



A NEW FUZZY UNIT COMMITMENT PROBLEM: APPLICATION TO SAUDI ELECTRICITY COMPANY

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ABSTRACT

This paper presents an advanced Simulated Annealing Algorithm (SAA) to solve a fuzzy Unit Commitment Problem (UCP) model. The uncertainties in the load demand and the spinning reserve constraints are treated in a fuzzy logic (FL) frame. The algorithm is based on a polynomial-time cooling schedule, which is advocated in the literature. The algorithm includes a step to find the initial control parameter at which any prespecified percentage of the trial solutions are accepted. A penalty factor extracted from fuzzy membership functions is used with the objective function to guide the optimal solution search process. The application of the algorithm to a sample of data selected from the Saudi Electricity Company (SEC) and two additional examples from the literature proves the efficiency of the algorithm.

Keywords: *unit commitment, simulating annealing, fuzzy logic.*

INTRODUCTION

Unit commitment is the problem of selecting the generating units to be in service during a scheduling period and for how long. The committed units must meet the system load and reserve requirements at minimum operating cost, subject to a variety of constraints. The Economic Dispatch problem (EDP) is to optimally allocate the load demand among the

running units while satisfying the power balance equations and units operating limits [Wood, Allen, 1984 and Mantawy, A.H. et al, 1998].

The solution of the UCP is generally treated in a crisp domain. However, some of the data used are not accurately specified such as load demand and spinning reserve. Since the load demand is only known through short-term load forecasting, errors are expected. Moreover, the spinning reserve constraint practically is based on the probability of abnormal conditions that might result in insufficient generation capacity to cover the load demand; hence this constraint could be a soft, not a hard limit, constraint. Consequently, it is advisable to formulate the problem within the uncertainty frame.

Fuzzy Logic (FL), which may be viewed as an extension of classical logical systems, provides an effective conceptual framework for dealing with the problem of knowledge representation in an environment of uncertainty and imprecision [Zadeh, L.A., 1965, Zimmermann, H.J, 1985 and Su, Chun-Ching et al., 1991]. The FL can be used to realize the expected error in the forecasted load demand and the soft limits of the spinning reserve requirements. Simulated Annealing (SA) is a powerful technique to solve combinatorial optimization problems [Mantawy, A.H. et al, 1998, Aarts, E et al., 1989 and Cerny V., 1985].

In this paper we propose an advanced algorithm (SAFL) based on SA and FL to solve the proposed FL model of the UCP. The combinatorial optimization subproblem is solved using the SAA while the EDP is solved via a quadratic programming routine. An advanced cooling schedule, called polynomial-time, is implemented. The parameters of this cooling schedule are determined based on a statistics calculated during the search.

The proposed SAFL is tested on a part of the Saudi Electric Company (SEC) in the eastern region system. A sample of 24 units with different capacities and a suitable load curve for this sample of units are selected. The results show the superiority of the proposed SAFL.

In the next section, a mathematical formulation of the problem is introduced. In Section 3, an overview of the proposed SAFL algorithm is presented, followed, in Section 4, by the details of FL implementation in the SAFL algorithm. Numerical results are presented in Section 5. Section 6 outlines the conclusions.

2. PROBLEM STATEMENT

In the UCP under consideration, one is interested in a solution that minimizes the total operating cost of the generating units during the scheduling time horizon while several constraints are satisfied [Wood, Allen, 1984 and Mantawy, A.H. et al, 1998].

2.1 The Objective Function

The overall objective function of the UCP of N generating units for a scheduling time horizon T is:

$$F_T = \sum_{t=1}^T \sum_{i=1}^N (U_{it}F_{it}(P_{it}) + V_{it}S_{it}) \text{ \$} \tag{1}$$

Where

U_{it} : is status of unit i at hour t (ON=1, OFF=0).

V_{it} : is start-up/shut-down status of unit i at hour t.

P_{it} : is the output power from unit i at time t.

The production cost, $F_{it}(P_{it})$, of a committed unit i, is conventionally taken in a quadratic form:

$$F_{it}(P_{it}) = A_i P_{it}^2 + B_i P_{it} + C_i \text{ \$/HR} \tag{2}$$

Where, A_i, B_i, C_i : are the cost function parameters of unit i.

The start-up cost, S_{it} , is a function of the down time of unit i [6]:

$$S_{it} = S_{oi} [1 - D_i \exp(-T_{off_i} / T_{down_i})] + E_i \text{ \$} \tag{3}$$

Where, S_{oi} : is unit i cold start-up cost, and

D_i, E_i : are start-up cost coefficients for unit i.

2.2 The Constraints

The constraints that have been taken into consideration in this work, may be classified into two main groups:

(i) System Constraints

1- Load demand constraints:

$$\sum_{i=1}^N U_{it} P_{it} = PD_t ; \forall t \tag{4}$$

Where PD_t : is the system peak demand at hour t (MW).

2- Spinning Reserve

Spinning reserve R_t , is the total amount of generation capacity available from all units synchronized (spinning) on the system minus the present load demand.

$$\sum_{i=1}^N U_{it} P_{max_i} \geq (PD_t + R_t); \forall t \quad (5)$$

(ii) Unit constraints:

The constraints on the generating units are

a- Generation limits

$$U_{it} P_{min_i} \leq P_{it} \leq P_{max_i} U_{it}; \forall i, t \quad (6)$$

Where, P_{min_i}, P_{max_i} is minimum and maximum generation limit (MW) of unit I.

b- Minimum up/down time

$$\begin{aligned} T_{off_i} &\geq T_{down_i}, \forall i \\ T_{on_i} &\geq T_{up_i} \end{aligned} \quad (7)$$

Where T_{up_i}, T_{down_i} are unit i minimum up/down time.

T_{on_i}, T_{off_i} are time periods during which unit i is continuously ON/OFF.

c- Unit initial status

d- Crew constraints

e- Unit availability; e.g., must run, unavailable, available, or fixed output (MW).

f- Unit derating

3. THE PROPOSED ALGORITHM

3.1 Overview

In the proposed algorithm we consider the load demand uncertainties and the reserve constraints as soft limits in a FL frame. The fuzzy load demand is calculated based on the error statistics and load membership function [Su, Chun-Ching et al., 1991]. The spinning reserve is calculated for each solution along with its membership function. A penalty factor is

then determined, as function of both the load demand and reserve membership functions to guide the search in the SAFL algorithm.

The major steps of the SAFL algorithm are summarized as follow:

- Apply FL rules to calculate the fuzzy load demand.
- Initialize the temperature of the SA cooling schedule algorithm, C_p^0 .
- Generate randomly an initial feasible solution and let it be the current and best solutions. For the k th iteration apply the following steps:
 - Calculate the new temperature for the SA algorithm $C_p^k = C_p^0(\beta)^k$, where $0 < \beta < 1$.
 - Generate randomly a trial solution as a neighbor to the current solution.
 - Calculate the objective function of the trial solution by solving the EDP.
 - Use the FL approach to calculate the penalty factor to be added to the objective function as reflection to the amount of reserve in the trial solution as follow:
 - Calculate the amount of spinning reserve in the trial solution.
 - Apply FL rules to calculate the reserve membership function.
 - Estimate the value of the penalty factor according the output of the load and reserve membership functions.
- Apply the SA test to accept or reject the trial solution.
- If the trial solution is accepted, let it be the current solution and update the best solution if needed.
- If the specified chain length reached go Step (k), otherwise go to Step (e).
- Check for stopping criteria. If satisfied stop, otherwise go to Step (d).

3.2 Stopping Criteria

There are several possible stopping conditions for the search. In our implementation, we stop the search if one of the following two conditions is satisfied in the order given:

- The number of iterations performed since the best solution last changed is greater than a prespecified maximum number of iterations, or
- Maximum allowable number of iterations is reached.

3.3 SA Test

Implementation steps of the SA test as applied in the k th iteration of the proposed algorithm are described as follow [Mantawy, A.H. et al, 1998]:

Step (1): At the same calculated temperature, c_p^k , apply the following acceptance test for the new trial solution.

Step (2): **Acceptance test:** If $E_j \leq E_i$, or
 if $\exp[(E_i - E_j) / C_p] \geq U(0,1)$, then accept the trial solution, set $x_i = x_j$ and $E_i = E_j$.
 Otherwise reject the trial solution. Where x_i, x_j, E_i, E_j are the SA current solution, the trial solution and their corresponding cost respectively.

Step (3): Go to the next step in the algorithm.

3.4 Cooling Schedule

A finite-time implementation of the SA algorithm can be realized by generating homogenous Markov chains of finite length for a finite sequence of descending values of the control parameter. To achieve this, one must specify a set of parameters that governs the convergence of the algorithm. These parameters form a cooling schedule. The parameters of the cooling schedules are: an initial value of the control parameter decrement function for decreasing the control parameter and a final value of the control parameter specified by the stopping criterion, and a finite length of each homogenous Markov chain. Details of the implemented cooling schedule are described in details in [Mantawy, A.H. et al, 1998].

4. FL IMPLEMENTATION IN THE SAFL ALGORITHM

In general, a fuzzy logic system, that is widely used, maps crisp inputs into crisp outputs. It comprises four principal components: *fuzzifier, rule base, inference engine, and defuzzifier* [Zadeh, L.A., 1965 and Zimmermann, H.J, 1985].

In the proposed algorithm FL is used to deals with the uncertainties in the forecasted load demand and the pre-specified spinning reserve requirements. The implemented fuzzy logic system consists of two inputs: the load demand and the spinning reserve, and two outputs: a fuzzy load demand and a penalty factor.

4.1 Membership Function For The Load Demand

The fuzzy set of input for the load demand is divided into six fuzzy values: low negative, LN, medium negative, MN, high negative, HN, low positive, LP, medium positive, MP, and high positive, HP). The membership function for load forecast error is taken as follow [Ho, K.-L. et al., 1990]:

$$\mu_L = \left\{ \begin{array}{ll} \frac{1}{1 + 2.33(\frac{\Delta I}{M_+})^2}, & \Delta I \geq 0 \\ \frac{1}{1 + 2.33(\frac{\Delta I}{M_-})^2}, & \Delta I < 0 \end{array} \right\} \tag{8}$$

where $\Delta I = \text{percentage error} = \frac{\Delta L}{L_{\text{forecasted}}} \times 100\%$

$$= \frac{L_{\text{actual}} - L_{\text{forecasted}}}{L_{\text{forecasted}}} \times 100\% \tag{9}$$

4.2 Membership Function for Spinning Reserve

The fuzzy set of input for the spinning reserve demand is divided into six fuzzy values: very low, VL, low, L, medium, M, high, H, very high, VH) as shown in Fig. (1). The membership function for the spinning reserve is taken as follow:

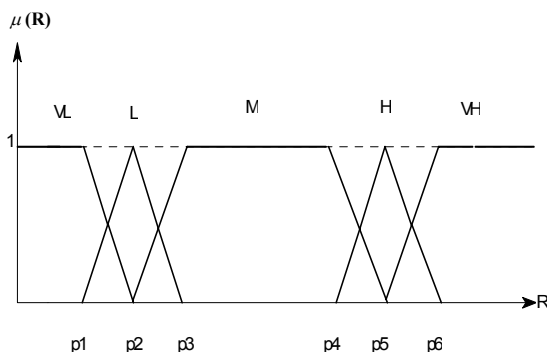


Fig. (1) Membership function for spinning reserve

where $p1=RR-d1, p2=RR-d1/2, p3=RR, p4=RR+d2, p5=RR+(d2+d3)/2,$ and $p6=RR+d3.$
 R: is the actual reserve in the schedule.
 RR is the required reserve.
 d1, d2, and d3 are selected percentage values of the spinning reserve.

5. NUMERICAL RESULTS

The proposed SAFL algorithm is applied to a sample of data from the SEC system. The sample of data includes 24 units with different capacities. A typical daily load curve from the winter season is chosen and modified to suit the selected sample of units. The amount of spinning reserve is taken as 400MW in all hours. The derating constraints are also taken into account and considered as function of the weather temperature. The units' data of the sample of SEC system is shown in Table (1).

A number of tests on the performance of the proposed SAFL have been carried out on the SEC Example to find the most suitable SAFL parameters settings for the cooling schedule.

The following control parameters have been chosen: $c_p=7000$, $\delta=0.3$, chain length=150 and the maximum number of chains=1000, error= $1e-6$.

Table (2) shows comparison of results obtained by the heuristic Dynamic Programming (DP) used in SEC, SAA [Mantawy, A.H. et al, 1998] and the proposed SAFL. It is obvious that a significant cost saving is achieved by SAFL related to DP and the SAA. The saving in cost for the SAFL is 9.66% as compared to DP, which is considered as big money if converted to daily saving.

Table (1) Units Data of SEC Example

Unit No.	A \$/MW.Sq	B \$/MW	C \$	Pmin MW	Pmax MW	Tup HR	TD HR
1	7.62E-03	13.728	605.779	250	625	8	8
2	7.62E-03	13.728	605.779	250	625	8	8
3	1.17E-02	14.346	1186.087	180	400	8	8
4	1.22E-02	14.020	1235.064	190	400	8	8
5	1.22E-02	14.020	1235.064	190	400	8	8
6	1.22E-02	14.020	1235.064	190	400	8	8
7	3.29E-02	15.240	671.691	33	75	4	2
8	3.26E-02	15.266	671.186	33	77	4	2
9	3.24E-02	15.278	670.939	33	78	4	2
10	4.95E-02	14.087	383.963	33	69	4	2
11	4.90E-02	14.126	383.200	33	67	4	2
12	4.55E-02	14.410	377.669	20	66	4	2
13	4.46E-02	14.483	376.284	15	68	4	2
14	4.46E-02	14.483	376.291	15	68	4	2
15	4.49E-02	14.468	376.461	15	69	4	2
16	1.07E-02	16.793	633.426	20	81	4	2
17	1.08E-02	16.782	633.584	20	82	4	2
18	1.06E-02	16.795	633.408	20	81	4	2
19	4.60E-02	13.710	317.410	15	79	4	2
20	4.60E-02	13.710	317.410	15	79	4	2
21	4.60E-02	13.710	317.410	10	54	4	2
22	4.60E-02	13.710	317.417	15	54	4	2
23	4.60E-02	13.709	317.426	15	61	4	2
24	4.60E-02	13.709	317.426	15	61	4	2

Pmin. , Min.: are maximum and minimum output limits of units. Tup, TD: are the Minimum up/down times of units.

Tables (3) & (4) present the detailed results for SEC Example. Table (3-a) & (3-b) show the load sharing in MW among the committed units in the 24 hours. Table (4) gives the hourly load demand, and the corresponding committed units capacities, economic dispatch costs, start-up costs, and total operating cost. In this table the start up costs are zeros, since there is no available data for calculation. Also the committed capacities are fractional number because of the derating constraints consideration.

Table (2) Comparison with the DP and the SAA Algorithms

	DP [SEC]	SAA *	SAFL
Total Cost (\$)	1375865	1319391	1242842
% Cost Saving	-	4.1	9.66

*Mantawy, A.H. et al, 1998

Table (3-a) Power Sharing (MW) of SEC Example

HR	Unit Number*					
	1	2	3	4	5	6
1	538.9	538.9	325.3	324.7	324.7	324.7
2	475.9	475.9	284.2	285.4	285.4	285.4
3	524.9	524.9	316.2	316.0	316.0	316.0
4	512.1	512.1	0.0	308.0	308.0	308.0
5	580.0	580.0	0.0	350.4	350.4	350.4
6	462.1	462.1	0.0	276.8	276.8	276.8
7	495.8	495.8	0.0	297.8	297.8	297.8
8	580.4	580.4	0.0	350.6	350.6	350.6
9	609.0	609.0	0.0	368.5	368.5	368.5
10	534.0	534.0	0.0	321.7	321.7	321.7
11	550.3	550.3	0.0	331.8	331.8	331.8
12	624.9	624.9	0.0	378.5	378.5	378.5
13	550.3	550.3	0.0	331.8	331.8	331.8
14	563.4	563.4	341.3	340.0	340.0	340.0
15	498.7	498.7	299.0	299.6	299.6	299.6
16	496.7	496.7	297.7	298.3	298.3	298.3
17	561.7	561.7	340.2	339.0	339.0	339.0
18	528.5	528.5	318.5	318.2	318.2	318.2
19	516.5	516.5	310.7	310.7	310.7	310.7
20	588.4	588.4	357.6	355.7	355.7	355.7
21	595.3	595.3	362.1	359.9	359.9	359.9
22	510.6	510.6	306.8	307.0	307.0	307.0
23	577.1	577.1	350.2	348.6	348.6	348.6
24	480.8	480.8	287.3	288.4	288.4	288.4

*Units 7-12 are OFF all hours

Table (3-b) Power Sharing (MW) of SEC Example

HR	Unit Number*					
	19	20	21	22	23	24
1	79	79	0	0	61	61
2	79	79	0	0	0	0
3	79	79	0	0	0	0
4	79	79	0	0	0	0
5	79	79	0	0	0	0
6	76.72	76.72	0	0	0	0
7	79	79	0	0	0	0
8	79	79	0	0	61	61
9	79	79	0	54	61	61
10	79	79	0	54	61	61
11	79	79	0	54	61	61
12	79	79	0	54	61	61
13	79	79	0	54	61	61
14	79	79	0	54	61	61
15	79	79	0	0	61	61
16	79	79	0	0	61	61
17	79	79	0	0	61	61
18	79	79	0	0	61	61
19	79	79	0	0	61	61
20	79	79	54	0	61	61
21	79	79	54	0	61	61
22	79	79	54	0	61	61
23	79	79	54	0	61	61
24	79	79	54	0	61	61

**Units 13-18 are OFF all hours

Table (4) Load, Capacities, and Hourly Costs of SEC Example

HR	Load (MW)	Cap. (MW)	ED-Cost (\$)	ST-Cost (\$)	T-Cost (\$)
1	2657.40	3130.00	54775.40	0.00	54775.40
2	2250.00	3008.00	45999.60	0.00	45999.60
3	2472.00	3008.00	50741.00	0.00	50741.00
4	2106.00	2608.00	42773.60	0.00	42773.60
5	2369.00	2608.00	48573.60	0.00	48573.60
6	1908.00	2608.00	38586.20	0.00	38586.20
7	2043.00	2608.00	41424.70	0.00	41424.70
8	2492.60	2730.00	51259.50	0.00	51259.50
9	2657.40	2784.00	54977.10	0.00	54977.10
10	2367.00	2784.00	48460.10	0.00	48460.10
11	2430.00	2784.00	49845.70	0.00	49845.70
12	2719.20	2784.00	56406.80	0.00	56406.80
13	2430.00	2784.00	49845.70	0.00	49845.70
14	2822.20	3184.00	58419.50	0.00	58419.50
15	2475.00	3130.00	50828.60	0.00	50828.60
16	2466.00	3130.00	50636.70	0.00	50636.70
17	2760.40	3130.00	57053.60	0.00	57053.60
18	2610.00	3130.00	53738.90	0.00	53738.90
19	2556.00	3130.00	52567.40	0.00	52567.40
20	2935.50	3184.00	60969.70	0.00	60969.70
21	2966.40	3184.00	61672.80	0.00	61672.80
22	2583.00	3184.00	53177.20	0.00	53177.20
23	2884.00	3184.00	59805.10	0.00	59805.10
24	2448.00	3184.00	50303.70	0.00	50303.70

6. CONCLUSIONS

In this paper we considered a fuzzy model for the UCP. The model treats the uncertainty for the load demand and the spinning reserve in a FL frame. The problem is highly combinatorial. Even moderate size problems are being solved with great difficulty.

The paper presents a simulated annealing-based algorithm (SAFL) for solving the problem. The algorithm includes a step to find an initial control parameter at which virtually all-trial solutions are accepted. It uses a polynomial-time cooling schedule that is advocated in the SA

literature. Moreover, a penalty factor is calculated based in the membership functions of the load demand and spinning reserve. This penalty factor is use with the objective function in the optimization procedure to guide the search for optimal solution.

The proposed algorithm has been applied to a sample of the SEC system. A cost saving of 9.66% has been achieved over that obtained by a dynamic programming-based algorithm implemented in the SEC. The proposed algorithm is also compared with the SAA without fuzzy and showed better results [9].

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