EFFECT OF INSPECTION ERRORS ON THE PERFORMANCE OF INSPECTION PLANS IN QUALITY CONTROL SYSTEMS

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ABSTRACT

Quality control systems can be divided into process control and product control. Process control is accomplished using process-control-techniques such as control charts and process targeting. Product control is achieved by inspection plans. Inspection plans play a vital role in product control and must be used prudently. In this paper, performance measures of inspection will be presented for single, double and repeat inspection plans. The impact of inspection error on these plans will be evaluated through the plans' performance measures. Recommendations will be made in order to guard against the effect of inspection errors and develop a better strategy to control it.

Keywords: *Quality Control, Inspection Error, Double Sampling Plans, Repeat Inspection Plans.*

الملخص

تنقسم أنظمة ضبط الجودة إلى كل من ضبط العمليات وضبط المنتجات. تنجز مهام ضبط العمليات بأساليب ضبط العمليات مثل خرائط التحكم وتحديد قيم العمليات. ضبط المنتجات يتم من خلال خطط الفحص. تلعب خطط الفحص دوراً أساسياً في ضبط المنتجات ويجب استخدامها بحرص، تعرض هذه الورقة مقاييس الأداء لكل من خطط الفحص المفرد والمزدوج والمتكرر. يتم تقييم آثار أخطاء الفحص على هذه الخطط بواسطة مقاييس أداء الفحص. وتقدم بعض التوصيات لتلافي أخطاء الفحص وتطوير استراتيجية أفضل للتحكم في آثار الأخطاء على فعالية خطط الفحص.

1. ROLE OF INSPECTION PLANS

Inspection of raw materials, semi finished products, or finished products are an important part of quality control. Inspection plans are designed for the purpose of acceptance or rejection of a product, based on adherence to specifications. In the literature, there are several types of inspection plans. The widely used classes of inspection plans are the single, double and repeat inspection plans. These plans are used to ensure product quality. Several factors are usually considered in designing an inspection plan [Tang, K. and Tang, J., 1994]. These factors include the goal to be accomplished, the nature of the performance variables, available information on the population, and economical and manufacturing environments. As a result, the complexity of the design issue is affected be these factors. For example, it can be as simple as designing a single sampling plan, or as complicated as designing a system of screening for a multi-stage manufacturing process.

Two separate objectives have been commonly used to design inspection plans. One is to optimize the expected total profit associated with an inspection procedure, and the other is to use inspection to reach certain statistical goals, such as controlling the outgoing nonconforming rate of the product. The methods using these objectives are known as economic and statistical designs of inspection plans, respectively. In an economic design three cost components are commonly considered: the cost of inspection, cost of rejection, and the cost of acceptance. The cost of inspection may include expenses of testing materials, labor, equipment, and so forth. The cost of rejection is incurred by false rejection of good components (i.e. classifying these components as scrap or rework) and by corrective actions taken on these items, such as repairing, scrapping, or returning the items to the supplier. The cost of acceptance is caused by the items of imperfect quality that reach the customers. This may include damage caused by product failure, warranty cost, handling cost, loss in sales, loss in goodwill, and so forth. On the other hand, in the statistical design of an inspection plan, the most commonly used criterion is the outgoing conforming rate. Note that when inspection is error-free, the outgoing conforming rate should be 100% after inspection. However, the outgoing conforming rate becomes a meaningful and important design criterion when nonconforming items may not he detected because of inspection error or for other reasons. Further note that economic factors are usually considered implicitly in selecting statistical goals. For example, the outgoing conforming rate should be set at a high level when the cost of accepting nonconforming items is large. In fact, it also possible to incorporate both the economical and statistical criteria in designing an inspection plan. For example, one may want to minimize the total related cost and, at the same time, require the outgoing conforming rate to be above a given level [Tang, K. and Tang, J., 1994].

The performance of an inspection plan is greatly influenced by inspection errors. An inspector can commit two types of errors. Type I error is the probability of classifying a non-defective item as defective and type II error is the probability of classifying a defective item as non-defective. These errors may have an adverse effect on the ability of an inspection plan to ensure product quality. The novel aspect of this paper is to develop a unified approach for evaluating inspection plan and based on the evaluation, strategies are devised to improve the ability of these plans to provide effective product control.

The purpose of the paper is to provide an effective product control strategy in the presence of inspection errors. In this paper, performance measures of inspection will be presented for single, double and repeat inspection plans. The impact of inspection error on these plans will be evaluated through the plans' performance measures. Recommendations will be made in order to guard against the effect of inspection errors.

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2. PERFORMANCE MEASURES

The objective of the performance measures is to examine how well an inspection plan is accomplishing the required results. There are many performance measures used in the literature but we will be using AOQ and ATI because they are critical to all inspection plans.

The performance measure average outgoing quality (AOQ) of the inspection process is defined as the ratio between the number of defective components going out of inspection and the number of accepted components. It measures the percentage defective items in a lot after inspection.

Another performance measure that is commonly used is the average total inspection (ATI). It is defined as the number of inspections conducted in the optimal inspection plan. This measure evaluates the inspection load.

3. INSPECTION ERRORS

Embedded within the design of acceptance-sampling plans is an assumption that the inspection procedures are error free. However, many inspection tasks are not error free; on the contrary, they may even be error prone [Montgomery, D.C., 1990]. Two types of errors are possible in attribute sampling. An item which is good may be classified as defective (type I error, e_1), or an item that is defective may be classified as good (type II error, e_2). So, for attribute sampling we define the apparent fraction of defective items in a lot as

$$p_e = p(1-e_2)+(1-p)e_1$$

where p represents the true fraction of defective items in the lot.

4. SAMPLING PLANS AND THE EFFECTS OF INSPECTION ERRORS

A single sampling plan for attributes is characterized by a sample size n and an acceptance number c. the procedure would operate as follows: select n items at random from the lot. If there are c or fewer defectives in the sample, accept the lot, and if there are more than c defective items in the sample, reject the lot. If N and p represent the lot size and the true fraction of defective items in the lot, we can write the average outgoing quality of the inspection with replacement as [Collins et. al, 1973]

$$AOQ=\frac{npe_2+p(N-n)(1-p_e)Pa_e+p(N-n)(1-Pa_e)e_2}{N-np_e-(1-Pa_e)(S-n)p_e}$$

where

n = Sample size

e_1 = Probability of type I error

e_2 = Probability of type II error

p_e = Apparent fraction of defective items

Pa_e = Probability of acceptance with inspection error, given by $\sum_{x=0}^{c}\binom{n}{x}p_e^{\,x}(1-p_e)^{n-x}$

Similarly we can write an expression for the average total inspection for the inspection process without replacement as

$$ATI = \frac{n + (1 - Pa_e)(S - n)}{1 - p_e}$$

We assume the lot size, sample size, and acceptance number as S = 4000, n = 150 and c = 5 respectively. For this plan, let us assume four error-pairs as (e_1,e_2) = (0,0), (0.01,0), (0,0.15) and (0.01,0.15) [Collins et. al, 1973]. We determine the AOQ and ATI as a function of incoming fraction defective for each error-pair.

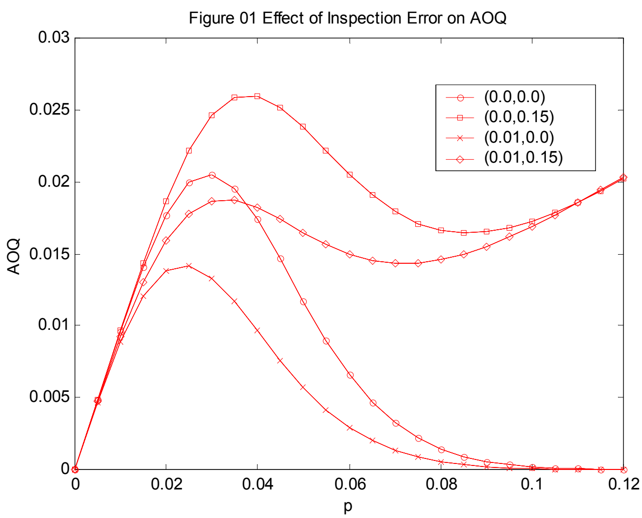

Figure 1 examines the average outgoing quality as a function of fraction defective and errors. Incorrect classification of a good item to defective ones (type I error) reduces the average outgoing quality due to the fact that more screening inspection takes place while incorrect classification of defective items to good ones (type II error) has the effect of causing higher AOQ for all values of p.

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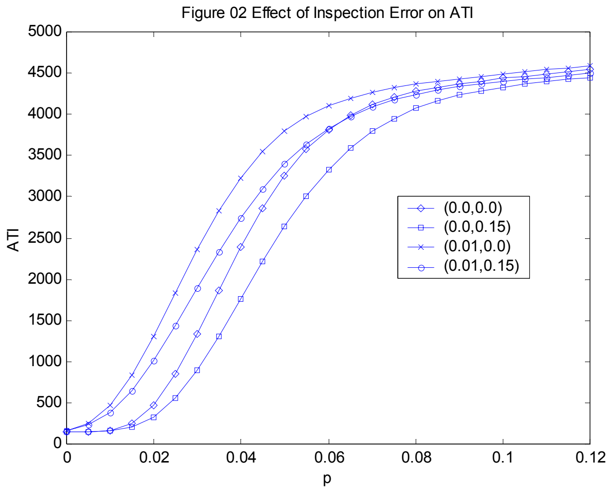

Figure 2 illustrates the average total inspection as a function of fraction defective and errors. As intuitively expected, the effect of type I error is to increase ATI and that of type II error is to decrease it.

On the other hand, a double-sampling plan is a procedure in which, under certain circumstances, a second sample is required before the lot can be sentenced. A double sampling plan is defined by four parameters:

n_1 = Sample size on the first sample
c_1 = Acceptance number of the first sample
n_2 = Sample size on the second sample
c_2 = Acceptance number for both samples

The procedure operates as follows: a random sample of n_1 items is selected from the lot, and the number of defectives in the sample, d1 observed. If $d_1 \le c_1$, the lot is accepted on the first sample. If $d_1 > c_2$, the lot is rejected on the first sample. If $c_1 < d_1 \le c_2$, a second random sample of size n_2 is drawn from the lot, and the number of defectives in this second sample, d_2, observed. Now the combined number of observed defectives from both the first and second sample, d_1+d_2, is used to determine the lot sentence. If $d_1+d_2 \le c_2$, the lot is accepted. However, $d_1+d_2 > c_2$, the lot is rejected. When rectifying inspection is performed with double sampling, the average outgoing quality, AOQ is given by [Montgomery, D.C., 1990]

$$AOQ = \frac{\left[P_a^I(N-n_1) + P_a^{II}(N-n_1-n_2)\right]p_e}{N}$$

where P_a^I and P_a^{II} denote the probability of acceptance on the first and second samples, respectively. The probability of acceptance of the lot, P_a would therefore be the sum of the above two probabilities. Assuming that all the defectives are replaced with good ones, the average total inspection, ATI is given by

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$$ATI = n_1 P_a^I + (n_1 + n_2) P_a^{II} + N(1 - P_a)$$

To illustrate the computation in this plan, we assume n_1=50, c_1=1, n_2=100, c_2=3. For the same set of four error-pairs as in the single sampling plan, we determine the AOQ and ATI as a function of incoming fraction defective.

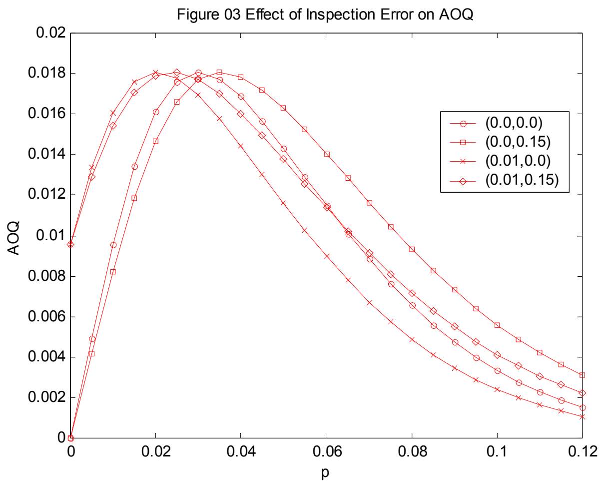

Figure 3 depicts that type I error reduces the average outgoing quality due to the fact that more screening inspection takes place while type II error has the effect of causing higher AOQ for all values of p.

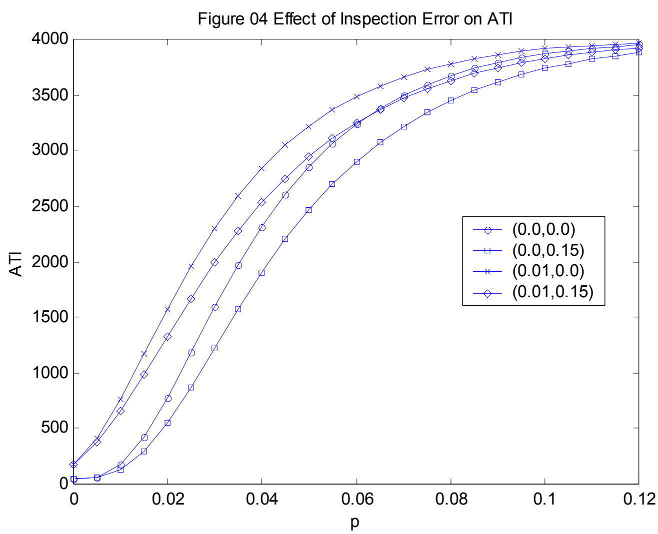

Figure 4 depicts that the effect of type I error is to increase average total inspection, i.e more inspection has to be done when a good items are classified as defective ones. While the type II error is to decreases average total inspection, i.e. the number of items under inspection goes down.

While inspecting components with several characteristics that can cause high cost or catastrophe i.e. multicharacteristic critical components, repeat inspection is likely to reduce the expected total cost of inspection. The repeat inspection plan for such components, where an inspector has to make a classification of good, rework and scrap components is applied as follows: an inspector inspects one particular characteristic for each component entering the inspection process, and classifies it as meeting specifications, scrap or rework. All the accepted components and the ones that are found to be meeting specifications at rework station, go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and this process is repeated a total of n times before the components are finally accepted. Here *n* is the optimal number of inspections necessary to minimize the total cost per accepted component [Duffuaa & Khan, 2002]. The average outgoing quality (AOQ) for this plan is given by the ratio of the number of defective components after inspection and the total number of accepted components. The average total inspection (ATI) here is defined as the total number of inspections conducted in the optimal inspection plan. For a batch of M components ATI is computed as:

$$ATI = \sum_{j=i}^{n}\left(\sum_{i=1}^{N}\left[\ \mathrm{M}_{\mathrm{i,j}} + \mathrm{M}_{\mathrm{j}}{}^{\mathrm{j}}\mathrm{PG}\left[\prod_{k=0}^{i-1}(1 - \mathrm{e}_{\mathrm{kgs}})\ \right]\mathrm{e}_{\mathrm{igr}} + \mathrm{M}_{\mathrm{i,j}}\mathrm{PS}_{\mathrm{i,j}}\mathrm{e}_{\mathrm{isr}} + M_{i,j}PR_{i,j}(1-e_{irg}-e_{irs})\right]\right)$$

where

j	=	Cycle under inspection
i	=	Stage or characteristic under inspection
e_i	=	Probability of misclassification an inspector can make for i^{th} characteristic
N	=	Number of characteristics
PG	=	Probability of a component to be good while entering a new cycle
PR	=	Probability of a component to be rework while entering a new cycle
PS	=	Probability of a component to be scrap while entering a new cycle

For this model we assume an inspection of a lot of 100 components with 3 characteristics. The probabilities of misclassification are taken to be 0.01, 0.10 and 0.15. We determine the AOQ and ATI as a function of type II error (classifying scrap to good) at a fixed value of type I error (classifying good to scrap). The other misclassification errors are fixed at 0.01or 0.15.

Figure 3 shows that AOQ increases as Egs increases. For Egs levels of 0.01, 0.10 and 0.15 the increase in AOQ is similar when Esg varies from 0.01 to 0.15. The other errors of misclassification are taken to be at 0.01.

Figure 05 Effect of the error on the Average Outgoing Quality (AOQ)
Egr = Erg = Ers = Esr = 0.01

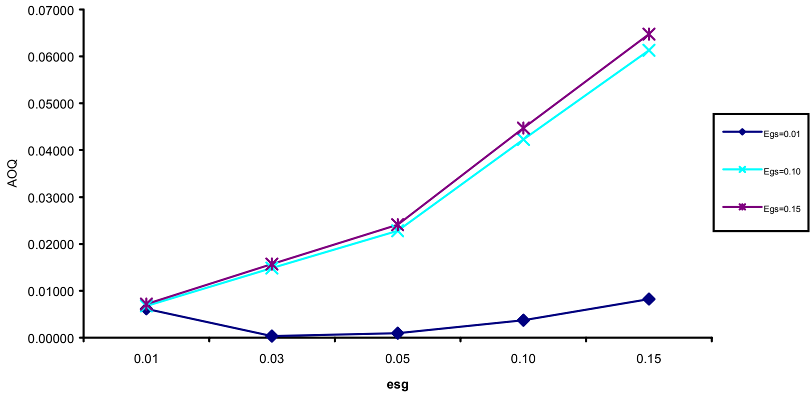

Figure 5 shows that AOQ increases as Egs increases. However, at Egs level of 0.01 or 0.03, AOQ decreases by 82% when Esg goes from 0.05 to 0.10. The other errors of misclassification are taken to be at 0.15 here.

Figure 06 Effect of the error Esg on the Average Outgoing Quality (AOQ)
Egr = Erg = Ers = Esr =0.15

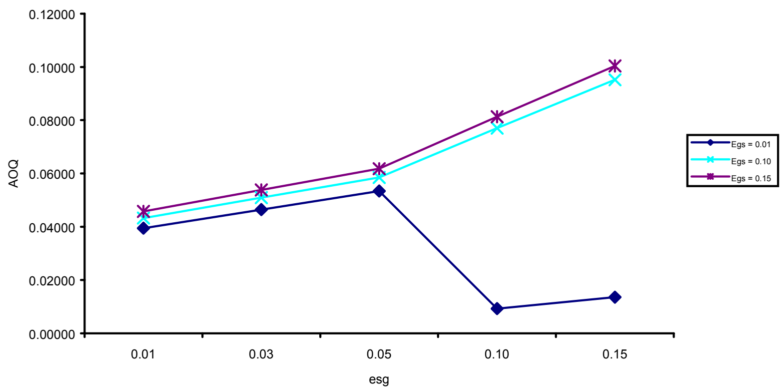

Figure 6 shows that the inspection load decreases with the increase in Egs. On the other hand, inspection load increases in a piece wise linear fashion as Esg increases. The other errors of misclassification are taken to be at 0.01.

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Figure 07 Effect of the error on the Average Total Inspection (ATI)
Egr = Erg = Ers = Esr =0.01

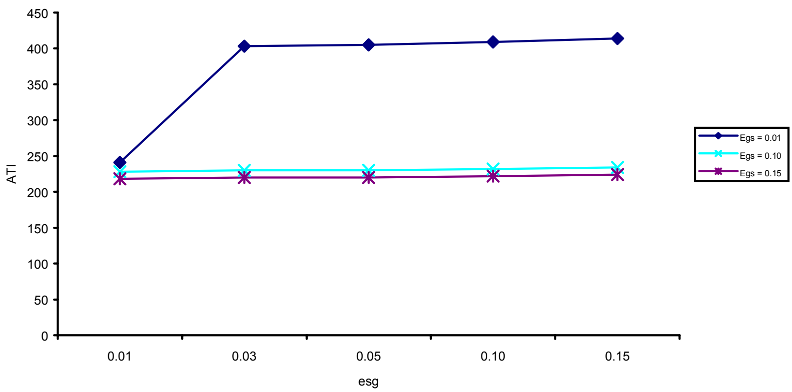
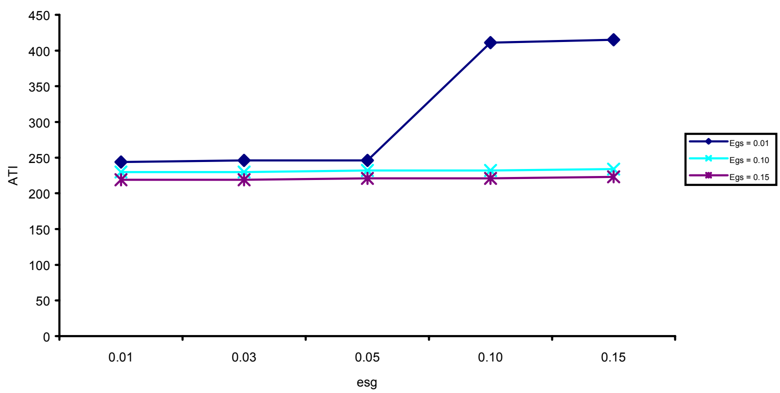


Figure 7 shows that the inspection load decreases with the increase in Egs. On the other hand, inspection load increases Esg increases. At Egs level of 0.01.

Figure 08 Effect of the error on the Average Total Inspection (ATI)
Egr = Erg = Ers = Esr = 0.15



Similarly, Figure 8 shows that the inspection load decreases with the increase in Egs while it increases with the variation in Esg from 0.01 to 0.15.

5. CONCLUSIONS AND RECOMMENDATIONS

In this paper, we have described inspection plans, their performance measures and the inspection errors while implementing them. Besides, we have investigated the effect of inspection errors on single, double sampling and repeat inspection plans.

It is clear from the above section that the inspection error has a drastic impact on the performance of inspections and could result to misleading conclusions about product quality.

Consumers may believe that the incoming quality is acceptable while it is not because they are not taking into consideration the effect of the inspection errors. The following actions are recommended to mitigate the effect of these errors:

1) Estimate the inspection error for the inspections by using well-designed experiments.
2) If the level of errors is high, train the inspectors to minimize the errors or to reach to an acceptable level of errors.
3) Use inspection plans based that incorporate the average error of the inspectors instead of depending on the inspection plans that assume an error free inspection process.

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