

# A Mathematical Model Identifies End-Users

Masaud M. Jahromi (mjahromi@ahliauniversity.org)

Department of Computer Sciences & Engineering, College of MS & IT, Ahlia University, Bahrain

**Abstract** — This paper introduces a mathematical model obtained from binomial coefficient. It encodes and decodes the multicasting-multiplexing information to and from two integer variables. The first variable represents the number of subscribers while the second variable identifies the desired end-users.

**Index Terms** — Multicasting, Multiplexing, Switching, Binomial coefficient, Encoder, Decoder.

## I. INTRODUCTION

In general, broadband telecommunication signals can be categorised into broadcasting signals and multicasting signals according to the possible number of subscribers. The public signal or the signal that all the end-users are subscribed to is called a broadcasting signal while the term multicasting signal is applied to the private signal or the signal in which the number of subscribers can be any value between 1 to  $m$ , where  $m$  is the number of connected end-users. To transmit any of these signals over the existing networks, the multicasting-multiplexing information (number of subscribers and their addresses) needs to be saved in the routing table within the involved switch(es) during connection negotiation and setup. Then, from the cells identifiers the signal will be recognised and the desired multicasting-multiplexing information will be called from the routing table. According to this information the switch multicasts or broadcasts the received cells and identifies them with the desired end-users addresses. [1-3]

## II. THE THEORY

Let us consider a set  $S$  with  $m$  elements that represented using sequence numbers:

$$S = \{1, 2, 3, \dots, m\} \quad (1)$$

From this set, it is possible to form several subsets with different number of elements. If the number of subset elements denoted as a variable parameter  $X$ , the number of possible subsets (combinations of size  $X$ ) that can be selected from the set  $S$  ( $m$  elements) is given by binomial coefficient: [4-5]

$$SN = \binom{m}{X} = \frac{m!}{X!(m-X)!} \quad (2)$$

If the possible subsets are formed successively and identified using sequence codes, then a desired subset can be denoted as a variable parameter  $Y$ . For example if  $m$  in set  $S$  is 4, then the variable parameter  $X$  has a possible range of 0 to 4. Moreover, table 1 lists the entire possible subsets for each possible value of  $X$ . Therefore, from the given parameters  $X$  and  $Y$  the desired subset is identified. For instant, given  $X$  equal to 2, parameter  $Y$  has a possible range of 1 to 6; if the given value of  $Y$  is 4 then the desired subset is {B, C}.

Table I  
POSSIBLE SUBSETS FOR A SET WITH 4 ELEMENTS

X	0	1				2					
Y	1	1	2	3	4	1	2	3	4	5	
Elements	$\emptyset$	A	B	C	D	A & B	A & C	A & D	B & C	B & D	
X	2		3					4			
Y	6		1		2		3		4		1
Elements	C & D		A&B&C		A&B&D		A&C&D		B&C&D		A&B&C&D

A similar technique can be applied for switch multicasting function where each access network (set  $S$ ) serves  $m$ -connected end-users (elements). Hence, each

multicasting signal transmits to a certain number of end-users (parameter  $X$ ) and labelled with their desired identifiers (parameter  $Y$ ). This can be illustrated using a

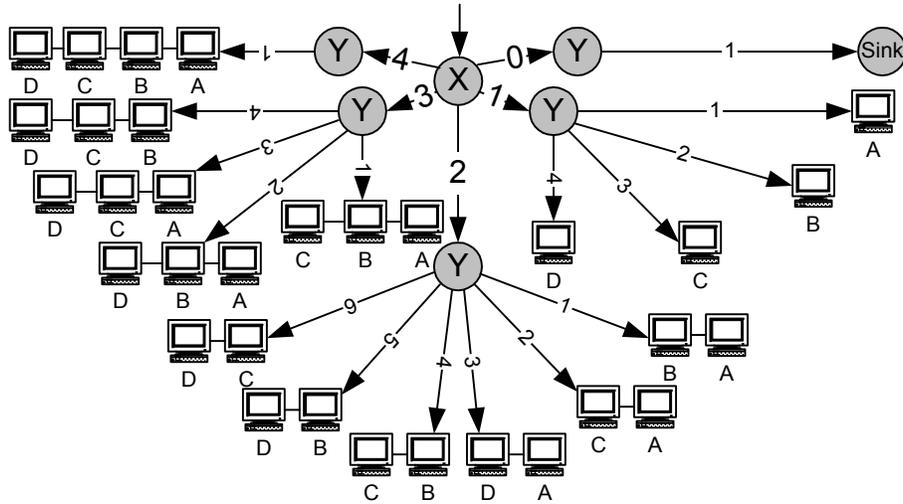


Figure 1: Two-Stage Switching Node Tree Diagram for an Access Network with 4 End-Users

two-stage switching node tree diagram as shown in figure 1. The switch observes the desired number of cell copies from the value of parameter X and using the corresponding value of parameter Y it identifies the subscribed end-users.

However, for an access network with large number of connected end-users this technique will be very complicated unless it can be converted into a general mathematical formula. Let us consider the same access network with four connected end-users. From equation (2) it is clear that for  $X = 0$  and  $X = m$  the parameter has only a single possible value, 1, so the possible range of parameter X can be converted into three conditions:  $X = 0$ ,  $X = m$  or  $0 < X < m$ . Logically the first condition declares the simple case of non-subscribed end-users while the second condition gives the feeling of m-subscribed end-users (broadcasting). Now let us expand the third condition as follows:

For  $X = 1$ ,

$$SN = \binom{4}{1} = \frac{4!}{1!(4-1)!} = \frac{4*3*2*1}{1*(3*2*1)} = 4 \quad (3)$$

$$P = [(1), (2), (3), (4)] \quad (4)$$

$$\text{Range or Domain of } Y = [Y] = [1-4] \quad (5)$$

For  $X = 2$ ,

$$SN = \binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4*3*2*1}{2*1*(2*1)} = 6 \quad (6)$$

$$P = [(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)] \quad (7)$$

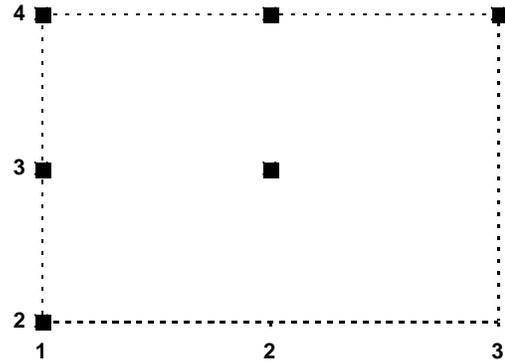


Figure 2: A 2-Dimensional plot for possible combinations of size 2 from a set with 4 elements

$$\text{Range or Domain of } Y = [Y] = [1-6] \quad (8)$$

These combinations can be illustrated on a 2-dimensional plot as shown in figure 2. From this figure the following points can be noticed:

- The maximum number of units in each axis is given by  $r = m - (X - 1)$  (9)
- The horizontal axis units (first element) starts from 1,
- The vertical axis units (2<sup>nd</sup> element) starts from 2,
- The minimum value of first element gives the maximum number of possible combinations, as it (1<sup>st</sup> element) increases the number of possible combinations decreases with equal amount.

Therefore, the number of possible combinations (SN) can be written in the following form, where each component represents a unit of horizontal axis and its value declares the corresponding number of possible combinations:

$$SN = 3 + 2 + 1 = r + (r - 1) + (r - 2) \quad (10)$$

For example, the value of the second component indicates that unit 2 has two (m-2) possible combinations: (2, 3) and (3, 4). Hence, for Y equal to 5 it is clear that the desired combination is within the second component, which has two possible combinations 4 and 5.

For X = 3,

$$SN = \binom{4}{3} = \frac{4!}{3!(4-3)!} = \frac{4*3*2*1}{3*2*1*(1)} = 4 \quad (11)$$

$$P = [(1,2,3), (1,2,4), (1,3,4), (2,3,4)] \quad (12)$$

$$\text{Range or Domain of } Y = [Y] = [1-4] \quad (13)$$

By taking the first element as a common factor, these combinations can be arranged into multiple groups of combinations so P can be written as (12). The number of possible combinations (SN) for each group can be written as the form of second case (X = 2) by ignoring the common factor (first element) as follows:

$$SN_1 = 2 + 1 \quad (14)$$

$$SN_2 = 1 \quad (15)$$

From the combination of these two equations and corresponding value of r, SN can be written in the following form:

$$SN = [2 + 1] + [1] = [r + (r - 1)] + [(r - 1)] \quad (16)$$

Hence, the number of possible combinations (SN) for each corresponding X parameter can be written in the following general form:

$$SN = \sum_{i=1}^r \sum_{j=i}^r \dots \sum_{z=v}^r r - (z - 1) \quad (17)$$

Where:  $r = m - (X - 1)$  and number of  $\sum$ s is equal to  $(X - 1)$ . Therefore, the overall SN equation that covers the full possible ranges of parameter X is:

$$\begin{aligned} SN &= 1, & X &= 0; \\ SN &= m, & X &= 1; \\ SN &= \sum_{i=1}^r \sum_{j=i}^r \dots \sum_{z=v}^r r - (z - 1), & X &> 1 \end{aligned} \quad (18)$$

### III. THE ENCODER

The above equation has been used in converting the multicasting-multiplexing information to two integer variables X and Y. The variable X represents number of subscribers or members while the parameter Y refers to the desired combination within the possible combination list.

The encoder which is modeled using Matlab programming forms the desired combination, C, as an array of size (1 x X) from the multicasting-multiplexing information; where X is the number of subscribed nodes and each element of the array represents the physical location (address) of the corresponding subscribed node.

Then the encoder searches for this array within the list of possible combination arrays that are formed using the following Matlab statement:

$$List = \text{combntrns}(1 : m, X) \quad (19)$$

The combntrns command in Matlab provides the possible combination arrays in form of a matrix based on the following parameters:

- m, number of the connected node to the server, and
- X, number of the subscribed nodes to the signal.

Hence, the resultant matrix has the size (SN x X), where SN is the binomial coefficient given by equation (2).

The encoder search engine operates based on a for-loop statement that determines the corresponding row number, Y, which has the same elements of the desired combination according to the following statements:

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for i = 1 : length(List)
    if (isequal(C, List(i,1 : X)))
        Y = i
    else
        continue
    end
end

```

Therefore, the multicasting-multiplexing information has been converted within the encoder to two variables:

X, number of subscribed nodes, and Y, the desired row number of the matrix List that has the same elements of the combination array C.

#### IV. THE DECODER

The decoder recovers the desired multicasting-multiplexing information from the received variables X and Y.

First from the received variable X and the fixed parameter m (number of the connected nodes to the server) the decoder forms the matrix List using the statement given by (19):

$$List = combntns(1:m, X) \quad (19)$$

Then by using the received variable Y, it recovers the desired combination array C from the formed matrix List via the following Matlab statement:

$$C = List(Y, 1:X) \quad (20)$$

#### V. CONCLUSION

From this paper it can be concluded that the proposed mathematical formula is able to convert and recover the multicasting-multiplexing information to and from two integer variables X and Y. X is the number of subscribed nodes and Y is the row number of the desired combination within the list of possible combination matrix.

The proposed mathematical model is a general formula, which is obtained from the binomial coefficient, covers all the possible conditions for a multimedia signal. In addition, it is not limited to a

specific network type and can be used wherever the multicasting-multiplexing function exists.

#### DEDICATION

I would like to dedicate this work to the World Reformer, the son of holy Prophet, "Imam Mahdi" peace be upon him. "M. Jahromi"

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