# AN IMPROVED ECONOMIC DISPATCH USING FAST DECOUPLED TECHNIQUE

**S. E. T. Mohamed[1], Osama M.A. Kalantan[2]**

*1: Professor, Electrical and Computer Engineering Department, King Abdul Aziz University.*
*2: Engineer, Electrical and Computer Engineering Department, King Abdul Aziz University.*

*E-mail:* osamaktn@awalnet.net.sa

## ABSTRACT

*Economic Dispatch (ED) plays an important role in operational planning. The principal objective of the economic dispatch is to determine the economic loadings of the generators so that the load demand can be met economically and the loadings are within the feasible operating margins of the generators. The analysis should also take in to consideration the effect of transmission network losses on the economic dispatch evaluation. An improved algorithm is presented in this paper for economic dispatch. The algorithm is based on a proposed method of closed form solution using Newton-Raphson technique. Application of the Fast Decoupled technique [Kwang Y. Lee and June Ho Park, 1998] for load flow analysis has been found to offer better prospects for the efficient solution of the economic dispatch problem. The improved technique offers a fast closed form solution of the economic dispatch problem. The technique enhance the solution of economic dispatch problem of the heavily interconnected networks and finds it's applications on-line control of system operation with enhance digital processing time and good convergence properties.*

**Keywords:** *Real Time Economic Dispatch, Closed Form Solution, Improved Closed Form Solution, Automatic generation control.*

## الملخص

إن التوزيع الاقتصادي للأحمال ليلعب دوراً هاماً وحيوياً في عملية التشغيل الأمثل للمحطات، والهدف الرئيسي للتوزيع الاقتصادي هو تحديد التشغيل الأمثل والاقتصادي للمولدات وذلك لمقابلة الأحمال الفعلية في الشبكة مع الأخذ في الاعتبار مسألة تأثير الفقدان للطاقة الكهربائية أثناء مسارها في الشبكة عند عمليات التحليل. في هذه الدراسة تم تقديم الصيغة المطورة لتحليل الشبكات للوصول إلى التوزيع الاقتصادي والذي بني على طريقة صيغة الحل المغلق باستخدام قانون نيوتن، والمعدل على طريقة أسلوب الارتباط السريع. وقد وجد أن صيغة الارتباط السريع فعالاً في تحليل الشبكات وخاصةً في مسائل التوزيع الاقتصادي. هذا الأسلوب حسن الحل في الشبكات الكبيرة ووجد أن تطبيقاتها مفيدة وخاصة عند ارتباطها بأجهزة تتحكم في مستوى التشغيل في الشبكة مع تحسين العمليات الرقمية للتحليل بالنسبة للزمن وأنها أيضاً ذات خاصية تقاربية .

## 1. INTRODUCTION

ED plays an important role in operation planning and real time control of modern power systems. Real Time Economic Dispatch (RTED) directly communicates with automatic generation control (AGC) by providing the most economic dispatch for the available on-line units [Fadhel Hasan, 1992]. Conventionally, the RTED determines the MW loading levels of the available units for minimum operating cost while meeting the system load demand, in a short time and without considering other constraints such as the line flow limits and emission allowances. Recently there has been a wide spread and increasing trend towards installation of new Energy Management systems for real time monitoring and control of power systems [Ji. Yuan Fan and Lan Zhang, 1998]. This trend has encouraged significant research effort towards development of more sophisticated, fast, reliable and efficient algorithms.

The purpose of this paper is to present a fast, simple and reliable ED algorithm. The important factors for economic operation of the system are operating efficiencies of generating plants, fuel cost, operating cost and transmission line losses. All these factors are included in algorithm. It is also obvious that the most efficient generators in the system necessarily yields the best economic operation cost, this operation may be too far from the load or the fuel cost may be high in the region.

The economic dispatch programs which are installed today in the most modern control centers use as well known classical set of coordination equations. The coordination requires that the incremental cost of generator power multiplied by a penalty factor for each generating unit should be same [Ramanation, 1985]. The main difference between different classical techniques is the method used to solve the coordination equations. The coordination equations are generally solved by iteratively adjusting the value of lambda until the sum of the generator outputs matches the system load plus loss to satisfy the power balance equation. The transmission loss penalty factors have been implemented using one of the several loss formulas which are calculated off-line or on-line at periodic intervals and on request.

To save computer time and improve convergence, this paper presents a method to find the value of lambda in closed form taking in to account total transmission loss change due to the generation changes. As no iterative process is employed to find the lambda, the algorithm is fast and has no convergence problems [Ramanation, 1985]. The method can be used with any type of transmission loss penalty factor calculation. In this paper penalty factors are implemented based upon the Newton's' method. The above algorithm is implemented and tested on the standard test systems.

Top

## 2. BASIC RELASHINSHIPS

As compared with the classical methods, this iterative technique is implemented to adjust the value of $\lambda'$ until the power balance equation is satisfied.

$$\sum_{i=1}^{N} P_{Gi} - P_D - P_L = 0 \tag{1}$$

where

$P_G$ …. Power generation for Unit in MW

$P_D$ …. Total system demand

$P_L$ …. Network losses

$N$ …. Total number of busses.

The objective function for the generation cost is given by:

$$F = \sum_{I=1}^{m} F_i \tag{2}$$

Where $F_i$ is the cost of the $i_{th}$ generator.

$m$ is the number of generation units.

This fuel cost function should be minimized.

For the system, the basic equation is

$$\lambda_i' = \frac{\lambda_i}{1 - \frac{\partial P_L}{\partial P_i}} \tag{3}$$

Where $\lambda'$ is the generation unit incremental cost.

$\frac{\partial P_L}{\partial P_{Gi}}$ is the incremental loss.

The polynomial of unit generation may be represented as follows:

$$P_{Gi} = \sum_{k=0}^{L} \alpha_{ik} \lambda_i^k \tag{4}$$

where $\alpha_{ik}$ is the coefficient of incremental cost terms in the polynomial

For *L=2*, the generation will be a second order equation. So $\alpha_{i0}, \alpha_{i1}, \alpha_{i2}$ are coefficients that may be obtained by curve fitting.

The number of term *L* term depends on the accuracy required for simulating the plant characteristics.

Now substituting equ. (3) in equ. (4): $$P_{Gi} = \sum_{k=0}^{L} \alpha_{ik} \lambda^{/k} (1 - \frac{\partial P_L}{\partial P_{Gi}})^k \tag{5}$$

Top

Then the total system generation is given by:

$$P_G = \sum_{i=1}^{m} P_{Gi} \tag{6}$$

From equ. ( 5 ) and equ. ( 6 )

$$P_G = \sum_{i=1}^{m} \sum_{k=0}^{L} (\alpha_{ik} \lambda^{/k} (1 - \frac{\partial P_L}{\partial P_{Gi}})^k ) \tag{7}$$

The total transmission losses for a network configuration are a function of busloads and generation. From the power balance equation:

$$P_{Loss} = \sum_{i=1}^{n} (P_{Gi} - P_{Di}) = \sum_{i=1}^{n} P_i \tag{8}$$

Where :

$\sum Pi$ …. Bus power at bus i

$P_{Di}$ …. Bus power demand at bus i

$n$ …. Total number of buses in system.

$\sum_{i=1}^{n} (P_{Gi} - P_{Di})$ …. The sum of the difference between generation and load at each bus.

**3. FAST DECOUPLED TECHNIQUE RELATIONS**

The general equation to solve the first step should use the Newton Raphson method in polar form as follows :

$$\left[\frac{dP}{dQ}\right] = [J]^* \left[\frac{d\,\theta}{d\,|V|}\right] \tag{9}$$

The active and reactive mismatches are given by:

$$\left[\frac{dP}{dQ}\right]^t = [dp_1 \ \ dp_2 \ \ dp_3 \ \ dp_4 ....... \ dp_n \ \ , \ \ dQ_{m+1} \ \ dQ_{m+2} ........ \ dQ_n \ \ ] \tag{10}$$

The Jacobian J does not contain terms for the reference (or slack) bus as well as generator reactive power. Decoupling active and reactive mismatches is in indicated in the network Jacobian in equation (11)

$$[J] = \left[\begin{array}{ccc|ccc} \frac{dp_2}{d\theta_2} & \cdots & \frac{dp_2}{d\theta_n} & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{dp_n}{d\theta_2} & \cdots & \frac{dp_n}{d\theta_n} & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & \frac{dQ_{m+1}}{d|V_{m+1}|} & \cdots & \frac{dQ_{m+1}}{d|V_n|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \frac{dQ_n}{d|V_{m+1}|} & \cdots & \frac{dQ_n}{d|V_n|} \end{array}\right] \tag{11}$$

Top

The active power and reactive power equations may be derived for any bus $k$ from the following relationships.

$$P_k - jQ_k = V_k^* I_k \quad \text{and} \quad I_k = \sum_{m=1}^{n} Y_{km} V_m \rightarrow P_k - jQ_k = V_k^* \sum_{m=1}^{n} Y_{km} V_m \tag{12}$$

The voltage and admittance in polar coordinates are expressed as:

$$V_k = |V_k| \times e^{-i\delta_k} \quad \text{and} \quad Y_{km} = |Y_{km}| \times e^{-i\theta_{km}} \tag{13}$$

So

$$P_k = \sum_{m=1}^{n} |V_k V_m Y_{km}| \cos(\theta_{km} + \delta_k - \delta_m) \quad \text{and} \quad Q_k = \sum_{m=1}^{n} |V_k V_m Y_{km}| \sin(\theta_{km} + \delta_k - \delta_m)$$

The slack bus active power mismatch is as follows:

$$dP_1 = \begin{bmatrix} \frac{dp_1}{d\theta_2} & \ldots\ldots & \frac{dp_1}{d\theta_n} & \vdots & 0 & \ldots\ldots & 0 \end{bmatrix} * \begin{bmatrix} d\theta_2 \\ \vdots \\ d\theta_n \\ -- \\ d|v_{m+1}| \\ \vdots \\ d|v_n| \end{bmatrix} \tag{14}$$

But:

$$\begin{bmatrix} d\theta_2 \\ \vdots \\ d\theta_n \\ -- \\ d|v_{m+1}| \\ \vdots \\ d|v_n| \end{bmatrix} = [J]^{-1} * \begin{bmatrix} dp_2 \\ \vdots \\ dp_n \\ -- \\ 0 \\ \vdots \\ 0 \end{bmatrix} = [J]^{-1} * \begin{bmatrix} dp \\ -- \\ 0 \end{bmatrix} \tag{15}$$

The slack bus power mismatch becomes:

$$dP_1 = \begin{bmatrix} \frac{dp_1}{d\theta_2} & \ldots\ldots & \frac{dp_1}{d\theta_n} & \vdots & 0 & \ldots\ldots & 0 \end{bmatrix} * [J]^{-1} * \begin{bmatrix} dp \\ -- \\ 0 \end{bmatrix} \tag{16}$$

Let

$$\phi = [J^{-1}] * \begin{bmatrix} \frac{dp_1}{d\theta} \\ --- \\ 0 \end{bmatrix} \tag{17}$$

where $\phi$ is a row vector of the form

Then

$$\phi^t = \begin{bmatrix} \frac{dp_1}{d\theta_2} & \ldots\ldots & \frac{dp_1}{d\theta_n} & \vdots & \frac{dp_1}{d|v_{m+1}|} & \ldots\ldots & \frac{dp_1}{d|v_n|} \end{bmatrix} * [J]^{-1} \tag{18}$$

Top

This may be expanded by Taylor's series around the initial bus power as follows, assuming higher order terms in the expansion are ignored.

This may be expanding in Taylor's series around the initial power.

$$P_L(\overline{p}) = P_L(p^o + dp) = P_L(\overline{p}^o) + \overline{P_L}(\overline{p}^o)dp + ....... + (HOD)$$

$$dP_L = dP_1 + \sum_{i=2}^{n} dP_i \qquad (19)$$

From equation (14) and (19), the change in total transmission loss is:

$$dP_1 = [\gamma_2 \quad ........ \quad \gamma_n \quad | \quad 0 \quad ........ \quad 0]^* \begin{bmatrix} dp_2 \\ \downarrow \\ dp_n \\ - \; - \\ 0 \\ \downarrow \\ 0 \end{bmatrix}$$

Where $\gamma$ is a result of multiply $\frac{dp}{d\theta}$ by Jacobian matrix.

So $$dP_1 = [\gamma_2 dp_2 + ........ + \gamma_n dp_n + 0 + ........ + 0] \qquad (20)$$

And $$dP_{Loss} = \sum_{i=2}^{n} (1 + \gamma_i) dp_i \qquad (21)$$

From last equation (21), ITL is given by:

$$ITL_i = \frac{dp_L}{dp_{Gi}} = \frac{dp_L}{dp_i} = 1 + \gamma_i \qquad (22)$$

The angle (θ) of the slack bus is constant so the incremental transmission loss (ITL) will be zero.

i.e. $$ITL_1 = \frac{dp_L}{dp_1} = 0 \qquad (23)$$

the penalty factor for the $i^{th}$ generator is

$$\frac{1}{1 - ITL_i} = \frac{1}{1 - (1 + \gamma_i)} = -\frac{1}{\gamma_i} \qquad (24)$$

For economic dispatch real and reactive loads are assumed to be constant. From equation (21) the total transmission loss at optimum dispatch is

$$P_L(\overline{P}) = P_L(\overline{p}^o) + dp_L \qquad (25)$$

substituting equ. (5) and equ. (25) in equ. (3)

$$\sum_{i=1}^{m} P_{Gi} - P_D - P_L^o - \sum_{i=2}^{m} P_{Gi} + \sum_{i=2}^{m} (1+\gamma_i) P_{Gi}^o - \sum_{i=2}^{m} \gamma_i P_{Gi} = 0 \quad (26)$$

But For slack bus k=0;

$$P_{G1} = \sum_{k=0}^{L} \alpha_{1k} * \lambda'^{k} (-\gamma_1)^k$$

Then most of generation characteristics may be taken as equations second order (L = 2);

$$P_{Gi} = \alpha_{i0} - \alpha_{i1}\lambda' + \alpha_{i2}\lambda'^2 * \gamma_i$$

Substituting these expression in equation (26)

$$\alpha_{10} + \alpha_{11}\lambda' + \alpha_{12}\lambda'^2 - P_D - P_L^o + \sum_{i=2}^{m}(1+\gamma_i)P_{Gi}^o - \sum_{i=2}^{m}\gamma_i\alpha_{i0} + \sum_{i=2}^{m}\alpha_{i1}\lambda'\gamma_i^2 - \sum_{i=2}^{m}\alpha_{i2}\lambda'^2\gamma_i^3 = 0$$

Which can be written as:

$$A + B\lambda' + C\lambda'^2 = 0 \quad (27)$$

Where:

$$\lambda' = \frac{-B \pm \sqrt{B^2 - 4CA}}{2C}$$

In this algorithm a major portion of the execution time is spent performing penalty factor calculation and is the same irrespective of the lambda calculation technique. In order to minimize execution time in the penalty factor step the following modifications will be helpful. Renumber all the buss connected to the slack bus towards the bottom of the Jacobian and also the generator buses more or less towards the bottom of the Jacobian to eliminate the forward and backward substitution steps of the transposed repeated solution to a relatively small number of operations.

## 4. MODIFICATION OF INCREMENTAL COST

In order to expedite the evaluation of the incremental cost of economic dispatch, the deviation of demand from total generation production is used as a measure to improve the solution. This has been found to yield a faster solution for the evaluation of $\lambda'$. This can be realized by the following modification for the evaluation of $\lambda'$.

$$\lambda'_{new} = \lambda'_{old}\left(1 \pm \sigma\left[\frac{|P_G - P_D|}{P_D}\right]\right) \quad (28)$$

Where:

$P_G$ = total generation.

$P_D$ = total demand.

$\lambda^{/}_{new}$ = new incremental cost.

$\lambda^{/}_{old}$ = old incremental cost.

$\sigma$ = adjustment factor, its value depends on the power system investigated for certain networks and this varies from 0.85 to 1.2rithm

## 5. RESULTS AND DISCUSSION

The Merits of ICFS can be indicated by comparing the processing time for the solution of incremental cost of economic dispatch of the three standard systems described in previous sections of this chapter. The solution is curried out by both CFS and ICFS, with using personnel computer of 400 MHz processor.

### 5.1 Processing Time for 10-Bus Network.

The given example of 10 bus system will solve the incremental cost by using Closed Form Solution and fast decoupled technique, the system increased the demand power from it's normal operation to 50 %. Solve calculation of Lamda by Closed Form Solution (CFS) and (ICFS) in Pentium II, 400 Hz, give the time showed in Table 1.

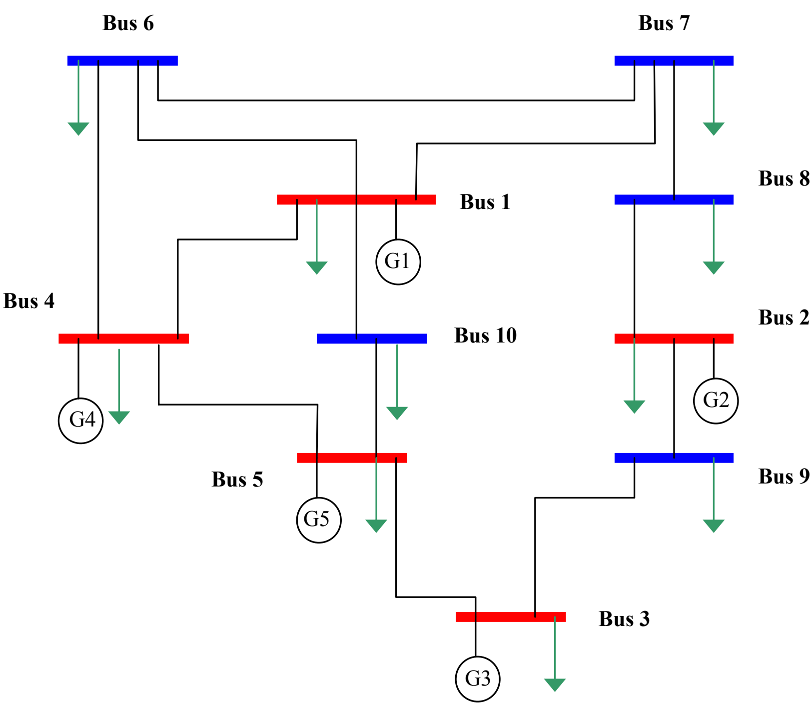



**Figure 1:** 10-bus system

Top

**Table (1):** The Results of (CFS) and (ICFS) for 10 bus system

| Load | Technique | Iteration | time | λ |
|---|---|---|---|---|
| **340 MW 115 Mvar** | CFS | 238 | 25.21 | 9.74 |
| | ICFS | 201 | 20.62 | 9.74 |
| **10 % additional A power 5 % additional R Power** | CFS | 245 | 26.11 | 9.87 |
| | ICFS | 215 | 22.16 | 9.87 |
| **25% additional A power 10 % additional R Power** | CFS | 248 | 36.17 | 10.17 |
| | ICFS | 216 | 30.61 | 10.17 |
| **50 % additional A power 30 % additional R Power** | CFS | 312 | 45.89 | 10.73 |
| | ICFS | 270 | 31.72 | 10.73 |

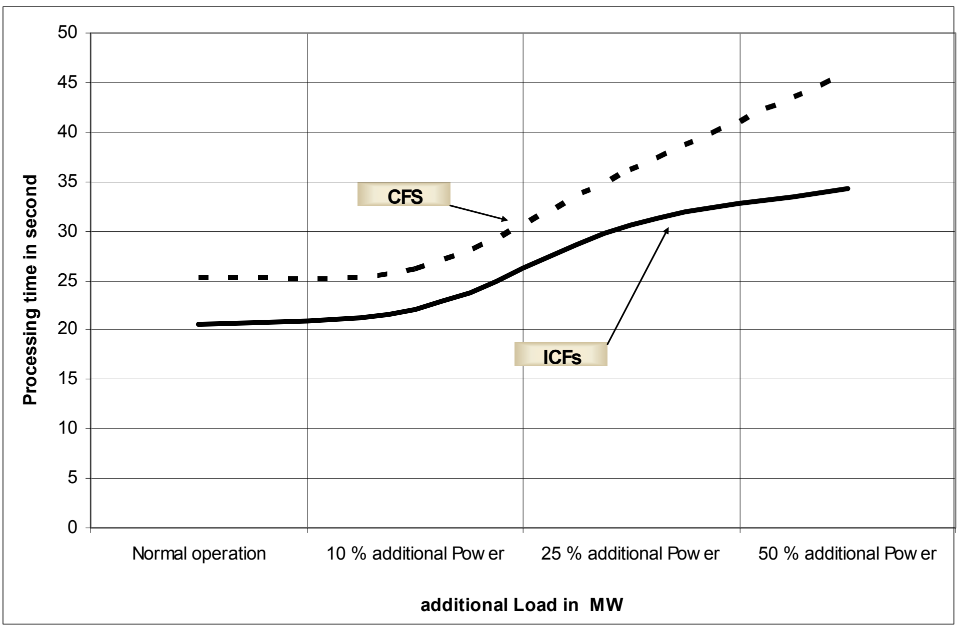



**Figure 2:** Improved and Closed Form Solution Processing Time

Top

### 5.2 The Result of 30 Bus Systems

An economic dispatch solution is carried out on an IEEE 30-bus network for different load demand conditions starting with base case and increasing the demand by regular intervals as shown in table (2).

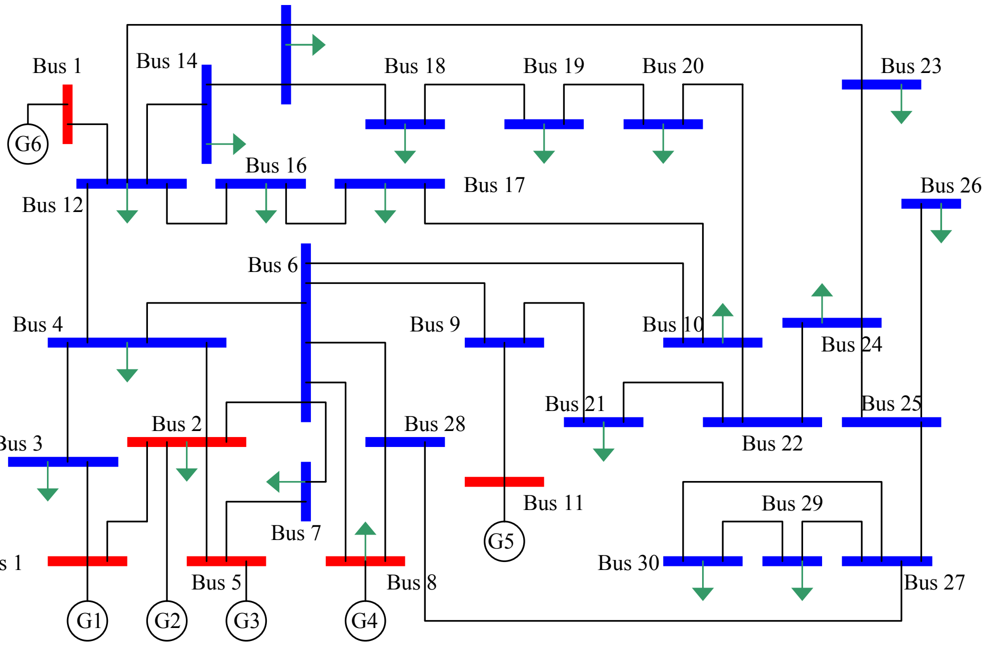


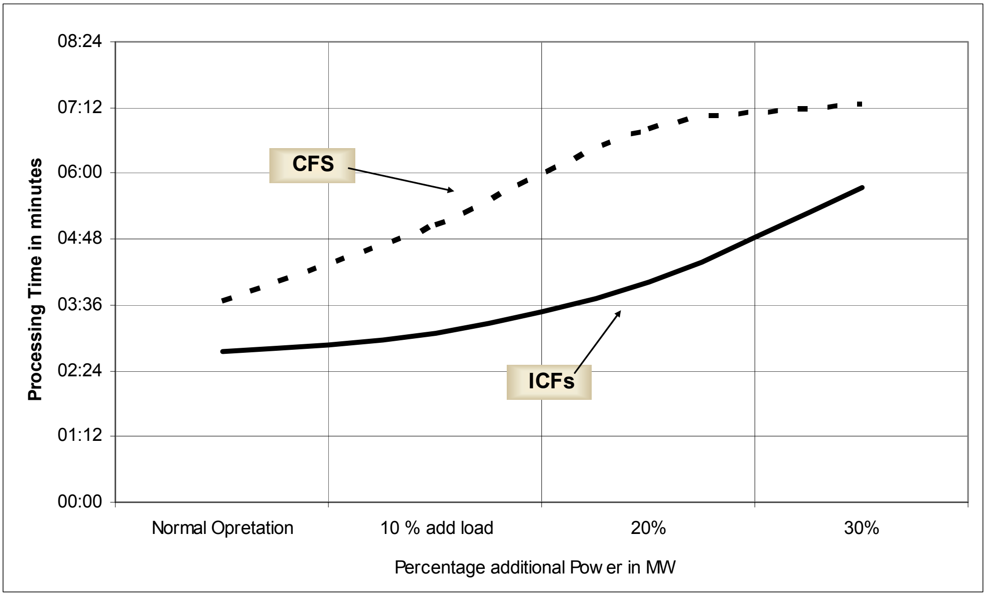

**Figure 3**: Single Line Diagram of the 30-bus Network

The given example of 30 bus system will solve the incremental cost by using Closed Form Solution and Fast de coupled technique, the system increased the demand power from it's normal operation to 30 %. Solve calculation of Lamda by Closed Form Solution (CFS) and improved (CFS) with time shown in Table 2.

**Table 2** : The Results of (CFS) and (ICFS) for 30 bus system

| Load | Technique | time | λ |
|---|---|---|---|
| **283.4 MW** **126.2 Mvar** | CFS | 03:40 | 3.33 |
| | ICFS | 02:45 | 3.33 |
| **10 % additional A power** | CFS | 05:03 | 3.42 |
| | ICFS | 03:05 | 3.43 |
| **20% additional A power** | CFS | 07:18 | 3.52 |
| | ICFS | 04:01 | 3.52 |
| **30 % additional A power** | CFS | 08:10 | 3.61 |
| | ICFS | 05:45 | 3.62 |

Top




**Additional load demand**

**Figure 4**: Improved and Closed Form Solution Processing Time

The advantage of the ICFS over the CFS is seen from Figures (3 and 4) where a significant improvement is indicated in the processing time of the improved CFS technique.

## 6. CONLUSIONS

The new algorithm presented in this paper solves the economic dispatch problem reliably and at a speed suitable for real-time on-line ED. The performance of the developed improved closed form solution algorithm has been amply discussed and demonstrated by application to a standard bench mark IEEE application of a 30-bus network composed of 6-generation units. The improved closed form solution of this power system has shown fairly accurate results for incremental and total cost of generation as compared with the results of the conventional Newton's closed form solution, but at comparatively shorter processing time.

The computation time for the improved solution technique has indicated a saving of up to 55% for (IEEE 30-bus system model). This is distinctly shorter than that of the conventional technique. This renders the improved solution technique significantly useful for on-line applications of economic dispatch investigations.

The current research investigation, the results of which are presented in this paper, has dealt with the problem of the influence of power transmission lines capability on economic load dispatch. The problem has been analyzed on the basis of the improved developed algorithm of closed form solution. The technique has been thoroughly validated.

Top

**REFFERENCES**

1. Abdullatif Mobarak Al-Zahrani," Load Flow Studies," BSc. Project, 1996 G.
2. Fadhel Abbas Salman Hasan, "Closed Form Solution For Economic Power Dispatch" BSc. Project, 1992.
3. Ji. Yuan Fan and Lan Zhang, "Real Time Economic Dispatch with Line Flow and Emission Constraints Using Quadratic Programming," IEEE Transactions on Power Systems, Vol. 13, No.2,May,1998.
4. Kwang Y. Lee and June Ho Park, "Adaptive Hopfield Neural Networks for Economic Load Dispatch," IEEE, Transactions on Power System, Vol. 13, No. 2, May 1998.
5. O. Alsac and B.Stott, "Optimal Load Flow With Steady-State Security," IEEE, Power system Engineering Committee, Canada, July 15-20, 1973.
6. R. Ramanation, "Fast Economic Dispatch Based On The Penalty Factors From Newton's Method," IEEE Transactions on Power Systems, Vol. PAS-104, No.7, July 1985.
7. R. Ramanation, "Fast Economic Dispatch Based On The Penalty Factors From Newton's Method," IEEE Transactions on Power Application and Systems, Vol. PAS-104, No.7, July 1985.

## APPENDIX A

### A.1. THE DATA FOR 10 BUS SYSTEM

**Table (4)** The Data for impedance and Line charging impedances for the system

| Bus i – j | Impedance Z ( pu. ) | Line charging Yc ( pu ) | Max P transfer |
|---|---|---|---|
| 1 – 2 | 0.02 + j 0.08 | 0.00 + j 0.01 | 60 MW |
| 1 – 6 | 0.06 + j 0.25 | 0.00 + j 0.02 | 30 MW |
| 1 – 9 | 0.04 + j 0.16 | 0.00 + j 0.02 | 30 MW |
| 2 – 3 | 0.06 + j 0.25 | 0.00 + j 0.02 | 30 MW |
| 2 – 6 | 0.06 + j 0.25 | 0.00 + j 0.02 | 30 MW |
| 3 – 7 | 0.06 + j 0.25 | 0.00 + j 0.02 | 30 MW |
| 4 – 7 | 0.04 + j 0.16 | 0.00 + j 0.02 | 30 MW |
| 4 – 8 | 0.06 + j 0.25 | 0.00 + j 0.02 | 30 MW |
| 5 – 6 | 0.04 + j 0.16 | 0.00 + j 0.02 | 30 MW |
| 5 – 10 | 0.06 + j 0.25 | 0.00 + j 0.02 | 30 MW |
| 6 – 9 | 0.02 + j 0.08 | 0.00 + j 0.01 | 60 MW |
| 8 – 10 | 0.04 + j 0.16 | 0.00 + j 0.02 | 30 MW |
| 9 – 10 | 0.08 + j 0.32 | 0.00 + j 0.025 | 20 MW |

**Table (5)** Generator characteristics

| Gen | Min P in MW | Max P in MW | a | B | c |
|---|---|---|---|---|---|
| 1 | 10 | 120 | 20 | 7 | 1.2 |
| 2 | 10 | 120 | 27 | 9 | 1.1 |
| 3 | 10 | 120 | 21 | 9 | 0.7 |
| 4 | 10 | 120 | 23 | 9 | 1.0 |
| 5 | 10 | 120 | 19 | 8 | 0.8 |

Where the Generator cost (F)= $a+bP+cP^2$ ( $ ), a, b, c are the generator constant.

**Table (6)** Load Data

| bus | Active Power ( MW) | Reactive Power ( Mvar) |
|---|---|---|
| 1 | 30 | 10 |
| 2 | 40 | 15 |
| 3 | 20 | 10 |
| 4 | 50 | 15 |
| 5 | 30 | 10 |
| 6 | 60 | 15 |
| 7 | 20 | 10 |
| 8 | 40 | 10 |
| 9 | 20 | 10 |
| 10 | 30 | 10 |

**A.2. THE DATA FOR 30 BUS SYSTEM :**

**Table (7)** The Data for impedance and Line charging impedances for the system

| Branch No. | Bus i – j | R p.u. | X p.u. | B ( Total ) | Max P transfer MW |
|---|---|---|---|---|---|
| 1. | 1-2 | 0.0192 | 0.0575 | 0.0264 | 130 |
| 2. | 1-3 | 0.0452 | 0.1852 | 0.0204 | 130 |
| 3. | 2-4 | 0.0570 | 0.1737 | 0.0184 | 65 |
| 4. | 3-4 | 0.0132 | 0.0379 | 0.0042 | 130 |
| 5. | 2-5 | 0.0472 | 0.1983 | 0.0209 | 130 |
| 6. | 2-6 | 0.0581 | 0.1763 | 0.0187 | 65 |
| 7. | 4-6 | 0.0119 | 0.0414 | 0.0045 | 90 |
| 8. | 5-7 | 0.0460 | 0.1160 | 0.0102 | 70 |
| 9. | 6-7 | 0.0267 | 0.0820 | 0.0085 | 130 |
| 10. | 6-8 | 0.0120 | 0.0420 | 0.0045 | 32 |
| 11. | 6-9 | 0.00 | 0.2080 | 0.00 | 65 |
| 12. | 6-10 | 0.00 | 0.5560 | 0.00 | 32 |
| 13. | 9-11 | 0.00 | 0.2080 | 0.00 | 65 |
| 14. | 9-10 | 0.00 | 0.1100 | 0.00 | 65 |
| 15. | 4-12 | 0.00 | 0.2560 | 0.00 | 65 |
| 16. | 12-13 | 0.00 | 0.1400 | 0.00 | 65 |
| 17. | 12-14 | 0.1231 | 0.2559 | 0.00 | 32 |
| 18. | 12-15 | 0.0662 | 0.1304 | 0.00 | 32 |
| 19. | 12-16 | 0.0945 | 0.1987 | 0.00 | 32 |
| 20. | 14-15 | 0.2210 | 0.1997 | 0.00 | 16 |
| 21. | 16-17 | 0.0824 | 0.1932 | 0.00 | 16 |
| 22. | 15-18 | 0.1070 | 0.2185 | 0.00 | 16 |
| 23. | 18-19 | 0.0639 | 0.1292 | 0.00 | 16 |
| 24. | 19-20 | 0.0340 | 0.0680 | 0.00 | 32 |
| 25. | 10-20 | 0.0936 | 0.2090 | 0.00 | 32 |
| 26. | 10-17 | 0.0324 | 0.0845 | 0.00 | 32 |
| 27. | 10-21 | 0.0348 | 0.0749 | 0.00 | 32 |
| 28. | 10-22 | 0.0727 | 0.1499 | 0.00 | 32 |
| 29. | 21-22 | 0.0116 | 0.0236 | 0.00 | 32 |
| 30. | 15-23 | 0.1000 | 0.2020 | 0.00 | 16 |
| 31. | 22-24 | 0.1150 | 0.1790 | 0.00 | 16 |
| 32. | 23-24 | 0.1320 | 0.2700 | 0.00 | 16 |
| 33. | 24-25 | 0.1885 | 0.3292 | 0.00 | 16 |
| 34. | 25-26 | 0.2544 | 0.3800 | 0.00 | 16 |
| 35. | 25-27 | 0.1093 | 0.2087 | 0.00 | 16 |
| 36. | 28-27 | 0.00 | 0.3960 | 0.00 | 65 |
| 37. | 27-29 | 0.2198 | 0.4153 | 0.00 | 16 |
| 38. | 27-30 | 0.3202 | 0.6027 | 0.00 | 16 |
| 39. | 29-30 | 0.2399 | 0.4533 | 0.00 | 16 |
| 40. | 8-28 | 0.0636 | 0.2000 | 0.0214 | 32 |
| 41. | 6-28 | 0.0169 | 0.05999 | 0.0065 | 32 |
| 42. | 10-10 | 0.00 | -5.2600 | | |
| 43. | 24-24 | 0.00 | -25.000 | | |

Top

**Table (8)** Generator characteristics.

| Gen | Min P (MW) | Max P (MW) | a | b | c |
|---|---|---|---|---|---|
| 1 | 50 | 350 | 0.00 | 2 | 0.00375 |
| 2 | 20 | 250 | 0.00 | 1.75 | 0.0175 |
| 3 | 15 | 220 | 0.00 | 1 | 0.0625 |
| 4 | 10 | 140 | 0.00 | 3.25 | 0.00834 |
| 5 | 10 | 140 | 0.00 | 3 | 0.025 |
| 6 | 12 | 140 | 0.00 | 3 | 0.025 |

**Table (9)** Load Data

| bus | Active Power (MW) | Reactive Power (Mvar) | bus | Active Power (MW) | Reactive Power ( Mvar) |
|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 16 | 3.5 | 1.8 |
| 2 | 21.7 | 12.7 | 17 | 9.0 | 5.8 |
| 3 | 2.4 | 1.2 | 18 | 3.2 | 0.9 |
| 4 | 7.6 | 1.6 | 19 | 9.5 | 3.4 |
| 5 | 94.2 | 19.0 | 20 | 2.2 | 0.7 |
| 6 | 0.00 | 0.0 | 21 | 17.5 | 11.2 |
| 7 | 22.8 | 10.9 | 22 | 0.0 | 0.00 |
| 8 | 30.0 | 30.0 | 23 | 3.2 | 1.6 |
| 9 | 0.0 | 0.0 | 24 | 8.7 | 6.7 |
| 10 | 5.8 | 2.0 | 25 | 0.00 | 0.00 |
| 11 | 0.0 | 0.0 | 26 | 3.5 | 2.3 |
| 12 | 11.2 | 7.5 | 27 | 0.00 | 0.00 |
| 13 | 0.0 | 0.0 | 28 | 0.00 | 0.00 |
| 14 | 6.2 | 1.6 | 29 | 2.4 | 0.9 |
| 15 | 8.2 | 2.5 | 30 | 10.6 | 1.9 |

Top