INTEGRATED PRODUCTION, INVENTORY, AND TRANSPORTATION PLANNING IN TWO-LAYER SUPPLY CHAIN

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ABSTRACT

Previous research on the joint vendor-buyer problem focused on the production shipment schedule in terms of number and the size of batches transferred between two parties. It is a fact that transportation cost is a major part of the total cost. However, the transportation cost is only considered implicitly as a part of fixed setup or ordering cost and thus the transportation cost is assumed to be independent of the size of the shipment. As such, the effect of the transportation cost is not adequately reflected in final planning decisions. There is a need for models involving transportation cost explicitly for better decision-making. In this study we analyze the vendor buyer lot-sizing problem under equal-size shipment policy. We introduce the complete solution of the problem in an explicit and extended manner that has not existed in the literature. We also consider the case where transportation cost is taken into account. The structure of the transportation cost is assumed to be an all-unit-discounted format. We develop a heuristic procedure to find a quality solution for the model with transportation cost. We give numerical examples to support the analysis.

Keywords: *Joint Economic Lot-Sizing, Inventory Control, Transportation, Freight Discounts.*

الملخص

إن البحوث السابقة في مسألة البائع-المشتري ركزت على حجم وعدد الشحنات التي يرسلها البائع إلى المشتري غير أن تكلفة النقل بين البائع والمشتري تشكل جزءا لا يستهان به من المجموع الكلي للتكلفه. و في هذه البحوث اعتبرت تكلفة النقل كجزء من تكلفة الطلب وبالتالي فهي ثابتة ولا تعتمد على حجم الكمية المشحونة وبالتالي فان تأثير تكلفة النقل لم تمثل في النماذج المطروحة بطريقة مناسبة مما يؤثر على طبيعة القرار المتخذ بشأن الانتاج. في هذه الدراسة نحلل مسألة البائع-المشتري مفترضين أن حجم الشحنات التي يرسلها البائع الى المشتري متساوية ونقدم حلا كاملا لهذه المسألة بشكل صريح وهذا لم يناقش في البحوث المنشورة من قبل. بالاضافة الى ذلك سنأخذ بعين الاعتبار كلفة النقل بين البائع والمشتري. نفترض في هذا النموذج ان طبيعة كلفة النقل تأخذ شكل الخصم على كل الوحدات. ثم نطور خطوات لإيجاد حل لهذا النموذج وأخيرا نقدم بعض الأمثلة لتوضيح الأفكار المعروضة.

1. INTRODUCTION

The significant interest in the supply chain management related research in the last decade has been due to the potential to improve the efficiency of operations and reduce the cost that each individual party in the supply chain experiences through closer collaboration of the parties and the integration of the decision processes. As it is the building block of any supply chain we focus on the vendor-buyer inventory problem in this study.

One of the major developments to improve the efficiency in the supply chains was the emergence of vendor managed inventory concept. Our model in this study can be employed to optimize the vendor managed inventory process.

In this study, we analyze the vendor-buyer inventory control problem under equal-size-shipments policy. This problem is known as joint economic lot-sizing problem (JELP) in the literature. The decisions to be determined are the production lot size of the vendor, shipment sizes from vendor to the buyer, and the number of shipments. We clarify and extend the solution of the problem given in the literature.

We also add a realistic dimension to the problem by incorporating the transportation cost issue into the model and develop a solution procedure for the model with transportation cost. The transportation cost issue has not been explicitly included in the models that currently exist in the literature.

In the next section we give a literature survey on JELP and the inventory models that considers the transportation costs issue in different ways. In section 3, we address the JELP under equal-size-shipments policy with the clarified and extended solution of the problem. Section 4 describes a JELP model with transportation cost and a solution procedure for the problem. The conclusion of the study can be found in section 5.

2. LITERATURE REVIEW

In this section first we will review the literature on the inventory problem we deal with in this study and in the second part we summarize some of the literature that considers the transportation cost issue.

2.1 Joint Economic Lot-Sizing Problem

In this literature review, we focus on the single-vendor single buyer joint economic lot-sizing problem (JELP) where the vendor is assumed to be a manufacturer and the buyer is a retailer or a central warehouse dispatching the goods. The literature we include here assumes the two

Top

stages to be a single system rather than taking the vendor and the buyer as completely independent. Problem variation in the literature differs in two aspects: i) production rate assumption for the vendor, ii) the shipment policy between two stages, i.e. whether shipment can start before the entire lot is produced by the vendor, single or multiple shipment lots, equal or different shipment lot sizes, and if different how shipment sizes are assumed to be changing.

One of the early papers related to JELP was Goyal (1977). He suggests a solution to the problem under the assumption of infinite production rate for the vendor and lot-for-lot policy for the shipments from the vendor to the buyer. Lot-for-lot policy equates the production and shipment sizes. This implies that the entire production lot should be ready before the shipment. Banerjee (1986) relaxes the infinite production rate assumption of Goyal (1977) but follows the lot-for-lot policy. That study coined the term JELP. Goyal (1988) contributes to the efforts of generalizing the problem by relaxing lot-for-lot policy. He assumes that the production lot is shipped in a number of equal size shipment lots, but only after the entire production lot is finished.

Many other studies eliminated the restriction of requiring the completion of the production lot before starting the shipments. The focus was looking at the policies where the shipment sizes increase by a factor geometrically for certain number of shipments and then remains constant for the shipments at the end. Goyal (1995) looks into a policy where the geometric growth of the shipments continues until the end without any constant shipment sizes. The geometric growth factor is set to the ratio of production rate to demand rate. In that short paper, he formulates the problem, gives the optimal expression for the first shipment size as a function of the number of shipments and solves a single example by searching on the number of shipments. Hill (1997) further generalizes Goyal (1995). He sticks with the all geometrically increasing shipment size policy but takes the geometric growth factor as a decision variable rather than fixed. Thus, the decision variables become the number of shipments, first shipment size, and the geometric growth factor. He suggests a solution method that does not in general guarantees the optimality. This method is based on exhaustive search for both the growth factor and the number of shipments in certain ranges. Numerically, he shows that his solution method outperforms both equal shipment sizes policy and the policy adopted by Goyal (1995), which is obviously expected since he turns the geometric growth factor into a variable whereas in both equal-size and Goyal (1995) policies, this factor is fixed. Goyal (2000) suggests a method to improve the solutions obtained by the method given in Hill (1997). In this procedure, as the first shipment size, number of shipments, and the production lot size, he uses the results of Hill (1997). All of the work listed above looks at the problem under a certain assumption on shipment policy. Hill (1999) found the optimal solution structure to the problem without any assumptions about the shipment policy. An extensive review on JELP and its variants can be found in Goyal and Gupta (1989).

Top

2.2 Integrated Analysis Of Inventory/Production/Transportation Planning

In this section, we will list some of the literature on lot sizing problems in multi stage inventory/production systems that consider the transportation costs explicitly. In general, transportation part of the problem has been taken into account as a routing problem where we distribute the allocated inventory to customers using a fleet of vehicles, or as a transportation mode and size selection problem under freight discount schedule where the effect of transportation cost is through the changing unit transportation costs depending on the mode of the transportation and the size of the shipments. The importance of transportation cost in inventory and production planning has been shown both numerically (Federgruen and Zipkin (1984), Chandra and Fisher (1994), Ertogral et al (1998) and based on business case surveys and analysis (Carter and Ferrin (1996)).

Traditionally, the transportation cost has not been explicitly incorporated into the inventory control/production planning models. It has been implicitly assumed that the transportation cost is part of fixed ordering cost or it is charged to the supplier. Neither of these assumptions is in general valid. Transportation costs are affected by the routing decision and selected shipment sizes. Even if the supplier incurs the transportation cost, it will be directly reflected in the unit purchase price charged by the supplier.

A well-referenced study that incorporates routing element into inventory allocation decision can be found in Federgruen and Zipkin (1984). They considered the problem of allocating a limited amount of one product in a depot to multiple customers in one period. They device a heuristic to decide both the shipment sizes to each customer and routing considering inventory related costs at each customer location and transportation cost. Their model and solution method was extended by Federgruen et al. (1986) to the case where the item to be distributed is a perishable one. Both papers present comparison between integrated versus sequential approaches and show the gain due to the integrated approach. The integrated approach refers to making inventory allocation and routing decisions simultaneously while the sequential one corresponds to making inventory allocation decision first and then the routing decision.

A common and transportation-wise costly way of avoiding shortages is expediting. In a two-stage system where the second stage satisfies the outside orders, Blumenfeld et al. (1985) studies the tradeoff between the cost of expediting at the first stage and holding safety stock at the second stage. Another interesting problem faced in logistics area is to determine the number of contracted vehicles to carry out shipments in such a way that a good tradeoff is achieved between the spare capacity in the contracted vehicles and the use of emergency shipments (expediting). Yano and Gerchak (1989) deal with this problem in a two-stage supplier customer system. They suggest a methodology to simultaneously determine the safety sock level at the customer, number of vehicles for regular delivery and the time between shipments considering the inventory holding and shortage cost at the customer location and regular and emergency shipment costs. The model of Yano and Gerchak was extended by Ernst and Pyke (1993) to include the consideration of the inventory cost at the first stage.

In the context of integrating production planning with transportation, both Chandra and Fisher (1994) and Ertogral et al. (1998) investigated the value of integration through a numerical study. In Chandra and Fisher (1994), the system is composed of a manufacturer and several customers. They heuristically integrate the production and routing decisions under different cost parameter sets and showed the value of integration. Ertogral et al. (1998) is the only study, to the best of our knowledge, that integrates the production and transportation routing decisions in multi layer systems. In that study, transportation occurs between the production facilities that are in supplier-user relation in a multi-layer supply chain. They devised a Lagrangian decomposition based schema to find the optimal integrated solution. A comprehensive survey on integrated analysis of production/inventory/distribution planning is given in Sarmiento and Nagi (1999).

Another way of incorporating the transportation costs into inventory control decisions has been through the use of freight discount schedule in the literature. In that context, rather than assuming the ownership of a fleet of vehicles and dealing with routing decisions, the assumption is that a carrier provides the transportation and the carrier gives a schedule of rates based on the size of the shipment. Here, we will list only some of the related literature.

One of the earliest papers on incorporating freight cost into an inventory model is Boumal and Vinod (1970). They introduce two models for inventory cost minimization and profit maximization. In those models, they include freight rates, speed, variance in speed, and the en-route lossage in an order-sizing model. They assume that unit shipping cost is fixed and is not dependent on the shipment size. But in reality, especially after the deregulation of the shipping industry in US in early 1990s, freight rate discounts depending on the shipment size became common practice (Carter and Ferrin (1996). Lee (1986) is one of the earliest studies that explicitly incorporate the discounted freight rate into the well-known EOQ model. He takes the shipment cost as a fixed cost which increases in a step function format depending on the order size. He presents an exact algorithm to solve this EOQ model with freight shipment cost.

Another type of discount usually offered by the suppliers is the price discount for larger orders to entice the buyer to buy more. Burwell et al. (1997) consider both discounts, namely freight discounts and price discounts, in a profit maximization model. Their model assumes a price dependent demand structure. Therefore, their decision variables are order size and the price to be charged to the end customer. They describe exact algorithms, based on the results of Abad (1988), for solving the problem under different scenarios for the discount structure of the shipping rate and unit price. A similar work that takes into account price and freight discount together is Tersine and Barman (1991). The main difference between the two studies is that in

Tersine and Barman (1991) the demand is assumed to be constant rather than price dependent. Finally in a recent study, Swenseth and Godfrey (2002), again the effect of freight rate discounts on ordering decision is studied. That paper looks into the question of when to over declare a shipment to exploit the reduced per unit transportation cost and how this possibility affects the order sizes using an inventory cost minimization model.

3 FORMULATION AND ANALYSIS OF THE EQUAL-SIZE-SHIPMENT POLICY

3.1 Model Formulation

The notation used in the formulation is as follows:

A_v: Production setup cost of the vendor.

A_b: Ordering cost of the vendor.

h_v: Inventory holding cost per unit per unit time for the vendor.

h_b: Inventory holding cost per unit per unit time for the buyer.

Δh: Difference between the inventory holding cost of the vendor and the buyer ($\Delta h = h_b - h_v$)

Q: Production lot size.

q: Shipment lot size.

n: Number of shipment lots from a lot of production run.

TC: The total cost incurred by the system, vendor and buyer, per unit time.

P: Production rate of the vendor.

D: Demand rate faced by the buyer.

$c(q)$: Unit transportation cost charged to the shipments from the vendor to the buyer.

It is assumed that P and D are constant over time and $P > D$. If $P \leq D$, then the problem would be trivial and decision would be to produce continuously with shipment lot size being one, in order to satisfy the demand as much as and as soon as possible. We will give the formulation for the total cost per unit time below without any explanation. For an explanation of the formulation, we refer the reader to Hill (1997).

$$TC(q,n) = (A_v + nA_b)\frac{D}{nq} + h_v(\frac{Dq}{P} + \frac{(P-D)nq}{2P}) + \Delta h\frac{q}{2} \qquad (1)$$

Total cost is convex in q and n for a given transportation cost. We omit the straightforward proof of convexity. The optimal shipment lot size, as given in Hill (1997), is;

Top

$$q^* = \left\{ (A_v + nA_b)\frac{D}{n} / \left[h_v\left(\frac{D}{P} + \frac{(P-D)n}{2P}\right) + \frac{\Delta h}{2} \right] \right\}^{1/2} \tag{2}$$

The results are given in equations 3, 4, and 5 are extensions to the solution given in Hill (1997). The optimal continuous value for the number of shipments is given by:

$$n^* = \frac{\sqrt{2h_v(P-D)DPA_v}}{h_v(P-D)q} \tag{3}$$

The optimal q and n values as a function of the problem parameters can be obtained by solving the equations 2 and 3 simultaneously. The resulting expressions for q and n are given by:

$$q^* = \frac{\sqrt{2KPD\left(h_v A_v (P-D) - K\right)\left(PA_b \Delta h + 2h_v A_b D + K\right)}}{-P^2 h_v A_v \Delta h + P\left(Dh_v A_v (h_b - 3h_v) - K\Delta h\right) - 2Dh_v\left(K + h_v DA_v\right)} \tag{4}$$

where

$$K = \sqrt{(P-D)(P\Delta h + 2Dh_v)h_v A_v A_b}$$

$$n^* = \frac{\sqrt{h_v A_v A_b (P-D)(P\Delta h + 2h_v D)}}{h_v A_b (P-D)} \tag{5}$$

It is clear from equations 2 and 5 that as the buyer inventory holding cost gets larger compared to vendor inventory holding cost, the optimal number of shipments will increase, and at the same time the optimal shipment size will decrease. This is an intuitive result since as the buyer side of the problem becomes more important, the optimal solution should favor smaller and frequent shipments since this improves the buyers' cost. From equation 2, we can also conclude that the optimal number shipments size gets larger as the setup cost of the vendor increases. The intuition behind this result is that, as the vendor setup cost gets larger the production lot size of the vendor will get larger and this large production lot, in turn, will require more shipments per production run.

3.2 The Procedure To Find The Optimal Solution:

Let $\overline{n}$ and $\underline{n}$ represent the largest integer less than or equal to n and the smallest integer greater than or equal to n respectively. Instead of the search procedure suggested in Hill (1997) we introduce the following one-pass heuristic to find the optimal solution:

1. Find the continuous optimal number of shipments, n^*, using equation 5.

2. Let $n = \underline{n}^*$

   2.1 Using n, find the optimal shipment size, q^*, from equation 2 and the corresponding total cost, TC_1, from equation 1.

3. Let $n = \overline{n^*}$

   3.1 Using n, find the optimal shipment lot size, q^*, from equation 2 and the corresponding total cost, TC_2 , from equation 1.

4. The optimal solution is $min\{ TC_1, TC_2\}$.

3.3 Numerical Example:

We will solve a numerical example used in several studies (see for example Hill (1997)). The problem data is as follows; h_v=4, h_b=5, A_v=400, A_b=25, P=3200, D=1000.

1. n^*=4.51

2. $n = \underline{n}^* = 4$

   q=131.3, TC_1=1903.94

3. $n = \overline{n^*} = 5$

   q=110.33, TC_2=1903.29

4. Optimal solution=$min\{1903.94, 1903.29\}$=1903.29, (n=5, q=110.33)

This is the same solution reported in Hill (1997).

4 FORMULATION AND ANALYSIS OF THE EQUAL-SIZE-SHIPMENT POLICY WITH TRANSPORTATION COST

4.1 Model Formulation

In this section, we will analyze the case where the transportation cost is explicitly considered. Rather than assuming it to be a part of the fixed ordering cost or to be insignificant, we will take the transportation cost as significant and as a function of the shipment lot size. We will consider the transportation cost to be in an all unit discounted cost format. The structure of the unit transportation cost is represented in the following way:

Top

Condition	Unit Transportation Cost
$0 \le q < M_1$	c_0
$M_1 \le q < M_2$	c_1
$M_2 \le q < M_3$	c_2
.	.
.	.
.	.
$M_{m\text{-}1} \le q < M_m$	$c_{m\text{-}1}$
$M_m \le q$	c_m

The total transportation cost per unit time is found by dividing the transportation cost per production lot cycle by the duration of the cycle as follows:

The transportation cost per unit time = $c_i nq / (D / nq) = c_i D$

We add the transportation cost per unit time to equation 1 to express the total cost per unit time for a given range of the shipment order size:

$$TC(q,n) = (A_v + nA_b)\frac{D}{nq} + h_v\left(\frac{Dq}{P} + \frac{(P-D)nq}{2P}\right) + \Delta h\frac{q}{2} + c_i D, \qquad q \in [M_i, M_{i+1}) \tag{6}$$

As we can see from equation 6, since the transportation cost part of the model is not written as a function of q, the expressions for the optimal shipment lot size and the number of shipments, and the rest of the analysis we give in section 3 are also valid for the model with transportation cost. Thus, the total cost expression is still convex in the two decision variables. The only difference is that we have to make sure that the shipment size falls in the indicated range for a given per unit transportation cost.

The optimal shipment size of the model with transportation will be always greater than or equal that of the model without transportation. Given the optimal shipment lot size of the model without transportation, we may improve the total cost including the transportation, by increasing the lot size. Note that the unit transportation cost decreases with the shipment lot size. Decreasing the lot size from optimal level will increase both the transportation cost and the inventory related costs.

The tradeoff to be checked is whether the saving in the transportation cost due to the increase in the shipment lot size is more than the increase in the inventory related costs. Based on this observation we develop the following heuristic procedure.

Top

4.2 Heuristic Solution Procedure

We will assume that index i represents the range $[M_i, M_{i+1})$ in following procedure:

1. Find n^* for the range using equation 5.

2. Let $n = \underline{n}^*$. Using n find q^* from equation 2. Let $\underline{q}^* = q$

    2.1 Find the range index j such that $M_j \le q^* < M_{j+1}$

    2.2 Find $TC(\underline{q}^*, \underline{n}^*)$ from equation 6 using c_j.

3. Let $n = \overline{n^*}$. Using n find q^* from equation 2. Let $\overline{q^*} = q^*$.

    3.1 Find the range index k such that $M_k \le \overline{q^*} < M_{k+1}$

    3.2 Find $TC(\overline{q^*}, \overline{n^*})$ from equation 6 using c_k.

4. Let $TC^* = min\{ TC(\underline{q}^*, \underline{n}^*), TC(\overline{q^*}, \overline{n^*}) \}$. Let l = the range index (j or k) associated with TC^*

5. For all the ranges with index $t \ge l$ apply the following

    5.1 Let $q = M_t$.

    5.2 Find n^* using equation 3.

    5.3 Let $n = \underline{n}^*$ and using n find q^* from equation 2 and let $\underline{q}^* = q$

        5.3.1 If $q^* < M_t$ let $\underline{q}^* = M_t$. If $q^* > M_{t+1}$ let $\underline{q}^* = M_{t+1}$.

        5.3.2 Find $TC(\underline{q}^*, \underline{n}^*)$ from equation 6 using c_t.

    5.4 Let $n = \overline{n^*}$ and using n find q^* from equation 2 and let $\overline{q^*} = q^*$

        5.4.1 If $q^* < M_t$ let $\overline{q^*} = M_t$. If $q^* > M_{t+1}$ let $\overline{q^*} = M_{t+1}$.

        5.4.2 Find $TC(\overline{q^*}, \overline{n^*})$ from equation 6 using c_t.

    5.5. Let $TC_t = min\{ TC(\underline{q}^*, \underline{n}^*), TC(\overline{q^*}, \overline{n^*}) \}$

6. Among the policies giving the costs TC^*, and TC_t for $t \ge l$, select one with the minimum cost.

In steps 1 through 3, we find the optimal solution of the problem without considering the transportation cost using equations 4 and 5. In step 5, we check if we can find a lower cost solution including the transportation cost by setting the shipment size equal to the minimum value in each range that follows the range where the optimal solution without considering the transportation cost falls in. We check only the policy of trying to set the shipment size equal to lower limit in each range since the total cost increases as we increase the shipment lot size and if setting the shipment lot size to the minimum value in a range does not give a better solution no other shipment size in the same range can give a better solution.

4.3 Numerical Example

We will use the example solved in section 2 with the following transportation cost structure:

Condition	Unit Transportation Cost
$0 \le q < 130$	2
$130 \le q < 250$	1.5
$250 \le q < 300$	1.25
$300 \le q$	1.2

The optimal solution found in steps 1-3 is n=5, q=110.33 with total cost of 3403 including the transportation cost. This is the same optimal policy we had in section 2. The optimal solution falls into the range [0,130). For the last three ranges, by applying the procedure described above, we obtain the following results:

$TC_1=TC(131.31,4)=3403.94$, $TC_2=TC(250,2)=3275.00$, $TC_3=TC(300,2)=3300.00$

Based on the results, we conclude that q=250 and n=2 is the policy to choose when we include the transportation cost in the model since it is the least cost solution. This policy is significantly different from the optimal policy we found in section 2. In the optimal policy, we increase the shipment lot size so that we can take the advantage of reduced unit transportation cost.

5. CONCLUSIONS

In this study we analyze the vendor buyer lot-sizing problem under equal-size shipment policy. We introduced the complete solution of the problem in an explicit manner that has not existed in the literature. We also considered the case where transportation cost is taken into account. The structure of the transportation cost is assumed to be an all-unit-discounted format. We develop a heuristic procedure to find a quality solution for the model with transportation cost. We give numerical examples to support the analysis. Future research will look into incorporating transportation cost into other JELP models using different transportation cost structures.

Top

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Top