previous iteration.

In general, it is reasonable to choose k as 0.5, but may be somewhat adjusted in accordance with the circumstances. The constant k is called the Section Reduction Factor.

The procedure for extremizing the problem of n generators is depicted in Figure (1).

THIRD SIMPLEX METHOD AND SOLUTION PROCEDURE

It is possible to develop the modified simplex method by reconstructing an identity matrix from the tables at every iteration by extracting the unit vectors and rearranging them. By using this developed method, the execution time and memory size are substantially reduced as a unit vector is generated at every iteration instead of the identity matrix.

In general, the linear programming problem is written as follows.

Minimize $J = CZ$
Subject to $[A] X \ge b$ with $X \ge 0$

where:

- A = [B N] is the constraint matrix.
- B = (C, C) is the Cost Coefficient Vector,
- X = (X, X) is the variable vector,
- b is the basic matrix,
- N is the non-basic matrix, and
- B, N are the underscripts pertaining to basic and non-basic variables respectively.

At the initialization step, the slack and surplus variables, and the artificial variable are introduced as shown in Table (1).

At the main step, the leaving and entering variables are determined, and, after the pivoting operation is carried out, Table (2) is obtained.

Comparing Tables (1) and (2), it is observed that the unit vector of the r-th column has been transferred to the k-th column. If we exchange the unit vector of the k-th column for the new r-th column vector, we obtain the same result by operating with only the right side part of the matrix [I B^{-1} N].

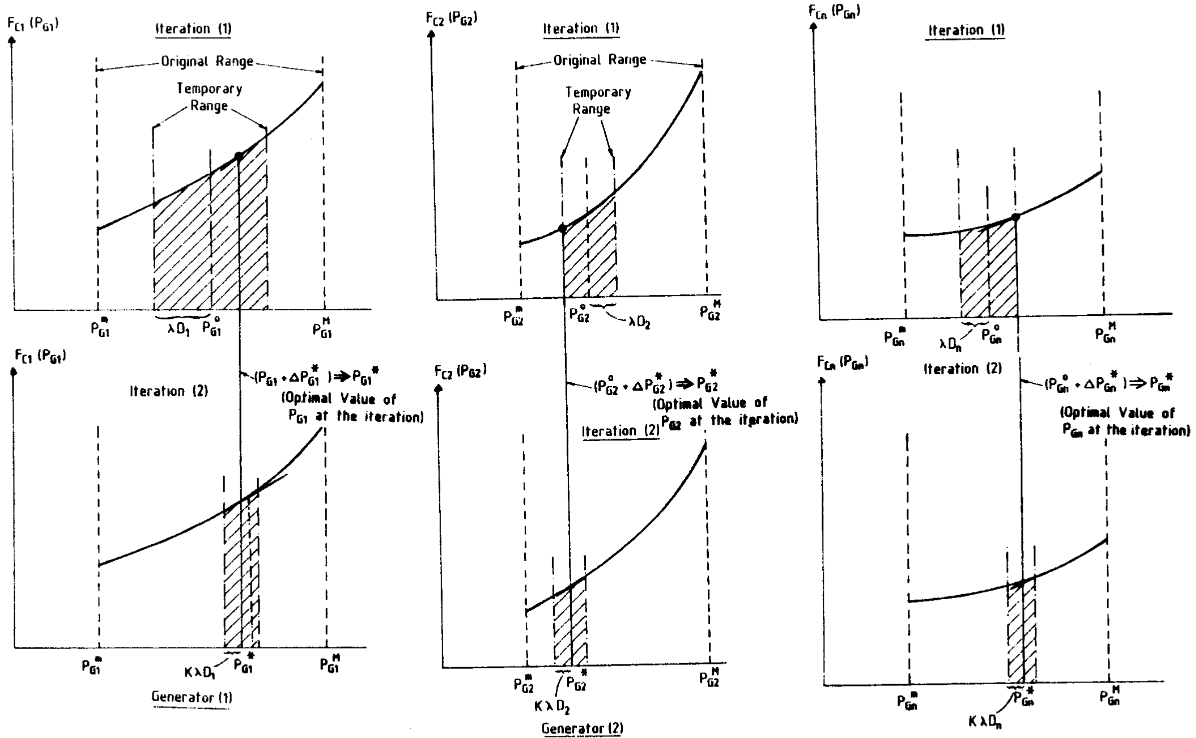

Figure (1) Graphic depiction of the Section reduction Method.

Table (1): Variables at the First Iteration

0	$C_B[B]^{-1}[N] - C_N$	$C_B\, b$
1	$[B]^{-1}[N]$	b

	0	0	0	...	0	0	$...Z_j-C_j$	...	Z_k-C_k	...	C_Bb
x_1	1	0	0	...	...	0	$...Y_{1j}$	...	Y_{1k}	...	b_1
x_2	0	1	0	...	...	0	$...Y_{2j}$	...	Y_{2k}	...	b_2
x_3	0	0	1	...	...	0	$...Y_{3j}$	...	Y_{3k}	...	b_3
⋮	⋮			⋱		⋮					⋮
x_r							$...Y_{rj}$	...	Y_{rk}	...	b_r
⋮	⋮										⋮
x_m	0	0	0	...	...	1	$...Y_{mj}$	...	Y_{mk}	...	b_m

Table (2) Variables at the Next Itertion

	0	0	..	C_k-Z_k/Y_{rk}	..	0	(Z_j-C_j) $-Y_{rj}(Z_k-C_k)/Y_{rk}$	0	$C_Bb-(Z_k-C_k)b_r/Y_{rk}$
x_1	1	0	..	$-Y_{1k}/Y_{rk}$	..	0	$..Y_{rj}-(Y_{rj}/Y_{rk})Y_{1k}$	0	$b_1-(Y_{1k}/Y_{rk})b_r$
x_2	0	1	..	⋮	..	⋮	.. ⋮	...	......
⋮									
x_r	..	..	..	$1/Y_{rk}$	..	0	.. Y_{rj}/Y_{rk}	1	b_r/Y_{rk}
⋮				⋮		⋮	⋮	⋮	
x_m	0	..	..	$-Y_{mk}/Y_{rk}$	..	1	.. $Y_{mj}-(Y_{rj}/Y_{rk})Y_{mk}$	0	$b_m-(Y_{mk}/Y_{rk})b_r$

The number of operations per iteration and the required memory size of the Simplex Method, the Revised Simplex Method, and the Third Simplex Method are shown in Table (3). From the table it can be seen that if m is significantly larger than n, the Third Simplex Method results in a substantial saving in the executing time and memory size.

The iterative technique for the economic load dispatch optimization procedure is shown in figure (2).

Table (3) Comparison of the Three Methods

Method	Operation				Memory size
	Calculation	Pivoting	Auxiliary	Total	
Simplex Method	Multiplication	(m+1) (m+n+1)		$(m+1)^2$+mn+n	(m+1) (m+n+1)
	Addition	m(m+n+1)		m(m+n+1)	
Revised Simplex Method	Multiplication	$(m+1)^2$	mn	$(m+1)^2$+mn	$(m+1)^2$+n+1
	Addition	m(m+1)	mn	m(m+n+1)	
Third Simplex Method	Multiplication	(m+1) (n+2)		(m+1) (n+2)	(m+1) (n+2)+m+n
	Addition	m(n+1)		m(n+1)	

where m is the number of constraints (number of rows of the constraint matrix A)
and n is the number of original variables.

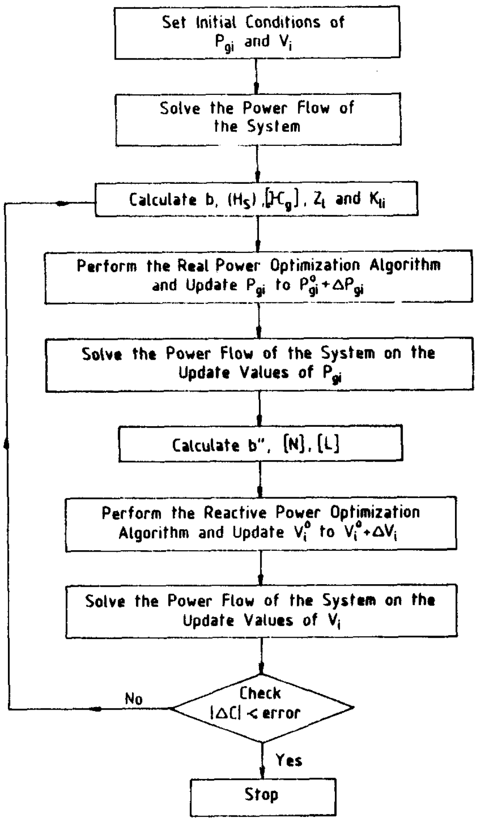

Figure (2): Iterative Procedure

RESULTS ON EXAMPLE (1) SYSTEM STUDIES

A 10-bus model system shown in Figure (3) was used to test the method of scheduling real and reactive power and to study the convergence characteristics of the optimization process prsented in the paper.

The impedance and line charging data is given in Table (4) and the bus voltage and load data are given in Table (5). The operating limits and cost data for each generator and real power generations of each iteration are shown in Table (6). In consequence of continuing the iteration procedure until the cost flucturation ΔC becomes smaller than 0.05% of cost, the number of load flow calculation is 3 times.

Table (4): Line Impedance and Charging Susceptance

Line Number	Bus Number	Line Impedance	Line charging susceptance
1	1-2	0.02 + j0.08	0.03
2	1-6	0.06 + j0.24	0.02
3	1-9	0.04 + j0.16	0.015
4	2-3	0.06 + j0.24	0.02
5	2-6	0.06 + j0.24	0.02
6	3-7	0.06 + j0.24	0.02
7	4-7	0.04 + j0.16	0.015
8	4-8	0.06 + j0.24	0.02
9	5-6	0.04 + j0.16	0.015
10	5-10	0.06 + j0.24	0.02
11	6-9	0.01 + j0.04	0.01
12	8-10	0.04 + j0.16	0.015
13	9-10	0.08 + j0.32	0.025

Table (5): Bus voltage and Connected Load

Bus No.	Bus Type	Bus Voltage Specified (Per unit)	Load	Bus voltage at Iteration # 3
1	2	Not specified	0.2 + j0.097	0.9793
2	2	Not specified	0.3 + j0.145	0.9737
3	2	Not specified	0.2 + j0.097	0.9650
4	2	Not specified	0.3 + j0.145	0.9726
5	2	Not specified	0.2 + j0.097	0.9996
6	3	1.0	0.3 + j0.145	1.0123
7	3	1.0	0.15 + j0.0726	0.9875
8	3	1.0	0.2 + j0.097	1.0125
9	3	1.0	0.2 + j0.097	0.9882
10	1	1.05	0.2 + j0.097	1.0175

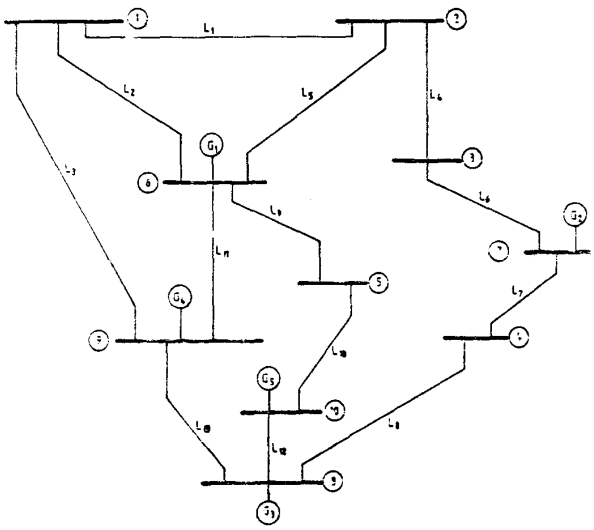

Figure (3): 10 Bus Model System

RESULTS ON EXAMPLE (2) SYSTEM STUDIES

The proposed optimization algorithm is applied to the IEEE 30 Bus test system with 6 generators and 41 lines given in Fig. (4). The system data are given in Tables (7) and (8), as obtained for Reference (9). The results using new algorithm are shown in Tables (9) and (10) for the optimal solution of the subproblem as compared to Table (11) of Reference (9). The numerical results on this power system have verified the validity of the algorithm with respect to the existing ones. The memory size and execution time using the Third Simplex Method has been reduced tremendously.

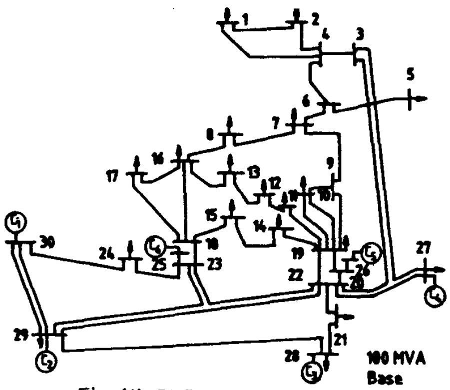

Fig. (4): 30 Bus Test System [9]

Table (6): Optimal Scheduling of Power Generation

Bus No.	Bus No.	Rating of each generator			Convergence of Power generation							
		Cost Function	P_{gi}^{M}	P_{gi}^{m}	Initial	Fuel Cost	Iter # 1	Fuel Cost	Iter # 2	Fuel Cost	Iter # 3	Cost
1	6	$27 + 6P_{g1} + P_{g1}^2$	1.5	0.05	0.6	30.960	0.05	27.303	0.4153	29.664	0.4141	29.656
2	7	$35 + 10P_{g2} + P_{g2}^2$	1.5	0.05	0.45	39.702	0.05	35.503	0.05	35.503	0.05	35.503
3	8	$29 + 5P_{g3} + 0.5P_{g3}^2$	1.5	0.05	0.35	30.811	1.075	34.953	1.0421	34.754	1.2235	35.866
4	9	$31 + 8P_{g4} + p_{g4}^2$	1.5	0.05	0.5	35.250	0.05	31.403	0.05	31.403	0.05	31.403
5	10	$28 + 6P_{g5} + 0.5P_{g5}^2$	1.5	0.05	0.37	30.289	1.1445	35.522	0.762	32.862	0.5863	31.689
Total Fuel Cost						167.012		164.682		164.185		164.177

CONCLUSIONS

An optimization technique has been presented for the economic allocation of real and reactive power applying the linear programming method and an algorithm has been developed for finding a post-emergency schedule with the minimum of load shedding.

Satisfactory results are obtained by adapting the program to the 10-bus model system and the 30-bus model system and found that it is very similar and even better to that of other optimum dispatch methods.

The technique of linear programming is shown to be a powerful and practical tool for obtaining an approximate solution of a linearized optimization problem. Above all, the Section Reduction Method and the Third Simplex Method may have applications elsewhere.

But, the confirmation of its usefulness must await further testing on actual power systems. Extensions and refinements on the present algorithm are expected to include reliability indices. As an example, the algorithm relies on the convergence of the power flow at each iteration and therefore may be interrupted by a single solution divergence. The efficiency can also be improved by exploiting sparsity of the problem.

Table (8): Line flow capacity

Line No.	P_i^S	Line No.	P_i^S	Line No.	P_i^S
1	0.20	15	0.70	29	0.20
2	0.20	16	0.50	30	0.20
3	0.20	17	0.70	31	0.20
4	0.20	18	0.50	32	0.50
5	0.20	19	0.50	33	0.50
6	0.20	20	0.50	34	0.50
7	0.10	21	0.50	35	0.15
8	0.15	22	0.50	36	0.50
9	0.15	23	0.50	37	0.50
10	0.50	24	0.50	38	0.50
11	0.50	25	0.20	39	0.50
12	0.50	26	0.20	40	0.50
13	0.30	27	0.20	41	0.50
14	0.50	28	0.20		[p.u.]

Line capacity is 110% of standard value (P_i^S)

Table (7): Specified bus data

Bus	Type	Active Power	Reactive Power	Bus Voltage
1	P-Q	-0.106	-0.019	-
2	P-Q	-0.024	-0.009	-
3	P-Q	0.000	0.000	-
4	P-Q	0.000	0.000	-
5	P-Q	-0.035	-0.023	-
6	P-Q	0.000	0.000	-
7	P-Q	-0.087	-0.067	-
8	P-Q	-0.032	-0.016	-
9	P-Q	0.000	0.000	-
10	P-Q	-0.175	-0.112	-
11	P-Q	-0.022	-0.007	-
12	P-Q	-0.095	-0.034	-
13	P-Q	-0.032	-0.009	-
14	P-Q	-0.090	-0.058	-
15	P-Q	-0.035	-0.018	-
16	P-Q	-0.082	-0.025	-
17	P-Q	-0.062	-0.016	-
18	P-Q	-0.112	-0.075	-
19	P-Q	-0.058	-0.020	-
20	P-Q	0.000	0.000	-
21	P-Q	-0.228	-0.109	-
22	P-Q	0.000	0.000	-
23	P-Q	-0.076	-0.016	-
24	P-Q	-0.024	-0.012	-
25	P-V	0.000	0.000	1.071
26	P-V	0.000	0.000	1.082
27	P-V	-0.300	-	1.010
28	P-V	-0.942	-	1.010
29	P-V	-0.217	-	1.045
30	S	0.000	0.000	1.060

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Table (9): Optimal Solution for Economy Subproblem

Gen No.	Bus No.	Rating of each generator: Cost function $F = A P_G^2 + B P_G + C$ ($/h)	P^M_{gi}	P^M_{gl}	Convergance of Power generation and Forced Outage: Initial	Cost	Iter # 1	Cost	Iter # 2	Cost	Iter # 3	Cost	Iter # 4	Cost	Iter # 5	Cost
1	30	$100P_G^2 + 200P_G + 10$	1.5	0.05	0.10	31.0	0.10	31.0	0.25	66.25	0.20	54.00	0.15	42.25	0.15	42.25
2	29	$120P_G^2 + 150P_G + 10$	1.5	0.05	0.25	55.00	0.10	26.20	0.30	65.80	0.30	65.80	0.30	65.80	0.30	65.80
3	28	$40P_G^2 + 180P_G + 20$	1.5	0.05	0.20	57.60	0.60	142.40	0.35	87.90	0.50	120.00	0.50	120.00	0.55	131.10
4	27	$60P_G^2 + 100P_G + 10$	1.5	0.05	1.35	254.35	1.20	216.40	1.10	192.60	1.10	192.60	1.10	192.60	1.05	181.15
5	26	$40P_G^2 + 180P_G + 20$	1.5	0.05	0.36	87.90	0.60	142.40	0.36	87.90	0.36	87.90	0.46	109.10	0.46	109.10
6	25	$100P_G^2 + 150P_G + 10$	1.5	0.05	0.6	136.00	0.26	55.76	0.50	110.00	0.40	86.00	0.35	74.75	0.35	74.75
Total Cost						621.85		614.16		610.45		606.30		604.50		604.15

Table (10) Optimal Solution for Emission Subproblem

Gen No.	Bus No.	Emission Characteristic of generators $F = a + b P_G + c P_G^2 + d \exp^{ePG}$	Initial	Total ton/h	Iter # 1	Total	Iter # 2	Total	Iter # 3	Total
1	30	$10^{-2}(4.091 - 5.554 P_G + 6.490 P_G^2) + 2*10^{-4} \exp^{2.857 PG}$	0.45	0.02978	0.38	0.02977	0.42	0.02970	0.40	0.02971
2	29	$10^{-2}(2.543 - 6.047 P_G + 5.638 P_G^2) + 5*10^{-4} \exp^{3.333 PG}$	0.50	0.01194	0.51	0.01199	0.48	0.01187	0.45	0.01188
3	28	$10^{-2}(4.258 - 5.094 P_G + 4.586 P_G^2) + 1*10^{-6} \exp^{8.0 PG}$	0.60	0.02685	0.57	0.02854	0.53	0.02853	0.55	0.02852
4	27	$10^{-2}(5.326 - 3.550 P_G + 3.38 P_G^2) + 2*10^{-3} \exp^{2.0 PG}$	0.40	0.04892	0.42	0.04895	0.37	0.04894	0.40	0.04892
5	26	$10^{-2}(4.258 - 5.094 P_G + 4.586 P_G^2) + 1*10^{-6} \exp^{8.0 PG}$	0.60	0.02865	0.53	0.02853	0.53	0.02853	0.55	0.02852
6	25	$10^{-2}(6.131 - 5.555 P_G + 5.151 P_G^2) + 1*10^{-5} \exp^{6.667 PG}$	0.30	0.04936	0.44	0.04694	0.52	0.04667	0.50	0.04669
Total				0.19730		0.19472		0.19424		0.19424
Total Cost				643.65		638.33		644.82		639.60

Table (11): Optimal solution of subproblems[9]

Minimized objectives		Generation cost minimum	Emission minimum	Line overload index minimum
Total generation cost ($/h)		606.04	645.88	640.62
Total emission (ton/h)		0.2215	0.1952	0.2350
Total flow deviation (p.u.)		0.0381	0.7778	0.0
Generator output (p.u.)	1	0.124	0.381	0.589
	2	0.310	0.515	0.300
	3	0.543	0.562	0.525
	4	1.016	0.399	1.055
	5	0.514	0.522	0.355

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