The 6th Saudi Engineering Conference, KFUPM, Dhahran, December 2002 Vol. 5. 591

THE BUOYANCY EFFECTS ON THE BOUNDARY LAYERS INDUCED BY CONTINUOUS SURFACES STRETCHED WITH RAPIDLY DECREASING VELOCITIES

Mohamed E. Ali[1]

1:Associate professor, Mechanical Engineering Department, KSU.

P. O. Box 800, Riyadh 11421, Saudi Arabia.
Email: mali@ksu.edu.sa, Fax: 966-1-467-6652.

ABSTRACT

Heat and mass transfer characteristics of the self-similar boundary layer flows induced by continuous surfaces stretched with rapidly decreasing power law velocities $U_w \propto x^m, m < -1$ *are considered for mixed convection flow. The effect of various governing parameters, such as Prandtl number Pr, temperature exponent n, dimensionless injection/suction velocity* f_w*, and the mixed convection parameter* $\lambda = Gr/Re^2$ *are studied. These parameters have great effects on velocity and temperature profiles, heat transfer coefficient, and skin friction coefficient at the moving surface. Results show that similarity solutions exist only where the condition* $n = 2m - 1$ *is satisfied. Critical values of* λ, $Nu/Re^{0.5}$ *and* $C_f Re^{0.5}$ *are obtained for predominate natural convection for different Prandtl numbers at* $m = -2$ *and* $n = -5$*. Results also show that the effect of buoyancy is remarkable for weak than for strong suction. Furthermore, for* $f_w < -3.5$ *as Pr increases* $Nu/Re^{0.5}$ *increases and for* $-2.9 \geq f_w \geq -3.5$ *there are critical Prandtl numbers for which as Pr increases* $Nu/Re^{0.5}$ *increases. Finally, critical values of* λ, $C_f Re^{0.5}$ *are also obtained for predominate natural convection.*

Keywords: *mixed convection, stretched surfaces, power law, self-similar flow*

الملخص

تم دراسة انتقال الحرارة والكتلة في الطبقة الجدارية الناتجة بواسطة سطح مستمر السحب بسرعة تناقصية طبقاً لقانون القوى $U_m \propto x^m$ حيث m أقل من −١ في حالة انتقال الحرارة المختلط. تم دراسة تأثير العوامل المختلفة مثل رقم براندل Pr وأس درجة الحرارة n وسرعة الحقن أو الشفط f_w عديمة الوحدات ومعامل الحمل المختلط $\lambda = Gr/Re^2$ التي تتحكم في هذه الدراسة. هذه العوامل لها تأثير كبير على توزيع السرعة ودرجات الحرارة ومعامل انتقال الحرارة و معا مل الاحتكاك على السطح. وقد بينت النتائج أن طريقة الحلول المتماثلة موجودة في حالة $n = 2m - 1$. تم الحصول على القيم الحرجة لـ λ, $C_f Re^{0.5}$, $Nu Re^{-0.5}$ في حالة ظهور تأثير الحمل الطبيعي لقيم براندل المختلفة في حالة $m = -2, n = -5$. كما أوضحت النتائج أيضاً أن تأثير الطفو ملحوظ في حالة الشفط الضعيف عنه في حالة الشفط القوي. كما بينت النتائج أن $Nu Re^{-0.5}$ يزداد كلما زاد Pr في حالة $f_w < -3.5$ ولكن لـ $f_w > -3.5$ يوجد هنـاك قيـم حرجـة لرقـم براندل التي عندها إذا زاد رقم برانديل يزداد $Nu Re^{-0.5}$. وأخيراً تم إيجاد قيم حرجة لـ, $C_f Re^{0.5}$, λ التي عندها يكون تأثير الحمل الطبيعي سائداً.

List of symbols

C	constant
C_{fx}	local skin friction coefficient
f	dimensionless stream function
Gr	Grashof number [$=g\beta(T_w - T_\infty)\, x^3 / \nu^2$]
k	thermal conductivity
m	velocity exponent parameter
n	temperature exponent parameter
Nu	Nusselt number [=hx/k]
Pr	Prandtl number [ν/α]
Re	Reynolds number [$U_o x^{(m+1)}/\nu$]
T	temperature
u	velocity component in x-direction
U_o	constant
v	velocity component in y-direction
x	coordinate in direction of surface motion
y	coordinate in direction normal to surface motion

Greek symbols

α	thermal diffusivity
η	dimensionless similarity variable $\left[= \left[(\lvert m\rvert - 1)U_o/(2\nu)\right]^{1/2} x^{\frac{m-1}{2}} y \right]$
θ	dimensionless temperature [$=(T-T_\infty)/(T_w - T_\infty)$]
ν	kinematics viscosity

Subscripts

w	condition at the surface
∞	condition at the ambient medium

1. INTRODUCTION

Continuous surfaces moving through a quiescent fluid medium are encountered in several industrial-manufacturing processes. Such processes are hot rolling, wire drawing, metal and polymer extrusion, crystal growing, continuous casting, glass fiber and paper production, drawing of plastic films, etc. [Altan et al., 1979], [Fisher 1976], and [Tadmor and Klein,1988]. Both the kinematics of stretching and the heat transfer during such processes have a decisive influence on the quality of the final products.

Following the pioneering work of Sakiadis (1961), a rapidly increasing number of papers investigating different aspects of this problem have been published. For a list of the key references of a vast literature concerning this subject one should refer to the recent papers by Ali (2000, 1996, 1995, 1994), and by Ali and Al-Yousef (1998, 1997). The great majority of the theoretical investigations in this field describe the heat and mass flow in the vicinity of the continuous stretching surface with the aid of similarity solutions of the two-dimensional boundary layer equations. The kinematics' driving conditions of the real processes have been modeled in most cases by different power-law variations of the stretching velocity $U_w(x) = U_o \cdot x^m$. The stretching surface is considered either as an impermeable [Sakiadis (1961), Ali (1994), Magyari and Keller (1999a, b), Tsou et al (1967), Crane (1970), Vleggaar (1977), Soundalgekar and Murty (1980), Grubka and Bobba (1985)] or as a permeable surface [Ali (1996, 1995), Ali and Al-Yousef (2001, 1998, 1997), Al-Sanea and Ali (2000), Erickson and Fox (1966), Gupta and Gupta (1977), Chen and Char (1988), and Elbashbeshy (1998)] with a lateral mass flux of velocity $V_w(x) = c \cdot x^{(m-1)/2}$ where $c < 0$ corresponds to the suction and $c > 0$ to the injection of the fluid. As shown by Banks (1983), in the range $-1 < m \leq -1/2$ of the stretching exponent the flow boundary value problem (and thus also the heat transfer problem) does not admit similarity solutions if the wall is impermeable ($c = 0$). Recently it has been shown by Magyari and keller (2000a, b) that if $c < 0$, the similarity solution also persists for $m = -1/2$. The proof has been given by deducing the corresponding exact solution in an implicit analytic form.

The rapidly decreasing stretching velocities $m < -1$ should be relevant for several industrial manufacturing processes as e.g. the drawing of plastic films from a viscous molten mass. In this case the film just issuing from the slot (x=small) is hot and thus rapidly stretching. For increasing x however, it hardens in the colder ambient progressively and, as a consequence, the local stretching velocity $U_w \propto x^m$ decreases rapidly. This case has been investigated recently by Magyari et al (2001) and it was obtained that, for m < -1 the boundary layer equations admit self-similar solutions only if a lateral suction applied and the dimensionless suction velocity $f_w < 0$ must be strong enough. It was also found that the flow problem admits a non-denumerable infinity of solutions corresponding to the values of the dimensionless skin friction at the surface for a finite interval. The paper includes several examples which illustrate the dependence of the heat and fluid flows induced by surfaces stretching with rapidly decreasing velocities on the physical parameters f_w, m, n, and Pr.

The aim of the present paper is to extend the work of Magyari et al (2001) when the buoyancy force exists and the surface is moving vertically upward with a power law velocity for m = -2, pr = 0.1, 1, and 5 and for f_w = -3, -5, -7, and –10.

The mathematical formulation of the problem is presented in section (2), followed by the analytical solutions in section (3). Numerical solution procedure is presented in section (4) and results and discussion are reported in (5) and finally conclusions are given in section (6).

2. MATHEMATICAL ANALYSIS

Consider the steady two dimensional motions of mixed convection boundary layer flow from a vertically moving upward surface with suction or injection at the surface. For incompressible viscous fluid environment with constant properties using Boussinesq approximation, the equations governing this convective flow are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta\left(T-T_\infty\right)+\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the following boundary conditions:

$$\begin{aligned}&u(x,0)=U_w(x),\quad u(x,\infty)=0,\ v(x,0)=v_w(x)\\&T(x,0)=T_w(x),\quad T(x,\infty)=T_\infty\end{aligned} \tag{4}$$

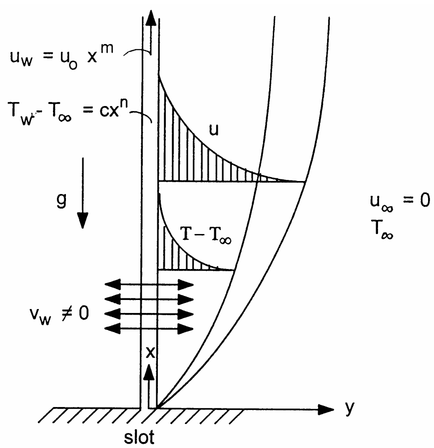

Figure 1. Schematic of the induced boundary layers close to a rapidly decreasing vertical surface moving with rapidly decreasing velocity.

The x coordinate is measured along the moving upward surface from the point where the surface originates, and the y coordinate is measured normal to it, as shown schematically in Fig. 1. u and v are the x and y components of the velocity field, respectively and $v_w(x)$ represents the lateral injection/suction velocity of the ambient fluid. Equations (1-3) admit similarity solutions only if the stretching velocity $U_w(x)$ and the temperature difference $T_w(x) - T_\infty$ as functions of the wall coordinate x are either of the power law type or of the exponential form. In the present paper the power law type is used for the case of m < -1 of the rapidly decreasing power law velocities when the buoyancy force exists. This range of the stretching velocities should be relevant for the drawing of plastic films from a viscous molten mass (see Magyari et al (2001) for the case of no buoyancy force). The usual similarity solutions arise when

$$u(x,y) = U_o\, x^m f'(\eta), \quad T(x,y) - T_\infty = C\, x^n \theta(\eta) \tag{5}$$

$$\eta = \left[\left(|m|-1\right)U_o/(2\nu)\right]^{1/2} \; x^{\frac{m-1}{2}} \; y \tag{6}$$

$$v(x,y) = \left[\frac{(m+1)\nu U_o)}{-2}\right]^{1/2} x^{\frac{m-1}{2}} \left[f(\eta) + \frac{m-1}{m+1}\eta f'(\eta)\right] \tag{7}$$

Where f' and θ are the dimensionless velocity and temperature respectively, and η is the similarity variable. Substitution in the governing equations gives rise to the following two-point boundary-value problem

$$f''' - f f'' + \frac{2m}{m+1} f'^2 - \frac{2}{m+1}\lambda\theta = 0 \tag{8}$$

$$\theta'' - \Pr\left(f\theta' - \frac{2n}{m+1} f'\theta\right) = 0 \tag{9}$$

The last term in Eq. (8) is due to the buoyancy force and $\lambda = Gr/\mathrm{Re}^2$ which serves as the buoyancy parameter, when λ = 0 the governing equations reduce to the forced convection limit given by Magyari et al (2001) for m < -1. However, when $\lambda \to \infty$ free convection should be dominated. Equations (8-9) are subject to the boundary conditions:

$$f'(0) = 1,\; f(0) = f_w,\; f'(\infty) \to 0 \tag{10}$$

$$\theta(0) = 1, \quad \theta(\infty) \to 0 \tag{11}$$

In driving the second of the boundary conditions given by Eq. (10) the horizontal injection or suction speed v_W must be a function of the distance (for m $\neq 1$) from the leading edge. Consequently, v_W for m < -1 is given by:

$$v_w(x) = \left[\frac{(|m|-1)\nu U_o}{2}\right]^{1/2} \quad x^{-(|m|+1)/2} \, f(0) \tag{12}$$

The quantity f(0) = f_w will be referred to as the dimensionless suction/injection velocity. Therefore, $f_w = 0$ corresponds to an impermeable surface, $f_w < 0$ to suction and $f_w > 0$ to lateral injection of the fluid through a permeable surface.

Expressions for the local skin friction coefficient, the local Reynolds and Nusselt numbers are given by:

$$C_{fx} = \left[\frac{2(|m|-1)\nu}{U_o}\right]^{1/2} \cdot x^{(|m|-1)/2} \cdot f''(0) \tag{13}$$

$$\mathrm{Re}_x = \frac{U_o}{\nu} x^{1-|m|}, \qquad \mathrm{Nu}_x = -\left[\frac{(|m|-1)U_o}{2\nu}\right]^{1/2} \cdot x^{-(|m|-1)/2} \cdot \theta'(0) \tag{14}$$

so that the dimensionless quantities

$$\frac{\mathrm{Nu}_x}{\sqrt{\mathrm{Re}_x}} = -\left(\frac{|m|-1}{2}\right)^{1/2} \cdot \theta'(0) \tag{15}$$

$$C_{fx} \cdot \sqrt{\mathrm{Re}_x} = [2(|m|-1)]^{1/2} \cdot f''(0) \tag{16}$$

becomes independent of the surface coordinate *x* and for m = -2 or for $|m|=2$ the above quantities reduce to

$$\frac{\mathrm{Nu}_x}{\sqrt{\mathrm{Re}_x}} = -\sqrt{\frac{1}{2}}\,\theta'(0), \qquad C_{fx}\sqrt{\mathrm{Re}_x} = \sqrt{2}\, f''(0) \tag{17}$$

It should be mentioned that the above analysis are valid for flows in which the Reynolds number is relatively large and in which there is no significant areas of reversed flow since the boundary layer approximations are used.

Top

3. NUMERICAL SOLUTION PROCEDURE

The coupled non linear ordinary differential equations (8) and (9) are solved numerically by using the fourth order Runge-Kutta method. Solutions of the differential Eqs. (8) and (9) subject to the boundary conditions (10, 11) were obtained for increasing values of λ at each constant f_w. At each new f_w we start from a known solution of the equations with $\lambda = 0$, (Magyari et al (2001)) where $f''(0)$ and $\theta'(0)$ are known. For a given value of λ the values of $f''(0)$ and $\theta'(0)$ were estimated and the differential equations (8) and (9) were integrated using Runge-Kutta method until the boundary conditions at infinity $f'(\eta)$ and $\theta(\eta)$ decay exponentially to zero. If the boundary conditions at infinity are not satisfied then the numerical routine uses a half interval method to calculate corrections to the estimated values of $f''(0)$ and $\theta'(0)$. These processes are repeated iteratively until exponentially decaying solutions in f' and θ were obtained. The value of η_∞ was chosen as large as possible depending upon the Prandtl number and the dimensionless suction/injection velocity f_w, without causing numerical oscillations in the values of f', and θ. The procedure was repeated for negative λ. The maximum and minimum λ were obtained for different values of f_w where the numerical solutions became more difficult to obtain as λ approached λ_{max} or λ_{min} for the corresponding f_w. It should be mentioned that beyond these values of λ the computation is difficult to obtain and unstable where the boundary layer assumptions are not valid and one should use another method of solution.

4. RESULTS AND DISCUSSION

Equations (8) and (9) were solved numerically, as described in section 3, for m = -2, f_w = -3, -5, -7, -10, Pr = 0.1, 1.0, 5, and for temperature exponent n = -5. It should be mentioned that, n = 2m –1 is the necessary and sufficient condition for the existence of the similarity solutions of power law type and for m = -2 gives n = -5. Samples of the resulting velocity and temperature profiles for Pr = 1, f_w = -3, and for λ = -1, -0.1, 0, 0.1 are presented in Fig. 2 and Fig. 3 respectively. It can be seen from Fig. 2 that for $\lambda < 0$, as λ decreases the velocity gradient at the surface decreases and this tends to reduce the shear stress at the surface as we will see in Fig. 6. However, for $\lambda >$ o as λ increases the slope at the surface increases and hence the shear stress at the surface increases. It worth mentioning that, more solutions for large values of $|\lambda|$ can be obtained but with low accuracy and they have been rejected.

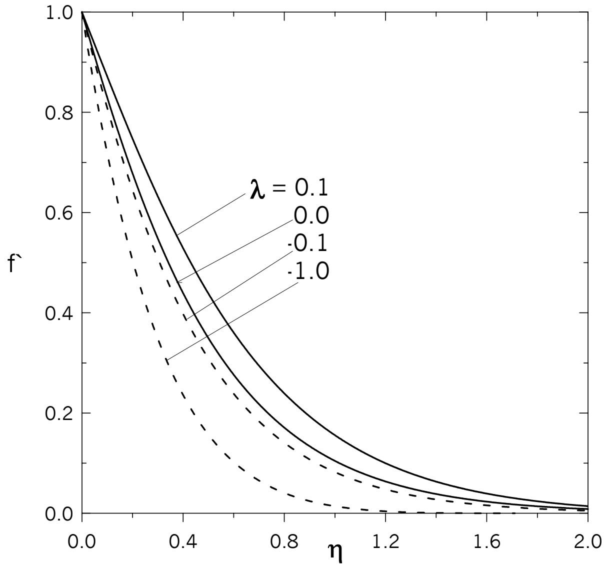

Figure 2. Samples of similarity velocity profiles for Pr = 1, f_w = -3, m = -2, n = -5, and for various values of λ, λ > 0 (solid) and λ < 0 (dashed).

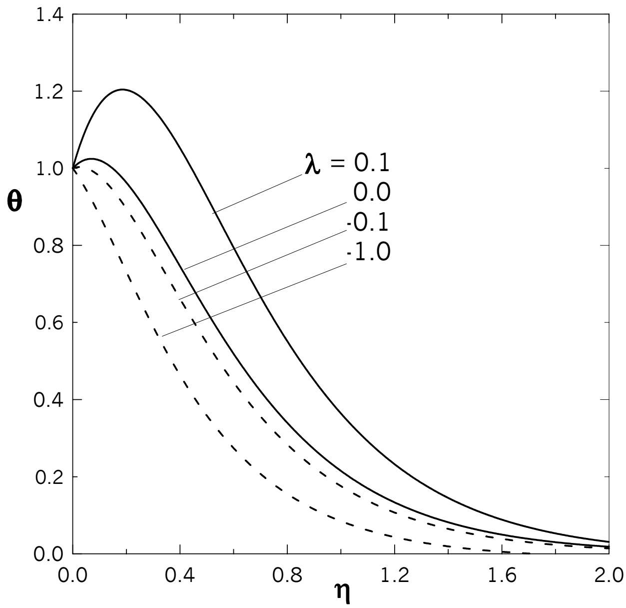

Figure 3. Temperature profiles for Pr = 1, f_w = -3, m = -2, n = -5, and for various values of λ, λ > 0 (solid) and λ < 0 (dashed).

Figure 3 shows the temperature profiles for the same parameters as in Fig. 2. In this figure it is clear that as λ increases the temperature gradient increases and for λ > 0 the slope is always positive therefore according to Eq. (15) the dimensionless quantity $Nu_x/\sqrt{Re_x}$ always decreases. Physically this is because the heat flows always into the surface despite the surface temperature's continual excess over the ambient temperature. This is a consequence of a fluid particle heated to nearly the surface temperature being convected downstream to a place at which surface temperature is lower. Then, since the fluid particle is warmer than the surface, heat flows into the surface. Such situation results in negative heat transfer coefficients and means only that the temperature gradient at the surface is no longer proportional to $T_w - T_\infty$ (see Burmeister (1983) for similar type of flows). However, for λ < 0 the temperature gradient is positive up to λ ≈ -0.185 where the slope is almost zero ($Nu_x/\sqrt{Re_x}$ = 0.00018) and for λ < -0.185, the slope at the surface is always negative and the heat is always transferred from the surface to the ambient medium.

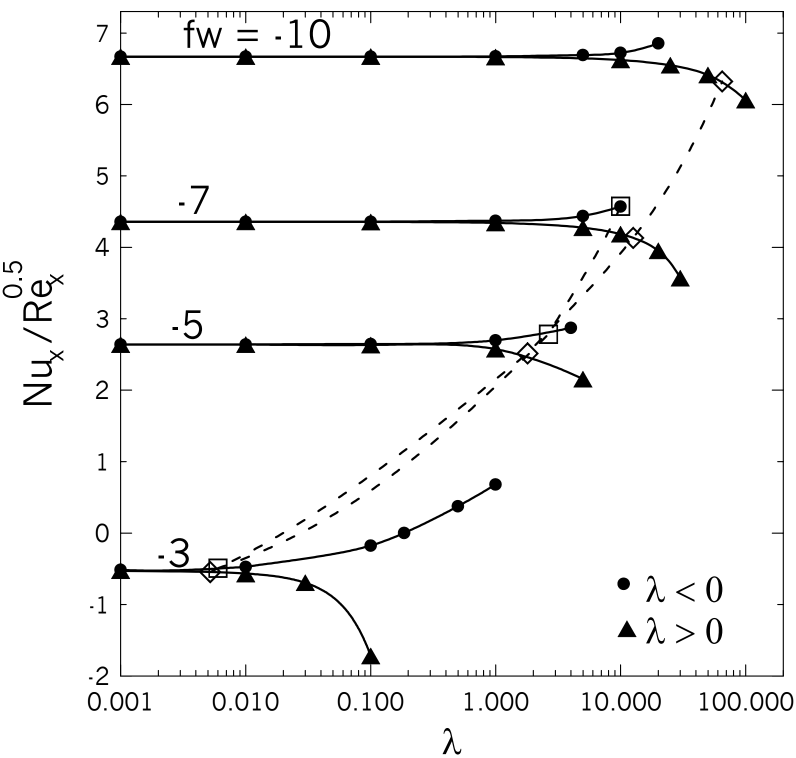

Figure 4. Local $Nu_x\ Re_x^{-0.5}$ distribution for the entire mixed convection at n = -5, m = -2, and for Pr = 1. Dashed lines present the locus separating the natural convection dominant region on the right and forced convection region on the left.

Table 1. Critical values of λ, $Nu_x\, Re_x^{-0.5}$, and $C_{fx}\, Re_x^{0.5}$ for predominate natural convection at different values of Pr and f_w for m = -2 and n = -5 for λ > 0. Stars mean that $C_{fx}\, Re_x^{0.5}$ has the same value corresponding to the same f_w at Pr = 0.1.

	Pr = 0.1				Pr = 1				Pr = 5	
f_w	λ	$Nu_x\, Re_x^{-0.5}$	λ	$C_{fx}Re_x^{0.5}$	λ	$Nu_x\, Re_x^{-0.5}$	λ	$C_{fx}Re_x^{0.5}$	λ	$C_{fx}\, Re_x^{0.5}$
-3	.00065	-0.14905	0.003	-2.4012	0.0052	-.54757	.028	*	0.4	*
-5	.0127	0.18794	0.038	-5.9818	1.8	2.51402	.42	*	2.5	*
-7	.070	0.37152	0.09	-8.9318	12.6	4.13137	1.0	*	5.4	*
-10	.33	0.60816	0.25	-13.002	65	6.32232	2.4	*	12	*

Table 2. Critical values of λ, $Nu_x\, Re_x^{-0.5}$, and $C_{fx}\, Re_x^{0.5}$ for predominate natural convection at different values of Pr and f_w for m = -2 and n = -5 for λ < 0.

	Pr = 1.0				Pr = 5			
f_w	λ	$Nu_x\, Re_x^{-0.5}$	λ	$C_{fx}Re_x^{0.5}$	λ	$Nu_x\, Re_x^{-0.5}$	λ	$C_{fx}Re_x^{0.5}$
-3	-.006	-0.49249	-0.03	-2.64395	-9.2	7.9896	-0.4	*
-5	-2.65	2.78267	-0.45	-6.62949	----	-------	-2.5	*
-7	-10	4.573863	-1.0	-9.82948	----	--------	-5.4	*
-10	-----	----------	-2.35	-14.48543	----	--------	-12.0	*

Similar profiles are obtained f_w = -5, -7, and –10. However, as the dimensionless suction/injection velocity f_w decreases (strong suction) the heat is always transferred from the surface to the ambient medium for λ ≤ 0. Figure 4 shows the effect of strong suction where f_w change from –3, to -10 for both negative and positive λ for Pr = 1, m = -2, and n = -5. As we see from the figure that as the suction increases the heat transfer to the medium increases and the effect of λ is remarkable for low suction only and as suction increases it sustains the buoyancy effects. A 5% increase or decrease has been taken from the value $Nu_x\, Re_x^{-0.5}$ at λ = 0 for both λ < 0 and λ > 0 respectively, to show the effect of predominate natural convection. Therefore, the dashed lines connected the squares and diamonds present the locus of these points for negative or positive λ respectively. The coordinates of these points, which we call them the critical values, are tabulated in Table 1 for various values of Pr and f_w for λ > 0 and in Table 2 for λ < 0. It should be mentioned that cells marked by dashed in Table 2 mean there are no solutions could be obtained with reasonable accuracies satisfying the boundary conditions at infinity. Furthermore in Fig. 4, the region on the left of the dashed lines presents predominate forced convection and the region on the right presents predominate natural convection.

Top

An intensive study has been made on the effect of Prandtl number on the heat transfer from the surface when the dimensionless suction velocity f_w is used as a parameter. Figure 5 shows these results where $Nu_x\, Re_x^{-0.5}$ decrease as Pr increase up to a critical value, which depends upon f_w and as f_w decreases (strong suction) these critical values of Pr decrease. However, beyond these critical Prandtl numbers as Pr increases $Nu_x\, Re_x^{-0.5}$ increases for any value of f_w. Dashed line on Fig. 5 connecting the critical Prandtl number designated by squares. These critical values of Prandtl numbers and $Nu_x\, Re_x^{-0.5}$ are given in Table 3. The following third order polynomial is used to fit the data of Table 3 with a maximum deviation between the numerical data points and the correlation of 1.11%. The polynomial coefficient A, B, C, and D are 0.0278, -0.296, -0.1708, and –1.1243, respectively.

$$Nu_x\, Re_x^{-0.5} \;=\; A + B\,Pr + C\,Pr^2 + D\,Pr^3, \quad -2.9 \geq f_w \geq -3.5 \tag{18}$$

Table 3. Critical values of Pr and $Nu_x\, Re_x^{-0.5}$ for $\lambda = 0$, where profiles of f_w in figure 5 have minimums designated by squares and connected by a dashed line.

f_w	-2.9	-3.0	-3.05	-3.1	-3.2	-3.3	-3.5
Pr	0.95	0.7	0.6	0.55	0.4	0.3	0.1
$Nu_x\, Re_x^{-0.5}$	-1.20491	-0.62328	-0.45927	-0.340	-0.18219	-0.0893	-0.00968

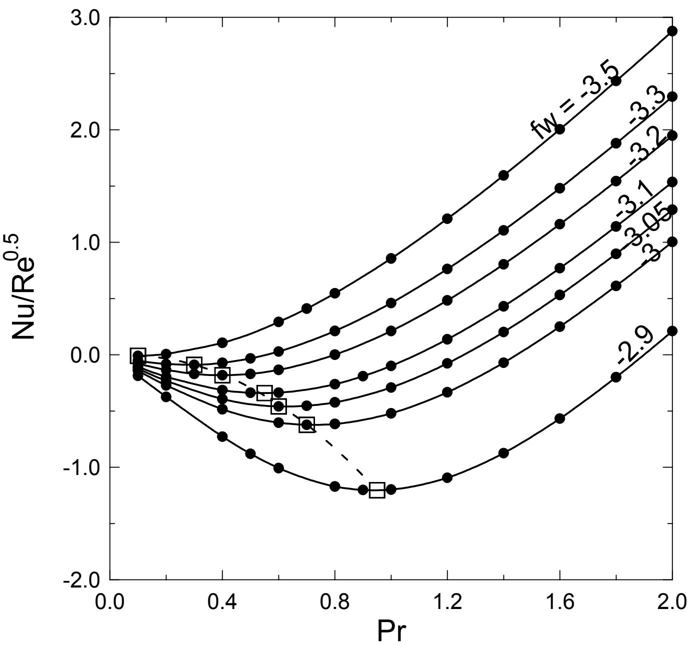

Figure 5. The effect of Prandtl number on $Nu_x\, Re_x^{-0.5}$ for different values of the dimensionless suction velocity f_w, dashed line connecting critical coordinates of Pr- $Nu_x\, Re_x^{-0.5}$ are designated by squares.

Top

Therefore, for $f_w < -3.5$ as Pr increases, the $Nu_x Re_x^{-0.5}$ increases but, for $-2.9 \geq f_w \geq -3.5$ one has to define the critical Prandtl number corresponding to each f_w. It should be mentioned that, the flow problem for $\lambda = 0$ does not admit solutions for $f_w > -2.75$ [see Magyari et al (2001)]. In spite that, Fig. 5 is constructed for $\lambda = 0$ but the critical values are still valid for $\lambda \neq 0$.

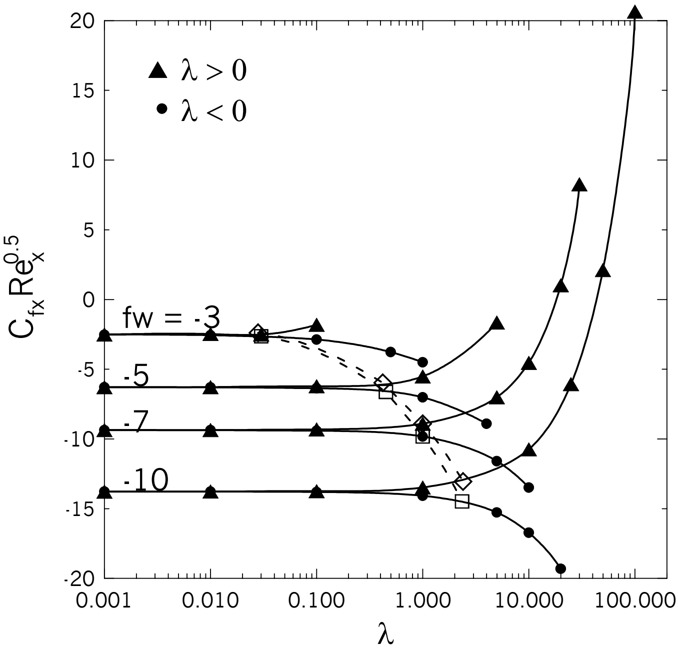

Figure 6. Shear stress profiles for different values of f_w and for Pr = 1. Dashed lines connecting the squares and diamonds are for $\lambda < 0$ and > 0, respectively for predominate natural convection.

Figure 6 shows the dimensionless shear stress profiles at the surface for various values of f_w and for positive or negative λ for m = -2, n = -5 and for Pr = 1. Five percent increase or decrease in $C_{fx} Re_x^{0.5}$ distributions at $\lambda = 0$ has been applied for $\lambda > 0$ and $\lambda < 0$ respectively. Dashed lines connecting squares and diamonds are the locus of these 5%, where on the right of these lines natural convection dominates and on the left forced convection dominates. Coordinates of squares and diamonds symbols, which we call them critical values of λ and $C_{fx} Re_x^{0.5}$, are given in Table 1 and 2 for positive and negative λ and for various values of Pr and f_w. In Table 1 and 2 stars mean that $C_{fx} Re_x^{0.5}$ has the same value corresponding to the same f_w at Pr = 0.1 and Pr = 1 respectively. In other words that, the dimensionless shear stress at the surface is independent of Prandtl number and the effect of Pr is only to sustain the effect of buoyancy. Furthermore, no solutions with reasonable accuracies to satisfy the boundary conditions at infinity for Pr = 0.1 are obtained therefore, results for Pr = 0.1 is not in the content of Table 2. It is clear from Fig. 6 that as the dimensionless suction velocity f_w decreases (strong suction) the dimensionless shear stress decreases, since the velocity profiles get steeper at the surface. Moreover, the dimensionless shear stress increases for positive increasing values of λ and decreasing for negative decreasing values of λ.

5. CONCLUSIONS

Mixed convection effect on a vertically moving surface stretched with rapidly decreasing velocity has reported for three Prandtl numbers of 0.1, 1, and 5 and for different values of the dimensionless suction velocity f_w at the surface. Similarity solutions were obtained for m = -2, and n = -5, where m and n are the exponent of the power law velocity and temperature respectively.

Numerical computation where performed for both λ > 0 and λ < 0. The results show that, for λ < 0: the dimensionless quantity at the surface $Nu_x Re_x^{-0.5}$ increases as λ decreases however, the dimensionless shear $C_{fx} Re_x^{0.5}$ decreases as λ decreases. For λ > 0: $Nu_x Re_x^{-0.5}$ decreases as λ increases but $C_{fx} Re_x^{0.5}$ increases as λ increases. Furthermore, as the dimensionless suction velocity f_w decreases (strong suction) the quantity $Nu_x Re_x^{-0.5}$ increases but $C_{fx} Re_x^{0.5}$ decreases.

Critical values of $Nu_x Re_x^{-0.5}$, $C_{fx} Re_x^{0.5}$, and λ are obtained for predominate natural convection and tabulated for various values of f_w and Pr in Table 1 and 2. Moreover, critical values of $Nu_x Re_x^{-0.5}$ and Pr, where profiles of f_w have minimums, are obtained to distinguish the behavior of $Nu_x Re_x^{-0.5}$ with Pr for $-2.9 \geq f_w \geq -3.5$. These critical values are tabulated in Table 3 and presented in a fitting polynomial form by Eq. (18).

ACKNOWLEDGEMENT

The author would like to acknowledge Professor Eugine Magyari at Swiss Federal Institute of Technology (ETH) Zurich in Switzerland for an interesting discussion on the subject of this paper.

REFERENCES

1. Ali, M. E., 2000, "Heat transfer and flow field of a continuously moving surface with suction or injection," *Trends in Heat, Mass & Momentum Transfer*, **6**, pp 135-140.
2. Ali, M. E., 1994, "Heat transfer characteristics of a continuous stretching surface," *Warme-und Stoffubertragung*," **29**, pp 227-234.

3. Ali, M. E., 1996, "The Effect of Suction or Injection on the Laminar boundary layer Development over a Stretched surface," *J. of King Saud University, Engineering Sciences*, **8** (1), pp 43-58.
4. Ali, M. E., 1995, "On thermal boundary layer on a power-law stretched surface with suction or injection," *Int. J. Heat and Fluid Flow*, **16** (4), pp 280-290.
5. Ali, M. E. and Al-Yousef, F., 1998, "Laminar mixed convection from a continuously moving vertical surface with suction or injection," *Heat and MassTransfer,* **33**, pp 301-306.
6. Ali, M. E. and Alyousef, F., 1997, "Heat transfer and flow field on an extruded vertical material," *Proceedings, The 10th Int. Conference of Mechanical Power Engineering*, p 207, Assiut, Egypt.
7. Ali, M. E. and Alyousef, F., 2001, "Laminar mixed convection boundary layers induced by a linearly stretching permeable surface," *Proceedings of ICFDP7, The 7th Int. Congress of Fluid Dynamics & Propulsion ,Paper No. ICFDP7-2001014*, Cairo, Egypt.
8. Al-Sanea, S. A. and Ali, M. E., 2000, " The effect of extrusion slit on the flow and heat transfer characteristics from a continuously moving material with suction or injection," *Int. J. of Heat and Fluid Flow*, **21**, pp 84-91.
9. Altan, T., Oh, S., and Gegel, H., 1979, Metal Forming Fundamentals and Applications, American Society of Metals, Metals Park, OH, USA.
10. Banks, W. H. H., 1983, "Similarity Solutions of the boundary-layer equations for a stretching wall," *Journal de Mecanique Theorique et Appliquee*, **2** (3), pp 375-392.
11. Burmeister, L. C., 1983, Convective heat transfer, John Wiley & Sons, New York, USA.
12. Chen, C. K. and Char, M. I., 1988, "Heat transfer of a continuous stretching surface with suction or blowing," *J. Math. Analysis and Applications*, **135**, pp 568-580.
13. Crane, L. J., 1970, "Flow paste a stretching plane," *Z. Amgew. Math. Phys*., **21**, pp 645-647.
14. Elbashbeshy, E. M. A., 1998, "Heat transfer over a stretching surface with variable surface heat flux," *J.Phys. D: Appl. Phys*. **31,** pp 1951-1954.
15. Erickson, L. E., Fan, L. T., and Fox, V. G., 1966, "Heat and Mass Transfer on a Moving Continuous Flat Plate With Suction or Injection," *Ind. Eng. Chem. Fundamentals*, **5**, pp 19-25.
16. Fisher, E.G., 1976, Extrusion of plastics, Wiley. New York, USA.
17. Grubka, L. G. and Bobba, K. M., 1985,"Heat transfer characteristics of a continuous stretching surface with variable temperature," *ASME J. of Heat Transfer*, **107**, pp 248-250.
18. Gupta, P. S., and Gupta, A. S., 1977, "Heat and mass transfer on a stretching sheet with suction or blowing," *Canadian Journal of Chemical Engineering*, **55**, (6), pp 744-746.
19. Magyari, E., Ali, M. E., and Keller, B., 2001, "Heat and mass transfer characteristics of the self-similar boundary-layer flows induced by continuous surface stretched with rapidly decreasing velocities," *Heat Mass Transfer,* **38,** pp 65-74.
20. Magyari, E. and Keller, B., 1999a, "Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface," *J. Phys. D: Appl. Phys*., **32**, pp577-585.
21. Magyari, E and Keller B., 1999b, "Heat transfer characteristics of the separation boundary flow induced by a continuous stretching surface," *J. Phys. D: Appl. Phys*. **32,** pp 2876-2881.
22. Magyari, E. and Keller, B., 2000a, "Exact solutions for self-similar boundary-layer flows induced by permeable stretching surfaces," *Eur.J.Mech.B-Fluids* **19,** pp 109-122.

23. Magyari, E. and Keller, B., 2000b, "Exact analytic solutions for free convection boundary-layers on a heated vertical plate with lateral mass flux embedded in a saturated porous medium," *Heat Mass Transfer* **36,** pp 109-116.
24. Sakiadis, B. C., 1961, "Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow," *A.I.Ch.E. Journal*, **7**, (1), pp 26-28.
25. Soundalgekar, V. M., and Ramana Murty, T. V., 1980, "Heat transfer past a continuous moving plate with variable temperature," *Warme- und Stoffubertragung*, **14**, pp 91-93.
26. Tadmor, Z., and Klein, I., 1970, Engineering Principles of Plasticating Extrusion, Polymer Science and Engineering Series, Van Nostrand Reinhold, New York, USA.
27. Tsou, F. K., Sparrow, E. M., and Goldstein, R. J., 1967, "Flow and heat transfer in the boundary layer on a continuous moving surface," *Int. J. Heat Mass Transfer*, **10**, pp 219-235
28. Vleggaar, J., 1977, "Laminar boundary-layer behavior on continuous accelerating surfaces", *Chemical Engineering Science*, **32**, pp 1517-1525.

Top