An Adaptive Algorithm for Sinusoidal Interference Suppression in UWB-IR Systems

Mauro Biagi, Cristian Pellizoni, Nicola Cordeschi, Fabio Garzia, Enzo Baccarelli
{biagi, pelcris, cordeschi, enzobac }@infocom.uniroma1.it
fabio.garzia@uniroma1.it

***Abstract*— Ultra Wide Band (UWB) Radio transmission is an emerging technology and up to now a lot of works are devoted to increase capacity and QoS of UWB-based systems. At this regard, a critical point still unresolved deals with the performance degradation induced in UWB systems by narrow-band interference, possibly generated by concurrent services. In this work we present a novel simple-to-implement algorithm to estimate and suppress narrow band interferences impairing UWB signals based on adaptive signal-processing techniques.**

I. Introduction and System Model

In these last years Ultra Wide Band radio is becoming an appealing technology for wireless networks. Starting from [1,2], a lot of works were focused on the utilization of this technology to plan communication systems. More in particular, several works deal with the utilization of Pulse Position Modulation (PPM), and by fact, this is the most adopted modulation technique in UWB. In this work we present an adaptive simple-to-implement algorithm to mitigate narrow band (sinusoidal) interference induced by concurrent services sharing the same band employed by an UWB system. This algorithm is able to estimate the number of interfering tones, their frequency allocations and amplitudes in an adaptive fashion. In the considered system, the transmitted signal $s(t)$ is the Ultra Wide Band pulse. After indicating the impulse response of the transmission channel by $h(t)$, the considered "filtered signal" $v(t)$ is given by the convolution between $s(t)$ and $h(t)$. We assume that the signal $v(t)$ is corrupted by a narrow band interfering signal $i_n(t)$. Some examples of $i_n(t)$ signals are those generated by concurrent telecommunication services as for example, GPS (1575.42 MHz). The last disturbing signal $n(t)$ is the Additive White Gaussian Noise (AWGN). The "monocycle" $s(t)$ signal is defined as in [1]. About the receiving correlation filter, its response is defined as $m(t) \doteq s(t) - s(t-\delta)$ with equal the PPM time shift [1]. An additional assumption we introduce is perfect synchronization between transmitter and receiver and by fact, for synchronization purposes, we may effectively use $m(t)$ and its output to detect the peak of the correlation function [1]. Therefore, the narrow band interference (also called external interference) $i_n(t)$ may be, at first, modeled as a cosinusoidal tone defined as

$$i_n(t) = \alpha \cos(\omega t + \varphi) \tag{1}$$

with an unknown amplitude α , angular frequency ω and phase φ. In the following, we relax this (simple) assumption by increasing the number of cosinusoidal tones in order to model the interference as

$$i_T(t) = \sum_{m=1}^{M} \alpha_m \cos(\omega_m t + \varphi_m) \tag{2}$$

Doing in so, (1) represents the particular case of (2). Therefore, according to the receiving scheme of Fig.1, main task concerns how to estimate the unknown parameters $\alpha, \omega\varphi$ presented in (1) in order to generate a reliable replica of the interfering tone and then subtract it from the received signal. In fact, after sampling the received signal r(t) and removing the estimated interfering tone, we have

$$r(k) - \tilde{i}_n(k) = s(k) * h(k) + n(k) + i_n(k) - \tilde{i}_n(k) \tag{3}$$

that can be also rewritten as

$$r(k) - \tilde{i}_n(k) = s(k) * h(k) + n(k) + \varepsilon_n(k) \tag{4}$$

where we indicate as $\varepsilon_n(k)$ the estimation error. Obviously, when this error becomes negligible, link (4) approaches the (ideal) AWGN channel when $h(t)$ approaches ISI-free channel.

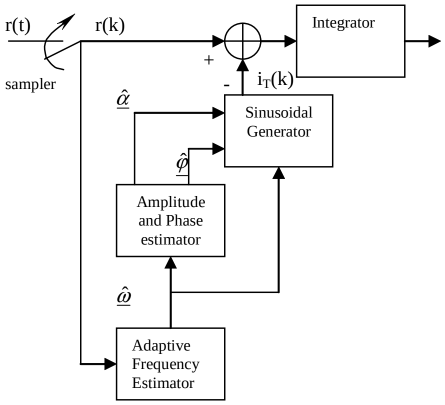

Fig. 1. The proposed receiver and interference suppressor scheme.

II. Frequency Estimation Stage

Before approaching the problem of estimation of the angular frequency ω in (1), it may be suitable to point out some considerations. Firstly, after modeling the narrow band interfering signal as a sinusoidal tone, receiver perfectly knows its spectral shape. Secondly, since the spectral properties of the received signal are known, from the outset it results that a frequency domain processing is an appealing approach for estimating ω in (1). Thus, we can try to estimate the frequency position of the interference tone via an FFT-based algorithm. For this purpose, after sampling the received signal r(t) to obtain r(k), we can apply FFT to the obtained sequence. Afterwards, the estimation algorithm tries to detect quick variations of the slope of the received signal spectrum induced by the presence of sinusoidal tones. To evaluate the change of the slope we can resort to the derivative operator so to arrive at the processed signal. Passing now to compare this frequency estimation method with the multi-parametric Non-Linear Least Square algorithm (NLS in the following), we recall that general goal of NLS algorithm is to minimize the parametric function

$$f(\omega, \alpha, \varphi) = \sum_{i=1}^{N} \left| r(t) - \sum_{m=1}^{M} \alpha_m \cdot e^{j(\omega_m \cdot t + \varphi_m)} \right|^2 \tag{5}$$

Mauro Biagi, Cristian Pelizzoni, Nicola Cordeschi and Enzo Baccarelli are with INFO-COM Dept., Università di Roma La Sapienza, via Eudossiana 18, 00184 Rome, Italy. Fabio Garzia is with ICMMPM depertement MP division, Università di Roma La Sapienza, via Eudossiana 18, 00184 Rome, Italy.

where ω_m is the angular frequency of the m-th interfering tone and α_m and φ_m are the corresponding amplitude and phase. The sum from 1 to N is referred to the N samples representing the sampled monocycle. In the simple case when only one cosinusoidal component is present, eq.(5) simplifies in

$$f(\omega, \alpha, \varphi) = \sum_{i=1}^{N} \left| r(t) - \alpha \cdot e^{j(\omega \cdot t + \varphi)} \right|^2 \tag{6}$$

Thus, the solution of (7) allow us to evaluate the estimated frequency as

$$\hat{\omega} \doteq \underset{\omega \in I_\omega}{\arg\max}\, \hat{P}_R(\omega) \tag{7}$$

where we indicate as $\hat{P}_R(\omega)$ the periodogram of the received signal, given by

$$\hat{P}_R(\omega) = \frac{1}{N} \left| r(t)\, e^{j \cdot \omega \cdot t} \right|^2 \tag{8}$$

and I_ω in (7) is the frequency sub-band we search for slope changes of the received signal. So, in the proposed algorithm the desired solution can be found by simply observing the change of slope of the received spectrum.

III. Phase Estimation Stage

We pass now to consider the phase estimation stage. The received sampled signal r(k) can be expressed as

$$r(k) = s(k) * h(k) + n(k) + i_n(k) \tag{9}$$

and we can rewrite the interfering tone (1) in the following equivalent form:

$$i_n(k) = a \sin(\omega k) + b \cos(\omega k) \tag{10}$$

By exploiting (10) we can resort to a vector form to collect the interference samples. More in particular, they can be lumped in the following vector

$$\mathbf{i}_n \doteq \left[i_n(k_1)\ \ i_n(k_2) \ldots i_n(k_N) \right]^T = \mathbf{A}\mathbf{x} \tag{11}$$

where $\mathbf{A}$ is defined as

$$\mathbf{A} \doteq \begin{bmatrix} \sin(\omega k_1) & \cos(\omega k_1) \\ \sin(\omega k_2) & \cos(\omega k_2) \\ \ldots & \ldots \\ \sin(\omega k_N) & \cos(\omega k_N) \end{bmatrix} \tag{12}$$

with $\mathbf{x}$ gathering the above coefficients *a* and *b* in (10) as in

$$\mathbf{x} \doteq [a\ b]^T \tag{13}$$

When the achieved frequency estimate is sufficiently reliable so that we may assume $\hat{\omega} \backsimeq \omega$, we can minimize the squared difference between the received sequence $r(k)$ and the sinusoidal interfering component defined in (11). This is equivalent to minimize the following quadratic form:

$$D(\mathbf{x}) \doteq \|\mathbf{r} - \mathbf{A}\mathbf{x}\|^2 = \\ (\mathbf{r} - \mathbf{A}\mathbf{x})^T (\mathbf{r} - \mathbf{A}\mathbf{x}) = \mathbf{r}^T\mathbf{r} - 2\mathbf{r}^T\mathbf{A}\mathbf{x} + \mathbf{x}^T\mathbf{A}^T\mathbf{A}\mathbf{x} \tag{14}$$

whose minimum may be directly evaluated via derivative of $D(\mathbf{x})$. In fact, after deriving (14), we arrive at the ML (Maximum Likelihood) equation

$$\partial D(\mathbf{x}) / \partial \mathbf{x} = -2\mathbf{r}^T\mathbf{A} + 2\mathbf{x}^T\mathbf{A}^T\mathbf{A} = 0 \tag{15}$$

that, in turns, leads to the desired ML solution

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{A} \right)^{-1} \mathbf{A}^T\mathbf{r} \tag{16}$$

for the unknown parameters a, b in (10). From a computational point of view, it is interesting to note that the symmetric matrix to be inverted in (16) is (2×2) so that the computational cost in any case is limited. Thus, the amplitude of the interfering tone in (1) can be directly estimated as

$$\hat{\alpha} = \sqrt{\hat{a}^2 + \hat{b}^2} \tag{17}$$

while the phase may at first be evaluated as

$$\hat{\varphi} = -\arctan\left(\hat{a} \big/ \hat{b} \right) \tag{18}$$

and then the π ambiguity may be resolved by retaining the sign of $\hat{a}$ in (17).

IV. Frequency and Phase Estimation Stage for Multiple Interfering Tones

When we consider the effect of M sinusoidal tones interfering the UWB signal, the expression (4) becomes

$$r(k) - \tilde{i}_T(k) = s(k) * h(k) + n(k) + \varepsilon_T(k) \tag{19}$$

where $\varepsilon_T(k)$ represents the resulting estimation error. Thus, in this case we can rewrite (10) as

$$i_T(k) = \sum_{m=1}^{M} a_m \sin(\omega_m k) + b_m \cos(\omega_m k) \tag{20}$$

Furthermore, to apply (14) we have to estimate accurately the frequency positions of the M tones, where also number M is a priori unknown. To solve this problem, we can resort to a frequency window where we apply the derivative to the received signal spectrum in order to count the number of tones by detecting the zero crossings of the spectral slope. Thus, after windowing the spectrum slope, we simply count the number of changes of sign encountered over the window scan and then we record them into a vector $\boldsymbol{\omega}$. This procedure can be summarized the pseudocode detailed in Table I. The pseudocode supports the adaptive feature of the proposed algorithm. In fact, we

```
counter=0;
set window length;
number of windows = number of samples / window length;
for n=1 to number of windows
{
if spectral slope presents quick variations then
counter=counter+1;
ω_counter = ω position;
}
M=counter;
```

TABLE I

A pseudo-code for the evaluation of number of tones and frequency positions.

are able to take into account M-interfering signals without any a priori knowledge of knowing M a priori. Afterwards, the adaptive frequency estimator block output in Fig.1 is the vector $\hat{\boldsymbol{\omega}}$ collecting the M estimated values for the frequency allocations of the interfering tones. After estimating these frequency allocations, we can apply the ML equation in (14). The general form of the resolving relationship (16) does not change but the matrix and vector dimensions in (16) increase respect to the simple case of only one interfering tone. In fact, now we have

$$\mathbf{A} = \begin{bmatrix} \sin(\omega_1 k_1) & \cos(\omega_1 k_1) & \ldots & \sin(\omega_M k_1) & \cos(\omega_M k_1) \\ \sin(\omega_1 k_2) & \cos(\omega_1 k_2) & \ldots & \sin(\omega_M k_2) & \cos(\omega_M k_1) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \sin(\omega_1 k_N) & \cos(\omega_1 k_N) & \ldots & \sin(\omega_M k_N) & \cos(\omega_M k_1) \end{bmatrix} \tag{21}$$

with

$$\mathbf{x} \doteq \begin{bmatrix} a_1 & b_1 & \ldots\ldots & a_M & b_M \end{bmatrix}^T \tag{22}$$

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in place of (12) and (13), respectively. The computational cost is of the order of $O(2M^2 + N)$.

V. Comparisons with conventional spectral estimation approaches

To test actual effectiveness of the proposed interference-suppression algorithm, in this Section we compare its performance with those of conventional spectral estimation methods such as MUSIC (MUlti SIgnal Classification) and Stochastic Approximation ones [3,4,6].

A. Application of the MUSIC method to UWB signals

The MUSIC method allows to estimate the frequency positions of interfering tones[1] by detecting peaks of the following function [6]:

$$\psi^*(\omega)\hat{\mathbf{G}}\hat{\mathbf{G}}^T\psi(\omega) \quad (23)$$

where $\hat{\mathbf{G}}$ is the matrix collecting the eigenvalues of the (possibly colored) ”disturb” affecting the received sequence in (4) and $\psi(\omega) \doteq \left[1 \; e^{-j\omega} \; ... \; e^{-j(n-1)\omega}\right]$ is defined as according to [6]. The dimension of squared matrix $\hat{\mathbf{G}}$ is set in agreement with the degrees of freedom desired for the MUSIC estimator (see Chap.IV of [3] for additional details about implementation of MUSIC algorithm). As it is known, by increasing the dimension of we are able to refine the desired estimates. However, since we have to detect the roots of (23) closest to the unit circle, due to the coloration of the disturbing term $\varepsilon_n(k)$ in (4) by increasing the degrees of freedom we find a lot of roots very close to the unit circle so that we may incur to ambiguity phenomena in choosing the right root (see Fig.2). By fact, we have experienced

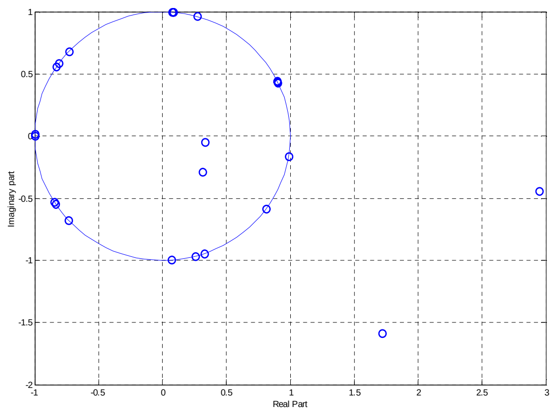

Fig. 2. Diagram of the roots for SIR=-29dB for three interfering tones at 900MHz, 1575.42MHz and 1800MHz, with 9degrees of freedom.

that a number of degrees of freedom for $\hat{\mathbf{G}}$ over three times the number of interfering tones does not allow to increase MUSIC performance in the considered UWB scenario. On the other hand, by reducing dimension of $\hat{\mathbf{G}}$ we decrease the corresponding degrees of freedom. When degrees of freedom equate two, MUSIC method collapses to Pisarenko one [4]. This last is less accurate for frequency estimation but does not present the ambiguity phenomena of MUSIC. The computational cost is of the order of $O(N^2 + N\log_2 N)$.

[1]MUSIC algorithm needs to know the number of M tones and this estimation can be performed by resorting to the so called Akaike Information Criterion (AIC) [9].

B. Application of Stochastic approximation approach to UWB signals

As it is known, the Stochastic Approximation approach is based on the recursive computation for the joined Maximum Likelihood estimation of frequency position, amplitude and phase of each interfering tone [9]. This estimation can be found via the following iteration in the k-index [5],

$$\hat{\mathbf{x}}(k) = \hat{\mathbf{x}}(k-1) + \frac{1}{k}\left[\mathbf{J}_0\left(\hat{\mathbf{x}}(k-1)\right)\right]^{-1} \cdot$$
$$\cdot \left[\underline{\nabla}_{\mathbf{x}} \log\left(p_0\left(r(k)|\mathbf{x}\right)\right)\right]_{| \; \mathbf{x}= \; \hat{\mathbf{x}}(k-1)} \quad (24)$$

where $\hat{\mathbf{x}} \doteq \left[\hat{\boldsymbol{\alpha}}^T\hat{\boldsymbol{\omega}}^T\hat{\boldsymbol{\varphi}}^T\right]$ is the vector of the parameters to be estimated, $\mathbf{J}_0\left(\hat{\mathbf{x}}(k-1)\right)$ is the $(3M \times 3M)$ Fisher's Information matrix per received sample evaluated at $\hat{\mathbf{x}}(k-1)$ [5] and $p_0\left(r(k)|\mathbf{x}\right)$ is the probability density function of the k-th observation conditioned on the parameters. The computational cost is of the order of $O(N^3)$.

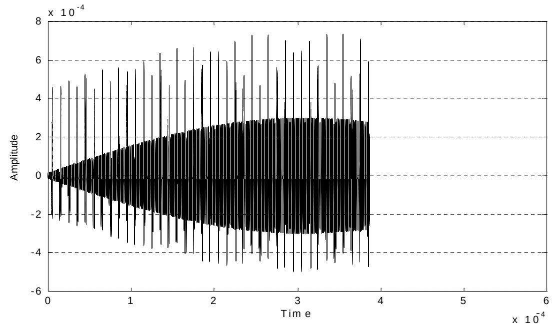

Fig. 3. Filtered signal after suppression (case 1).

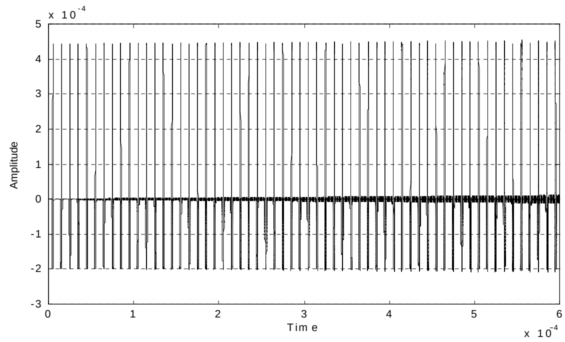

Fig. 4. Filtered signal after suppression (case 2).

VI. Comparisons and conclusive remarks

The comparisons are performed over the channel employed in [5] with pulse duration of $T = 0.5ns$, PPM shift $\delta = 0.5ns$. In Fig.3 the effect of estimation error is shown by considering an impaired estimation error in frequency detection. If this error become negligible the filtered signal becomes more similar to the transmit one (see Fig.5) An examination of Fig.5 shows that proposed algorithm is able to gain 3 dB over MUSIC-Pisarenko approaches and 6dB over Stochastic Approximation one with lower computational cost. Furthermore, performance loss of our algorithm with respect to the ideal AWGN-like is limited up to 1dB. Furthermore, Fig.5 also shows that both MUSIC-Pisarenko and Stochastic Approximation method fail to achieve reliable performance in the considered application scenarios and we have experienced that this drawback is mainly due to the colored and non-stationary features of the UWB signal s(k) presented in (4).

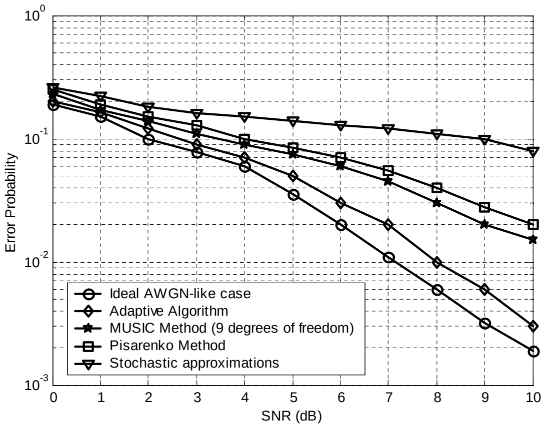

Fig. 5. Performance comparisons between the estimation methods for the same operating conditions of Fig.2.

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