A New Deterministic Interleaver for Turbo Codes

Muhammad Arif
University of Engineering and Technology Lahore, Pakistan
Email: muhammadarifwahla@gmail.com
Telephone: (92) (300) 444–2725

Noor Muhammad Sheikh
University of Engineering and Technology Lahore, Pakistan
Email: adstec@mailcity.com

Asrar.U.H Sheikh
King Fahed University of Petroleum and Minerals, Saudi Arabia
Email: asrarhaq@kfupm.edu

***Abstract*— Interleavers play a critical role in performance of turbo codes. The turbo code performance curve can change its slope in low bit error rate (BER) region, if the interleaver is not designed properly. The interleaver is also a cause of bottleneck for parallelization of turbo decoder. In this paper, we propose the design of interleaver that combines the regular permutation of the interleaver and de-correlation property of the decoder. The interleaver design is systematic and provides enlarged the minimum effective free distance d_{min}. The systematic design and low memory requirement make the interleaver best suited for parallelism. Spread spectrum distribution and simulation results are presented and compared with well established S-random interleaver for short block lengths.**

I. Introduction

Turbo codes have received plenty of attention since their introduction [1]. They are the parallel concatenated convolutional codes (PCCC), whose encoder is formed by two or more constituent systematic encoders connected through one or more interleavers. The basic form of turbo code is implemented as two recursive systematic convolutional (RSC) codes in parallel (Figure 1), where as the input to the second encoder is an interleaved version of the original information sequence. A codeword of parallel concatenated convolutional code consists of the input bits x_i to the first encoder followed by the parity bits y_i from both the constituent encoders. The turbo code considered in this paper consists of two identical rate 1/2 recursive systematic convolutional encoders (RSC1 and RSC2) concatenated in parallel through an interleaver as shown in Figure 1. The feed backward(fb) and feed forward (ff) polynomials of the constituent code are $(23, 35)_8$ respectively. Without puncturing, the overall code rate is 1/3. Other code rates can be achieved as required by puncturing the code parity bit y_i^1 and y_i^2 form RSC1 and RSC2 respectively.

Interleaving is a key component of turbo codes. Most random interleavers perform well for large block sizes interleavers. For short block size interleavers, the performance of turbo codes with random interleavers substantially degrades to an unacceptable level. To overcome this problem, deterministic interleavers have been proposed by many authors for short block length turbo codes [2], [3]. The deterministic interleavers are also preferred to reduce the hardware requirement for interleaving and de-interleaving operations. The design of interleaver is based on two major criteria: 1) the distance spectrum properties reflecting the weight distribution of the code, and 2) the correlation between the soft output of the decoder corresponding to its parity bits and the information data sequence. The second criterion is often referred to as iterative decoding suitability (IDS) criterion [4]. This is a measure of effectiveness of iterative decoding based on the fact that if two data sequences are less correlated, then the performance of the iterative decoding algorithm improves.

The performance of turbo codes at low BER is mainly dominated by minimum effective free distance (d_{min}) [5], [6]. The error floor, which usually appears in the region of BER curve of turbo codes below 10^{-5}, is the result of small d_{min} [8]. The error floor can be lowered by increasing either interleaver size or d_{min}. The increase in interleaver size may not be desirable to avoid latency in some applications, whereas the increase in d_{min} can be realized through an appropriate design and choice of the interleaver. It is well recognized that good spreading properties are desirable for achieving good distance properties. A sufficient d_{min} guarantees an improvement in residual BER at decoder output, when increasing the SNR.

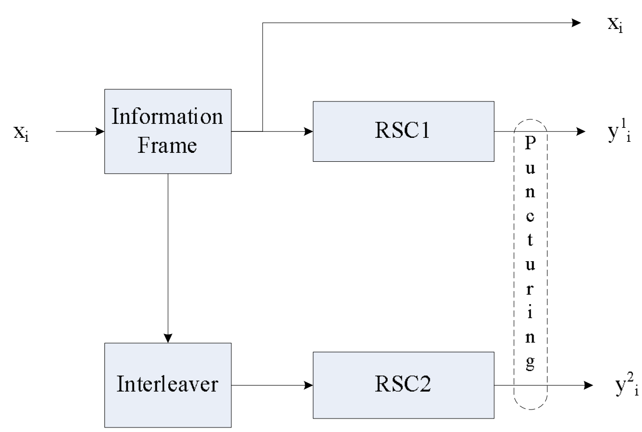

Fig. 1. 16 state turbo code with polynomial $(fb, ff) = (23, 35)_8$

The performance of iterative decoding improves if the information that is sent to each decoder from the other decoders is less correlated with the input information data sequence. John Hokfelt [4] proposed the IDS criterion based on this fact for designing an interleaver. The interleaver presented here is designed to exploit the advantages of both criteria simultaneously and also providing an interesting implementation advantage. The interleaver is defined by very simple rules and yet provides enhanced performance as compared long pseudo random interleavers. Section II describes the structure of the new deterministic interleaver and its spread spectrum. The S-random interleaver and spread definition are reviewed in section III. Simulation results on BER and FER performance of turbo code are compared for the new deterministic interleaver and S-random interleaver in section IV. The paper is concluded in section V.

II. Review of S-Random Interleaver and Spread Definition

The interleaver reads from a vector of input symbols or samples and writes to a vector of interleaved output samples. The output samples are written using a write index *i=0,...,N-1*, where as the read index is *j=0,...,N-1*. *N* denotes the length of the interleaver. The operator Π defines the mapping from order of the input samples to the position of the output vector.

The S-random interleaver is a semi-random interleaver, which is constructed by random selection, without replacement of *N* integers from $i = 0, ..., N-1$ with following constraint. Each randomly selected integer is compared to the S previously selected integers. If the absolute value of the difference between currently selected integer and S previously selected integers is smaller than S, then current selection is rejected. This process is repeated until all N integers are selected. S is an integers smaller than N that denotes the one sided span. The theoretical maximum spread for S-random interleaver is $\left\lfloor\sqrt{N}\right\rfloor$ but generally we keep $S < \sqrt{\frac{N}{2}}$ to have a good trade-off between interleaver performance and search time.

Recently a two step S-random interleaver [7] has been proposed for short block length turbo codes. The construction of two step S-random interleaver is based on S-random interleaver and an additional constraint is applied to reduce, or at least maintain the correlation between the input information and the extrinsic information. The two step S-random interleaver achieves better results than S-random at the cost of increased complexity and search time.

The spread associated with S-random interleaver can be defined as

$$Spread(i,j) = |\Pi(i) - \Pi(j)| \tag{1}$$

$$Spread_{min} = \underbrace{min}_{i,j} S(i,j) \tag{2}$$

where *i* and *j* are two write induces and $\Pi(i), \Pi(j)$ are their interleaved positions.

A more effective definition of spread that is closely related to turbo codes was presented in [9]. The same spread definition is used here. The spread measure associated with two write indices *i* and *j* is defined as

$$S(i,j) = |\Pi(i) - \Pi(j)| + |i-j| \tag{3}$$

$$S_{min} = \underbrace{min}_{i,j} S(i,j) \tag{4}$$

S_{min} is the minimum spread value of *S(i,j)* for all possible *i* and *j*. The theoretical maximum spread based on the above definition is $\left\lfloor\sqrt{2N}\right\rfloor$. As discussed earlier, maximum spread is required to achieve sufficient d_{min}. The design of the new interleaver presented in section III, achieves the maximum theoretical spread throughout its length.

III. Structure of New Deterministic Interleaver

The deterministic interleaver can be viewed as line wise writing and column wise reading, which provides regular permutation to the interleaved sequence. The interleaver reads from a vector of input symbols or samples and writes to a vector of interleaved output samples. The output samples are written using a write index *i=0,...,N-1*, where as the read index is *j=0,...,N-1*. The operator Π defines the order in which the samples are read from the input vector. That is the j^{th} element selected and written to position *i* in the output vector, is read from location Π(*j*) in the input vector. It can be written as

$$i = \Pi(j) \tag{5}$$

The increment step *k* to choose the next j^{th} element from the input vector is given by

$$i = \Pi(j) = jk \quad mod\ N \tag{6}$$

where step value *k* is an integer. To avoid duplication, *k* should be *prime* relative to *N*.

As discussed earlier, the maximum spread S_{min} is required to achieve sufficient d_{min}. The value of *k* that maximizes the S_{min} is an integer in close vicinity of the S_{min} but $k \neq S_{min}$, and *k* is prime relative to *N*. If we impose a constraint on the block length, so as to be $N = 2n^2$, where *n* is any integer, then the value of *k* in equation 6 can be calculated as

$$k = S_{min} \pm 1 \tag{7}$$

The interleaver designed in this manner provides regular permutations, which can efficiently handle the single error events. Another class of codewords is made up of multiple error events. These errors usually occur at low BER. Certain amount of de-correlation has to be introduced to the regular permutation function in equation 6 to improve the performance of the decoder at low BER. For information data sequences, *i* and *j*, $\Pi(i)$ and $\Pi(j)$ represent their interleaved locations in the output vector. The S-random interleaver which is believed to be the best in literature [7], guarantees that if

$$|i-j| \leq S \tag{8}$$

then

$$|\Pi(i) - \Pi(j)| > S \tag{9}$$

However, some information bits are mapped to itself in the interleaved output vector. That is, mapping of index $j \rightarrow \pi(j)$, where $j = \pi(j)$ is a valid assignment for S-random interleaver. Such assignment can degrade the performance of iterative decoding in turbo codes. The larger the displacement

$|j-\pi(j)|_{min}$, the smaller the correlation between the extrinsic information from one decoder to the input information data sequence at the input of the other decoder.

TABLE I

ACHIEVABLE SPREAD AND DISPLACEMENT FOR DIFFERENT BLOCK LENGTHS

N	k	**d**	S_{min}	$min(\|j-\pi(j)\|)$
4050	631	44	90	45
2048	449	31	64	32
512	97	15	32	16
408	43	2	20	3
392	85	13	28	14
200	61	9	20	10
128	49	7	16	8
50	31	4	10	5
32	9	3	8	4
8	5	1	4	2

Table .1: Results on spread spectrum distribution of the interleaver.

The interleavers designed for regular permutations avoids such assignments but the minimum displacement distance is always 1. That is $|j-\pi(j)|_{min}=1$. In the next step, we try to maximize the minimum displacement distance $|j-\pi(j)|_{min}$ by modifying equation 6 as

$$i=\Pi(j)=jk+d \quad mod\ N; \\ 0\leq d<S_{min}/2 \tag{10}$$

where *d* is the circular displacement distance. The value of *d*, which maximizes the $|j-\pi(j)|_{min}$ is an integer, which is prime relative to *N*. In practice if the block length meets the constraint, then the value of of d in equation 10 can be calculated as

$$d=\frac{S_{min}}{2}-1 \tag{11}$$

and the achievable $|j-\pi(j)|_{min}=S_{min}/2$. Table I shows the values of S_{min} and $|j-\pi(j)|_{min}$ for different block lengths. The values of *k* and *d* used in equation 10 are also shown in the third and fourth columns of Table I, respectively.

Recently some parallel architectures for turbo decoder has been proposed [10], [11], that is possibility to use several processors, without increasing the memory requirement. The interleaver is a cause of bottleneck for parallelization of turbo decoder. Our design of new deterministic interleaver inherits the property of low memory requirement, because all the indices can be calculated *'on the fly'* by storing very few parameters. This makes the natural parallelism possible for the decoder.

IV. SIMULATION RESULTS

To estimate the BER and FER performance of turbo codes applying binary phase shift keying (BPSK) and using the new deterministic interleaver, we have simulated the rate 1/3 turbo codes for short block length. The turbo code is comprised of two identical 16 state, rate 1/2 RSC encoders with feed forward (*ff*) and feed backward (*fb*) polynomials $(35,23)_8$ respectively. Both the encoders start in the all zero state. The trellis of the first RSC encoder is terminated to the initial state, where as the trellis of the second RSC encoder is left unterminated. To establish the reference point, the BER and FER results with S-random interleaver are also given for similar block sizes and code rate.

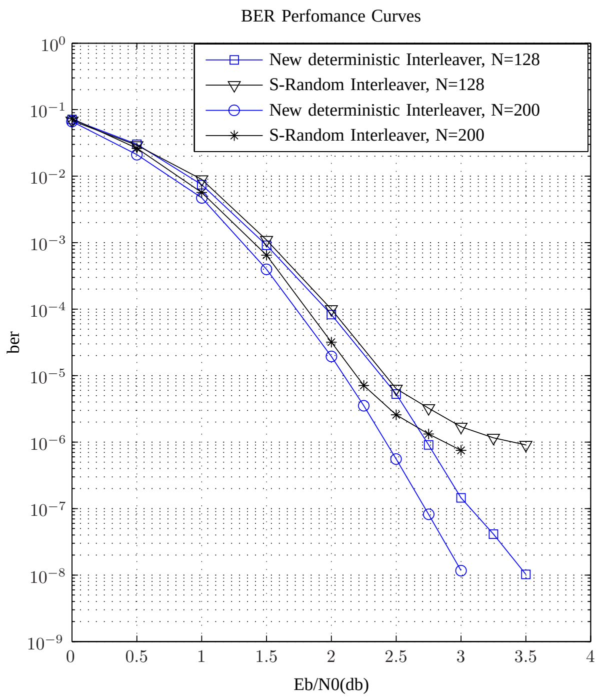

Fig. 2. Simulation (BER) results for 16 state turbo code

Figure 2 compares the BER performance of the new deterministic interleaver with S-random interleaver for block size of 128 and 200 data bits. In all the examples, the number of iterations using log-MAP decoding algorithm is 14. For the S-random interleaver, the value of S is initially set to 8 and 10 for for block size of 128 and 200 data bits respectively. However, in the search procedure the values reduce to 5 and 3 for the last few indices. The performance curve of S-random interleaver diverges below $BER=10^{-5}$, where as the new deterministic interleaver shows better convergence. It is also observed that the new deterministic interleaver does not show the error floor at error rates as low as $BER=10^{-8}$. Figure 3 compares the FER performance of the new deterministic interleaver with S-random interleaver for the same parameters as used for simulation in Figure 2. The performance curve of S-random interleaver diverges at $FER=10^{-4}$ for the block size of 200. The performance is even worse, when the block size is increased to 128. As expected the superiority of the new deterministic interleaver is evident in the error floor region (below $FER=10^{-4}$). From Figure 2 and Figure 3, it can be concluded that the new deterministic interleaver outperforms the S-random interleaver in low BER region because of the larger values of d_{min} and $|i-\pi(i)|_{min}$.

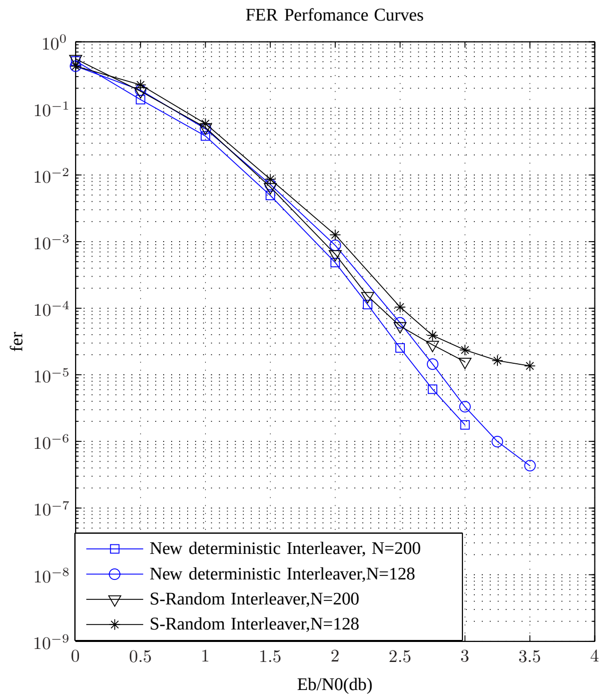

Fig. 3. Simulation (FER) results for 16 state turbo code

V. CONCLUSION

In this paper we have presented a solution to design an interleaver for short block length that exploits the distance spectrum properties of turbo codes and correlation properties of iterative decoding, simultaneously. A systematic method to calculate the parameters that characterize the interleaver was presented for block lengths of interest under certain constraint. The interleaving and de-interleaving operation can be performed "on the fly" by only storing few parameters. The interleaver designed in this manner suits the natural parallelism of the decoder, and yet provides impressive BER and FER performance.

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