



# ON THE INVESTIGATION OF VIBRATION SIGNALS USING JOINT TIME FREQUENCY ANALYSIS

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## ABSTRACT

*This paper addresses the problem of coupled blade bending and shaft torsional vibration signals using the Joint Time Frequency Analysis (JTFA). Simulation results for the blade bending and shaft torsional vibration are studied using the Fast Fourier Transform (FFT) and the JTFA. The FFT spectra showed little information on the nonlinear dynamic interaction between the blade bending and the shaft torsional vibration; and thus cannot be used as a tool for monitoring the blade vibration by looking into the shaft torsional vibration signal. In contrast, the JTFA in the form of Wigner Ville Distribution (WVD) has given more useful information and reflected the dynamic interaction between different vibration modes on one side and between the blade vibration and the shaft torsional vibration on the other side. The obtained WVD representations of the shaft torsional vibration showed frequency layers that represent blade vibration activity.*

**Keywords:** *Vibration, Rotating, Blades, Spectrums, Wigner, JTFA, Shaft, Torsional*

(FFT)

( JTFA )

## 1. INTRODUCTION

Vibration measurement process has been used extensively as a tool for machinery health monitoring and it has been playing an increasingly important role in maintenance programs and in maintenance planning. The process of vibration measurement is generally constituted of the vibration signal collection that is done by the vibration pick-up (transducer) and the data representation, which is done in the signal-processing unit. Machinery vibration problems are encountered some times in components that are unreachable for direct measurement and thus closer dynamically related component should be monitored. This problem occurs typically in the vibration of rotating blades in turbomachinery. Instead of direct monitoring of the blade, the carrying shaft torsional vibration can be thought of as the closest system that has direct dynamic interaction with the blades. To make up for this indirect method of measurement, and in order to extract useful information on the rotating blade vibration, more investment in the signal processing should be put.

The recognition of the dynamic interaction between the rotating blade and the shaft torsional vibrations was reported first by [Okabe et. al. 1991]. They introduced an equivalent reduced order model that coupled shaft torsional and blade tangential vibrations. Their model adopted the modal synthesis procedure, wherein the blade was modeled as simple mass-spring subsystem and the shaft as the other discrete subsystem. The two subsystems were coupled and the natural frequencies were obtained. [Okabe et. al. 1991] compared their model predictions to actual measurements and reported that they are in close agreement. [Huang and Ho 1996] reported results of study on the coupled shaft torsion and blade bending vibrations of a rotating shaft-disk-blade unit. The shaft torsional and blade bending deformations were modeled, separately, using the Assumed Modes Method. They used the weighted residual method and the receptances at the connection between the disk and the blade to couple the shaft-torsional and blade bending deformations. [Al-Bedoor 1998, 1999], based on the multi-body dynamic approach, developed a coupled model for shaft-torsional and blade-bending vibrations in rotors. The model employed the finite element method to discretize the blade deformations. The study identified the nonlinear interaction and the destabilizing effect that the blade and shaft could introduce to excite each other. Due to the difficulty encountered in quantifying the nature of nonlinear coupling when the finite element method is used, a reduced-order nonlinear dynamic model for shaft-torsional and blade-bending vibrations was reported by [Al-Bedoor, 2001]. The obtained model showed that the blade vibration and the shaft torsional vibration are coupled. However, an identification of the nature of coupling was not addressed.

The present work is devoted towards studying the vibration signals obtained from the model of [Al-Bedoor, 2001] using the Fast Fourier Transform (FFT), Short Time Fourier Transform (STFT) and Joint Time Frequency Analysis (JTFA). The objective of applying these signal processing techniques is to extract useful information on the problem of blade vibration from monitoring the shaft torsional vibration signals.

## 2. THE DYNAMIC MODEL

The equations of motion of the model, shown in Figures 1 and 2, reported by [Al-Bedoor, 2001] are taken as

$$\begin{bmatrix} m_{\theta\theta} & m_{\theta\psi} \\ m_{\psi\theta} & m_{\psi\psi} \\ m_{q\theta} & m_{q\psi} \end{bmatrix} \begin{bmatrix} m_{\theta q} \\ m_{\psi q} \\ I_{qq} \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{\psi} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} c_{\theta\theta} & c_{\theta\psi} & [c_{\theta q}] \\ c_{\psi\theta} & c_{\psi\psi} & [0] \\ [c_{q\theta}] & [0] & [c_{qq}] \end{bmatrix} \begin{Bmatrix} \dot{\theta} \\ \dot{\psi} \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & [0] \\ 0 & k_{\psi\psi} & [0] \\ [0] & [0] & [k_{qq}] \end{bmatrix} \begin{Bmatrix} \theta \\ \psi \\ q \end{Bmatrix} = \begin{Bmatrix} F_{\theta} \\ F_{\psi} \\ F_q \end{Bmatrix} \quad (1)$$

where,

$$m_{\theta\theta} = J_M + J_D + J_B(1 + \psi^2) + \{q\}^T [(1 + \psi^2)I] - [k_s] \{q\} \quad (2)$$

$$m_{\psi\psi} = m_{\theta\psi} = m_{\psi\theta} = J_D + J_B + \{q\}^T [I] - [k_s] \{q\} \quad (3)$$

$$[m_{\theta q}] = [m_{q\theta}]^T = (1 + \psi^2)[h] \quad (4)$$

$$[m_{\psi q}] = [m_{q\psi}]^T = [h] + \psi \{q\}^T [I] \quad (5)$$

$$[m_{qq}] = (1 + \psi^2)[I] \quad (6)$$

$$c_{\theta\theta} = 2\psi\dot{\psi}[J_B + \{q\}^T \{q\}] + 2\{q\}^T (1 + \psi^2)[I] - [k_s] \{\dot{q}\} \quad (7)$$

$$c_{\psi\psi} = c_{\theta\psi} = c_{\psi\theta} = 2\{q\}^T ([I] - [k_s]) \{\dot{q}\} \quad (8)$$

$$[c_{\theta q}] = [c_{q\theta}]^T = 2\psi\dot{\psi}[h] \quad (9)$$

$$[c_{qq}] = 2\psi\dot{\psi}[I] \quad (10)$$

$$k_{\psi\psi} = k_T - \dot{\theta}^2 (J_B + \{q\}^T \{q\}) - 2\dot{\theta}[h]\{\dot{q}\} \quad (11)$$

$$[k_{qq}] = \frac{EI}{\rho L^4} [k] + (\dot{\theta} + \dot{\psi})^2 [[k_s] - [I]] + (\dot{\psi}^2 - \psi^2 \dot{\theta}^2) [I] \quad (12)$$

$$J_B = \rho \left( \frac{L^3}{3} + R_D L^2 + R_D^2 L \right) \quad (13)$$

where  $\theta$  is the rigid body rotation,  $\psi$  is the torsional deformation angle and  $\{q\}$  is the blade modal deflections.

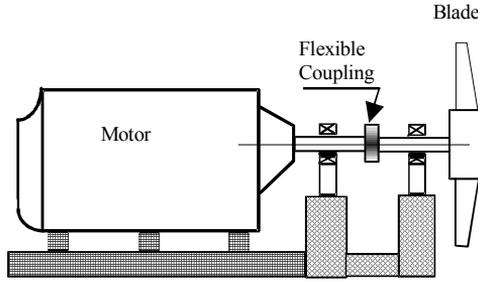


Figure 1: Schematic diagram of blade-disk-shaft system.

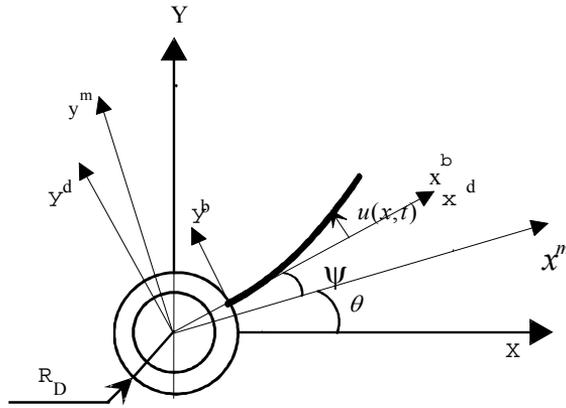


Figure 2: System deformed configuration and coordinate system.

The equations of motion, (1), are arranged in a way such that all nonlinear terms are in the position to contribute to the system inertia, damping and stiffness.

### 3. NUMERICAL SIMULATION AND DISCUSSION

The dimensions and material properties of the blade-shaft-disk system are given in Table 1. For the considered system data, the uncoupled natural frequencies, calculated for the shaft torsional and blade bending separately are compared to the coupled system natural frequencies in Table 2. The natural frequencies results show that the natural frequencies are tremendously changed for the shaft torsional and blade first bending vibration mode due to coupling.

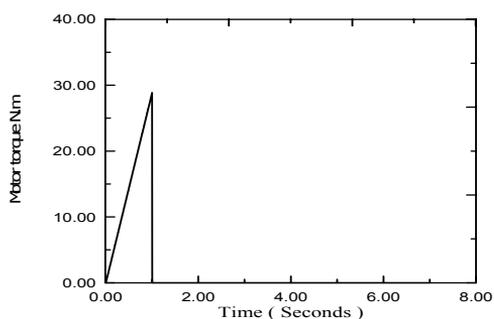
The motor torque that is designed using the inverse dynamic procedure to rotate the system to reach a constant speed of 1000 RPM in 1.0 second and the resulting system speeds are shown in Figure 3a and b, respectively.

Table 1: Blade-Disk-Shaft Data

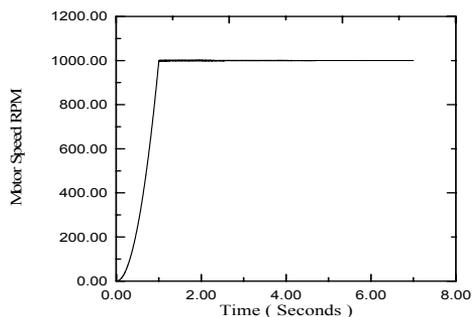
Property	Value
Blade length $L$	0.4 m
Blade mass per unit length, $\rho$	1.35 kg/m
Blade flexural rigidity, $EI$	$75N.m^2$
Disk Radius, $R_D$	0.05 m
Disk moment of inertia, $J_D$	$8.8 \times 10^{-3} kg.m^2$
Motor moment of inertia, $J_M$	$8.8 \times 10^{-2} kg.m^2$
Torsional stiffness $k_T$	9880 N.m/Rad

Table 2: System Natural Frequencies Hz

D.O.F	Un-coupled	Coupled
Torsional	70.9	25
Blade 1 <sup>st</sup>	26.8	119.6
Blade 2 <sup>nd</sup>	163.3	174.4
Blade 3 <sup>rd</sup>	457.43	460.1
Blade 4 <sup>th</sup>	896.38	898
Blade 5 <sup>th</sup>	1481.73	1482.9



(a)



(b)

Figure 3: (a) Motor torque to rotate the system to a speed of 1000 RPM in 1 second and (b) System speed with time.

Due to the application of the external torque to drive the system, the blade-blending and the shaft torsional vibration are monitored as shown in Figures 4 and 5, with their corresponding frequency spectra, respectively.

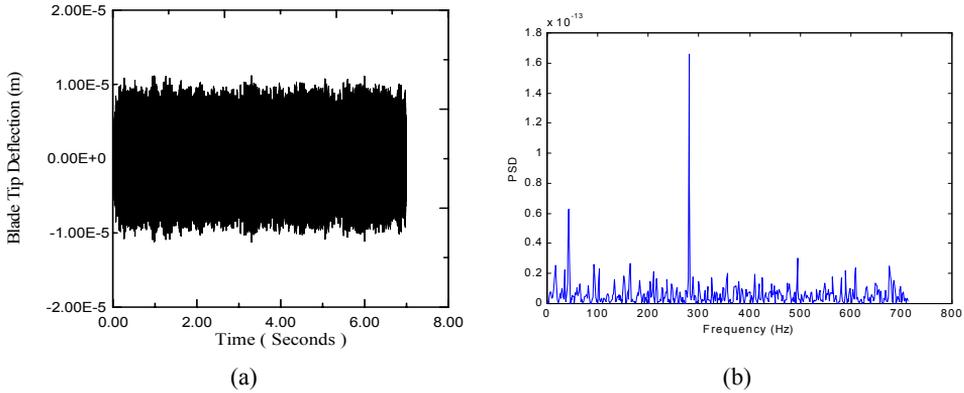


Figure 4: (a) Blade tip deflection with time and (b) Frequency spectrum using FFT.

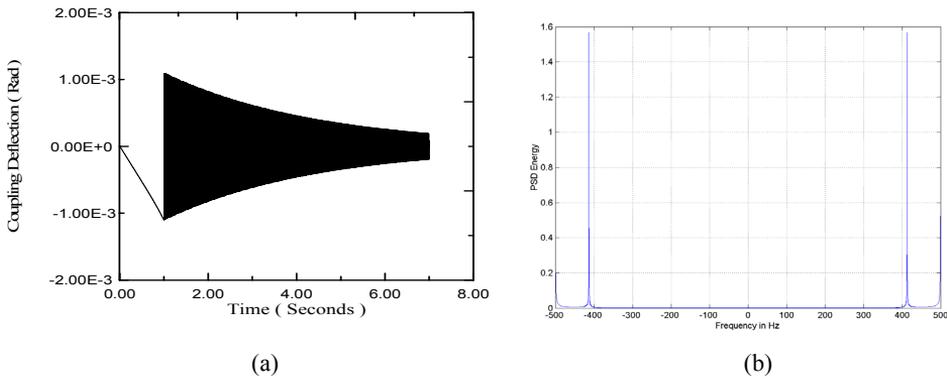
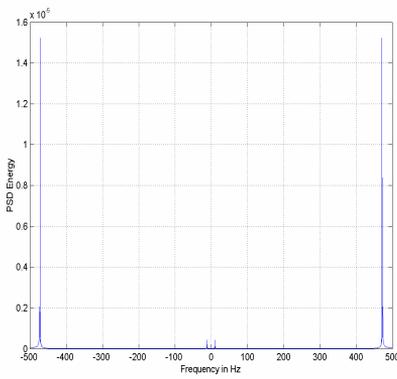


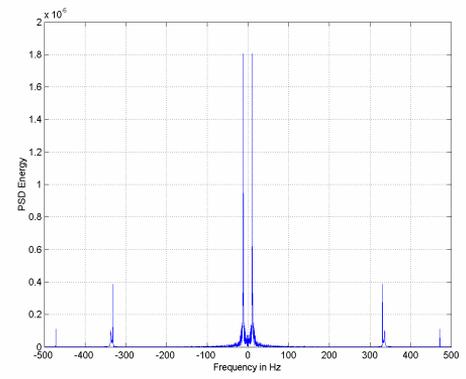
Figure 5: (a) Coupling torsional vibration with time and (b) Frequency spectrum using FFT.

The FFT of the blade vibration, Figure 4-b, shows frequency contents of about 12 Hz, 40 Hz and 290 Hz. In the same time, the frequency spectrum of the torsional vibration, Figure 5-b, shows a dominant component at around 410 Hz, which is neither the shaft running speed, nor the natural frequency of the shaft in torsion. In fact, the spectrum of the torsional vibration gives an indication that one of the blade higher modes natural frequency have appeared. In order to gain more insight into this result, the FFT of the first 4-modes of the blade vibration are shown in Figure 6. The spectrum of the first mode, Figure 6-a, contains frequency of

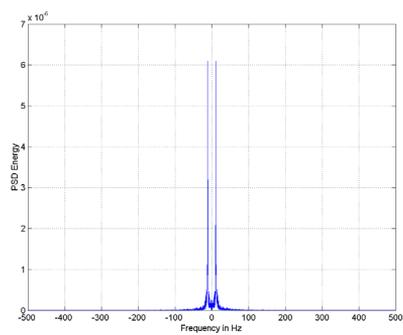
12 Hz and 472 Hz, the second mode's frequency spectrum, Figure 6-b contains 12 Hz, 432 Hz and 472 Hz, the third mode FFT, Figure 6-c, contains only the 12 Hz component and the fourth mode FFT, Figure 6-d, contains the 12 Hz and 396 Hz frequency components. As seen from the results of the FFT for the blade tip deflection, the torsional vibration and the particular modes, extracting useful information on how the system frequencies are appearing is not an easy task. The main reason for this behavior can be referred, partly, to contribution the FFT algorithm. The FFT algorithm is known to treat the whole signal as one entity and removes any time dependency, or frequency content as time is changing. In the nonlinear system at hand it is expected that the system reach certain vibration frequency starting from some initial frequency, which gets tuned as a result of the system nonlinearity and the dynamic interaction. To overcome this deficiency and to try to learn more about the system dynamic and nonlinear interaction, Joint Time Frequency Analysis (JTFA) will be applied to the tip deflection and the shaft torsional vibration signals in the coming section.



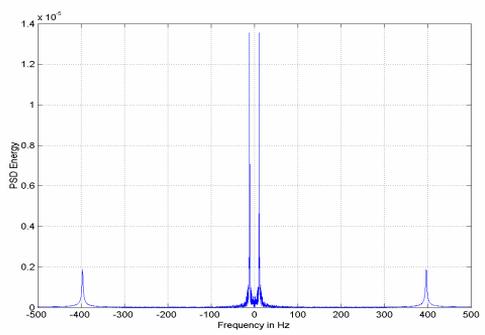
(a)



(b)



(c)



(d)

Figure 6: Frequency spectra of the blade modes: (a) First, (b) Second, (c) Third, and (d) Fourth.

#### 4. JOINT TIME FREQUENCY ANALYSIS (JTFA)

Given a signal,  $x(t)$ , one can readily see the energy distribution in time. However, by performing a Fourier Transform (FT) to obtain the signal spectrum,  $X(w)$ , one can take a look at the energy distribution and the frequency content in the frequency domain. For a stationary signal, there is usually no need to go beyond the time and frequency domains [Qian and Chen, 1998]. But most real signals, such as vibration signals, have characteristics that change over time at slow (quasi-stationary signals) or fast (transient signals) speed, such as the case of vibration problem we are currently addressing in this paper.

The distribution in the time defined as the squared magnitude of the signal,  $|x(t)|^2$ , and the distribution in frequency is defined in terms of the signal Fourier transform FT,  $|X(w)|^2$ .

$$X(w) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad (14)$$

It is worth noting that the FT representation, being a unitary operator [Oppenheim and Schaffer, 1989], provides an equivalent representation of the underlying signal in both time and frequency domains. This equivalent representation cannot indicate how the signal energy is distributed simultaneously in both domains. For illustration, consider the synthetic signals shown in Figure 7.

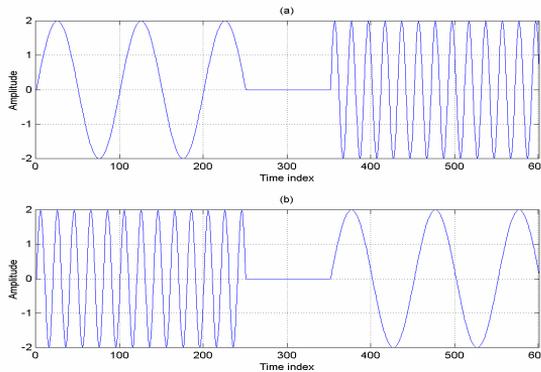


Figure 7: Time-Domain Representation of Composed Signals. (a) Low-High Frequency Signal. (b) High-Low Frequency Signal.

The behavior of the two signals is quite different in the time domain where the two frequency components can be easily discriminated from each other. However, the FT representation of the two signals is quite similar as shown in Figure 8.

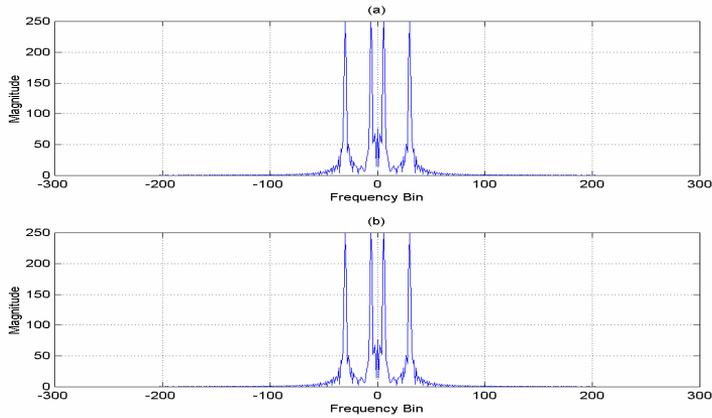


Figure 8: Frequency-Domain Representation of Composed Signals. (a) Low-High Frequency Signal. (b) High-Low Frequency Signal.

The loss of information due to the uncertainty principle can be remedied by using any kind of JTFA representation as illustrated in Figure 9.

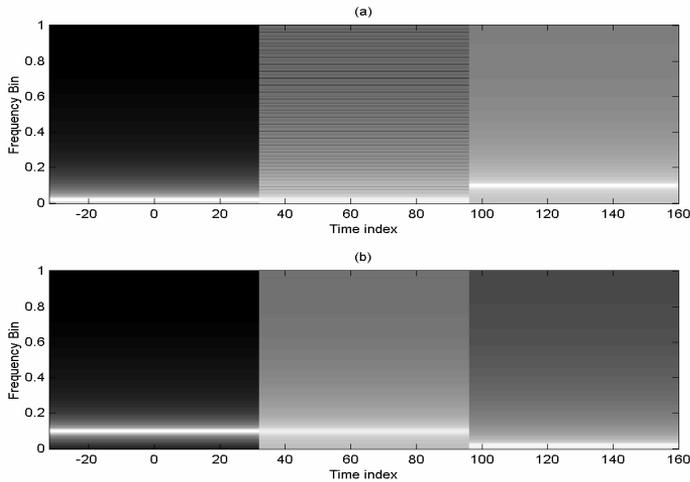


Figure 9<sup>1</sup>: JTFA Representation of Composed Signals. (a) Low-High Frequency Signal (b) High-Low Frequency Signal.

As expected, the JTFA representation successfully resolved the time-frequency ambiguity that is brought by the time-domain and the frequency-domain representations.

<sup>1</sup>We have used the simple spectrogram representation.

The JTFA analysis in general attempts to achieve a signal representation that is, in some sense, localized at time  $t_0$ , where the contributing frequency components can be extracted easily. For this purpose, an analysis window,  $h(t)$ , will “localize” the signal around time  $t_0$  as shown below:

$$x_{t_0}(t) = x(t)h(t - t_0) \tag{15}$$

To compute the localized spectrum one needs only to apply the FT analysis on the localized signal,  $x_{t_0}(t)$ . This is called the Short-Time Fourier Transform (STFT) at time  $t_0$ . To compute the STFT at other time centers, one simply shifts the window,  $h(t)$ , in time to localize the signal at another time center. The STFT is a linear function of the signal and will be a complex function of time and frequency. To obtain a real function, one often computes the squared magnitude of the STFT, which is called the *spectrogram* [1, 3]. The spectrogram is a quadratic function of the signal and is commonly written in the following form

$$S_x(t, w, h) = \left| \int_{-\infty}^{\infty} x(\tau)h(t - \tau)e^{-jw\tau} d\tau \right|^2 \tag{16}$$

The spectrogram, unlike many other JTFA representations, is always non-negative [Qian and Chen, 1998] and also has a straightforward computation. However, the spectrogram fails to satisfy many of the desirable properties that a JTFA representation should possess. The spectrogram also cannot have a good resolution in both time and frequency. This is a direct consequence of the time-frequency uncertainty principle. An example illustrating the tradeoffs in time and frequency resolutions is shown in Figure 10.

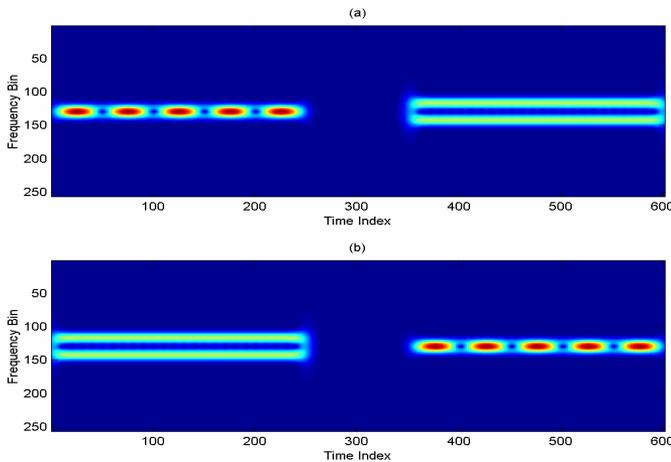


Figure 10: STFT Representation of Composed Signals. (a) Low-High Frequency Signal.  
 (b) High-Low Frequency Signal.

It can be clearly seen from Figure 4 that the STFT representation suffers from the lack of high resolution in both time and frequency domains.

An observation of the STFT representation is that the analysis window,  $h(t)$ , that provides the “optimal” concentration is the one that is similar to the signal itself. This provides an intuitive explanation of the Wigner-Ville Distribution (WVD) as defined below:

$$W_x(t, w) = \int_{-\infty}^{\infty} x\left(t + \frac{\tau}{2}\right)x^*\left(t - \frac{\tau}{2}\right)e^{-jw\tau} d\tau \tag{17}$$

The WVD representation was derived in the context of quantum mechanics by Wigner in 1932 and later extended to the signal-processing context by Ville in 1948 [Qian and Chen, 1998]. The WVD satisfies many of the desirable JTFA properties. Another interpretation of the WVD representation is given below [Boashash, 1991]:

$$W_x(t, w) = \int_{-\infty}^{\infty} X\left(w + \frac{v}{2}\right)X^*\left(w - \frac{v}{2}\right)e^{-jvw} dv \tag{18}$$

The most remarkable property of the WVD representation is that for a Gaussian windowed linear chirp, defined as [Boashash, 1991]:

$$x(t) = e^{-ct^2} e^{j(a_0 + a_1 t + a_2 t^2)} \tag{19}$$

The WVD concentrates the energy along the *instantaneous frequency* of the signal. The WVD formula for this signal is, [Boashash, 1991]:

$$W_x(t, w) = e^{-(w - a_1 - 2a_2 t)^2 / 2c} e^{-2ct^2} \tag{20}$$

This representation is shown in Figure 11.

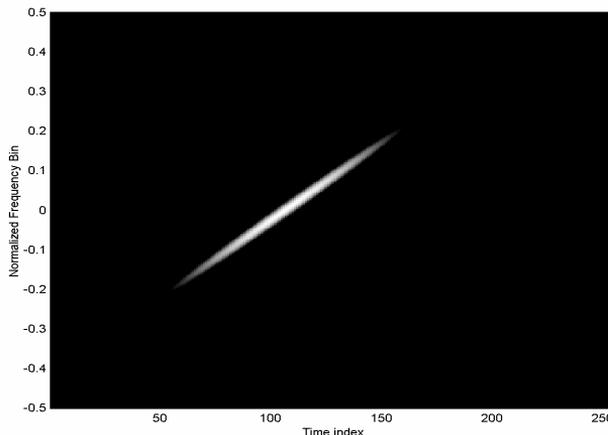


Figure 11: WVD Representation of a Chirp Signal.

As seen in Figure 11, the WVD has given continuous change of the frequency content with time. This feature has motivated using the WVD for the vibration signals of the problem of blade vibration and shaft torsional vibration.

The WVD of the blade tip deflection is shown in Figure 12. The frequency content of the tip deflection signal is shown to change as time passes that reflects the contributions of the different modal contributions to the blade vibration.

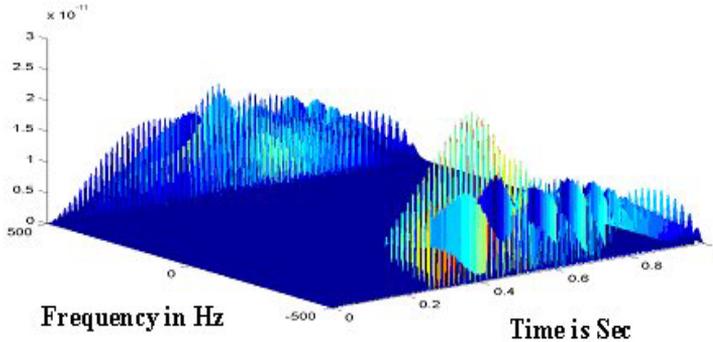


Figure 12: WVD Representation of blade tip deflection.

Comparing the WVD representation given in Figure 12 to the FFT of the blade tip deflection of Figure 4-b, shows that the WVD has given the evolution of frequency components with time.

The more important signal which is proposed to be used as a tool for monitoring blade vibration indirectly is the torsional vibration signal which as given in the FFT representation, Figure 5-b, shows a single frequency component at about 410 Hz. The result of applying the WVD to the torsional vibration signal is shown in Figure 13.

Although, the FFT of the shaft torsional vibration has shown one frequency component, the WVD of the same signal is showing at least 5 frequency layers with continuously time varying amplitudes as shown in Figure 13. The Layers can be referred to the contribution of the 5 modes of blade vibration, after tuned due to nonlinear interaction.

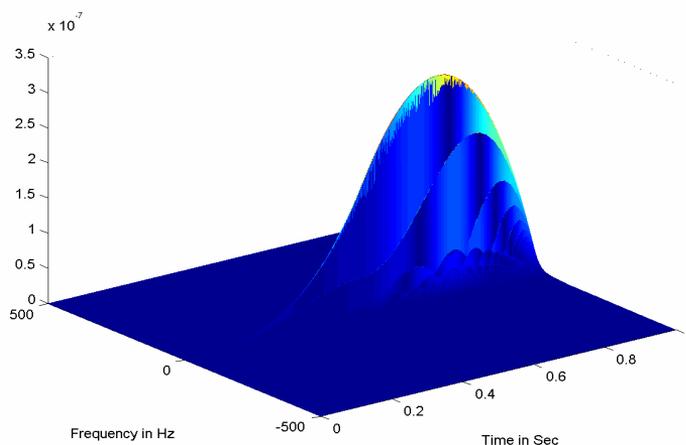


Figure 13: WVD Representation of shaft torsional vibration.

## 5. CONCLUSION

In this work, the vibration signals from the coupled blade-bending and shaft torsional model are studied. The Fast Fourier Transform is applied to the blade tip vibration signals and the shaft torsional vibration signals in a trial to monitor the blade vibration indirectly through the shaft torsional vibration signal. The FFT results have shown very little information on the coupling between the shaft torsional and blade bending vibration, due to the time dependency of this interaction. To aid in taking the time factor continuously into the picture, Joint Time Frequency technique (JTFA) is applied to the vibration signals. The results of the WVD representations have shown the changes in the frequency content of the vibrating blade. The shaft torsional vibration signal has preserved the nonlinear interaction and has shown the vibration of the rotating blade in the torsional vibration WVD representation. This result is encouraging for researchers who are working to measure the blade vibration through measuring the shaft torsional vibration. Finally, it is recommended that this signal processing approach be applied to actual experimental signals.

## ACKNOWLEDGEMENT

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