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Optimal Structure for one Type Informational Networks

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Abstract - In this paper a formal statement of the problem of optimal structure for one type informational networks has been given. The strategy of forming informational field is working set(WS)of P. Denning strategy. In this respect the main problem and auxiliary problem were introduced. The conditions under which acceptable solution of the main problem via solution of auxiliary problem also given.

I. Introduction

Let us suppose that the network consist of $n+1$ nodes informationally connected with one another. Among them one auxiliary (from the processing point of view) node V is singled out as a center which forms a fragment $F_\tau(\theta)$ an informational field at any moment t of the seance of transmitting and processing a random information θ from one of the initial nodes to a final node. Mind that any random information $\theta \in D$ in an active node from the nodes $\{v_1, v_2, \ldots, v_n\}$ is to be processed while a seance. At any moment t of the seance the fragment $F_\tau(\theta)$ of the information field includes some subset of nodes from $v_1, v_2, \ldots, v_n$ and the center V itself does not belong to information field but serves , also as transmitter of information θ between two nodes at times as well. The random initial information θ while a seance is transferred from a node to a node , but the chain (sequence) of the processing nodes is unknown in advance and information does not necessarily go through all the nodes $v_1, v_2, \ldots v_n$ but any node v_i can be active many times while one seance. The processing chain of the nodes depends on the initial random information θ and at any moment t of the seance the current active node v_i together with the random information θ is to determine the next processing node or determine that the seance is completed at this moment.

Transmitting information between two nodes within of the fragment of the information field is free of change and from the fragment of the information field to outside via center V is charged. And let us consider that network's nodes except the center V is covered by the sets $S_1, S_2, \ldots, S_m$ without intersections.

The main problem is to minimize the functional of average value (charge) of the transmission of information due to the lucky coverage of the nodes of network. The functional here is mathematical expectation of total charge of the transmitting of information for one seance [1].

II. Statement of the Problem and Results

Let to each node v_i a weight $l_i > 0$ is assigned, $i = 1,2,\ldots,n$. Some of the nodes $v_1, v_2, \ldots, v_n$ initiate processing and transfer information (we can denote the set of such nodes by M^o) and some of them receive information (the corresponding set is denoted by M^1). The chain of processing nodes from one of the initial nodes to one of the final nodes is called a seance.

We assume that parallel computing process in the network is organized so that any node can process different information simultaneously and the time of expectation for processing for any random information θ does not exceed some system constant L for any node. It is naturally that the expecting time is not taken into consideration during virtual interval of time .

We will suppose that the network was in working state for a sufficiently long time and the sets $S_1, S_2, \ldots, S_p$ are given so that

1) $\{v_1, v_2, \ldots, v_n\} = \bigcup_1^p S_r$

2) $S_i \cap S_j = \phi \quad , \quad i \neq j$

3) The total weight of the nodes belonging to S_j does not exceed the weight a_r of the set S_r , $r = 1,2,\ldots,p$.

Further the sets $S_1, S_2, \ldots, S_p$ are called segments and the whole collection of these sets is called a segmentation of the network. Let a segmentation is always supposed to satisfy 1-3.

Let D be a set of random information. Assume that to every information $\theta \in D$ under the condition of activating of processing θ there corresponds a seance C_θ.

Let us consider that due to limited resources the node V (center) at any moment t of the seance C_θ forms a fragment of the informational field which coincident with a working set of segments $R_\tau(t)$ consisting of those and only those segments from $S_1, S_2, \ldots, S_p$ which contained information θ in their nodes during the interval $[t-\tau, t)$, where τ is a parameter (the window size [4,5]).

From now on the duration of seance is assumed to be bounded for any $\theta \in D$. Let us pose the problem more exactly. Let the functional $\overset{\circ}{F}_\tau(x)$ be the mathematical expectation of the total cost of information transmitting for one seance, where x is a restructuring (segmentation) matrix defining the covering of the nodes $v_1, v_2, \ldots, v_n$ by the segments $S_1, S_2, \ldots, S_p$ where $x_{ri} = 1$ if node v_i belongs to segment S_r, and $x_{ri} = 0$ otherwise, $i = 1, 2, \ldots, n$.

Let G denote some set of such matrices. Conceptually to any matrix $x \in G$ there corresponds a structure(segmentation) of the network under which the operating of the network is feasible. So, the main problem for us is

$$\min_{x \in G} F_\tau^o(x) \tag{0.1}$$

As an auxiliary problem we consider the problem of minimizing the functional $F_\tau^k(x)$ of the average cost (over $k \geq 1$ seances) of the information transmitting ,i.e.

$$\min_{x \in G} F_\tau^k(x) \tag{1.1}$$

Here we can consider the following questions :

defining the analytical from of the functional F°, F^k description of the set G,defining the expression for the radius of stability of optimal solutions of the main problem (0.1) defining the conditions under which the optimal solution of the auxiliary problem(1.1) will be optimal solution of the main problem(0.1).

Let $R_\tau(t)$ be working set of segments and $q_t = \{i_1, i_2, \ldots, i_{m(q_t)}\}$ be "working set" for the nodes at t virtual moment i.e. q_t is defined as the set of nodes referred to in the interval $[t-\tau, t)$. Let's call q_t at t virtual moment the check state of the program (c.p.s.) at moment t .

Note. Let $t' \neq t''$ then we shall not differentiate between $q_{t'}$ and $q_{t''}$ as the subsets of $\{1, 2, \ldots, n\}$. Let q_0 denote c.p.s. $q_0 = \phi$, and let's consider $q_0 \notin Q$, but of course $q_0 \subset Q$.

The elements of the set Q may be defined by reference strings to the nodes for different seance . The check state $q_t \in Q$ and the matrix $x \in$ G generate working set $R(q_t, x)$, each segment of which contains at least one node belonging to $q_t = \{i_1, i_2, \ldots, i_{m(q_t)}\}$. $R(q_t, x)$ may include nodes differing from the above mentioned ones. Let's denote set of nodes belonging to segments from $R(q_t, x)$ by $\hat{R}(q_t, x)$.

The nature of the main problem and Note 1 allow to omit the index t in the notation $q_t = \{i_1, i_2, \ldots, i_{m(q_t)}\}$ i.e. $q = \{i_1, i_2, \ldots, i_{m(q)}\}$. The same check state of the program q may occur at different virtual moments throughout the seance.

Let random variable ξ_{qi} - be the number of references to the node i while the execution of q for one seance. The same c.p.s. q may occur many times while the frame (window) moves along the reference string to nodes in one seance. The value of random variable ξ_{qi} is clearly independent of the change of matrix x , i.e. of the network structure change. Let $\forall q \in Q$, $1 \leq i \leq n$ and $M(\xi_{qi}) = E_{qi}$. The random variables ξ_{qi} ($q \in Q$, $1 \leq i \leq n$) are defined by reference string to the node in one seance.

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Let $\xi_{qi}^{(j)}$ be the same as ξ_{qi}, but in *j*-th seance of the network *j =1,2,...,k* and let $\xi_{qi}^{(1)}$, $\xi_{qi}^{(2)}$,... , $\xi_{qi}^{(k)}$ - are pare wise independent

$M(\xi_{qi}^{(j)}) = E_{qi}$, $j = 1,2,\ldots, k$

$Var(\xi_{qi}^{(j)}) = b_{qi}$, $j = 1, 2, \ldots, k$

Let $q = \{i_1, i_2, \ldots, i_m\}$ and i - be an arbitrary node: $1 \le i \le n$; the function $\delta_{qi}(x)$ is defined as

$$\begin{cases} 0 , & \text{if node } i \in \hat{R}(q,x) \\ 1 , & \text{otherwise} \end{cases}$$

By this function we will see if the reference from R(*q*,*x*) cause the segment fault or it will not. Let *i* , *x* - be fixed let's show the way to calculate the function $\delta_{qi}(x)$ by the elements of the matrix *x* under the conditions 1-3,

$$\delta_{qi}(x) = \prod_{j=1}^{m(q)} \left(1 - \sum_{r=1}^{p} x_{ri} \cdot x_{ri_j} \right)$$

Here it is evident that if the node $i \in q = \{i_1, i_2, \ldots, i_{m(q_t)}\}$ then $\delta_{qi}(x) \equiv 0 \ \forall x \in G$.

Lemma 1. Let time is taken for any seance for the network with WS strategy of the forming the informational field is finite, then for the functional $\overset{\circ}{F}_{\tau}(x)$ there exists representation

$$F_{\tau}^{\circ}(x) = \sum_{q \in Q} \sum_{i=1}^{n} E_{qi} \cdot \delta_{qi}(x) + \sum_{i=1}^{n} E_{q_{\circ}i} \qquad (1.2)$$

Corollary 1. Let time is taken for any seance for the network with WS strategy is finite, then for functional $F^{(k)}(x,\tau)$ there exists representation

$$F_{\tau}^{k}(x) = \sum_{q \in Q} \sum_{i=1}^{n} E_{qi}^{(k)} \cdot \delta_{qi}(x) + \sum_{i=1}^{n} E_{q_{\circ}i}^{(k)} ,$$

where $E_{qi}^{(k)} = 1/k \sum_{j=1}^{k} \xi_{qi}^{(j)} \quad \forall q \in Q, i = 1,2,\ldots,n$.

Let $H_{qr}(x)$ be characteristic function of working set R(q,x) i.e.

$$H_{qr}(x) = \begin{cases} 1 , & \text{if segment } S_r \in R(q,x) \\ 0 , & \text{otherwise} \end{cases}$$

then set G is described by the following constraints

$$\sum_{i=1}^{n} l_i x_{ri} \le a_r \qquad r = 1,2,\ldots, p \qquad (1.3)$$

$$\sum_{r=1}^{p} x_{ri} = 1 , \quad i = 1,2,\ldots, n \qquad (1.4)$$

$$\sum_{r=1}^{p} a_r \cdot H_{qr}(x) \le N_{1q} , \quad q \in Q \qquad (1.5)$$

$$x_{ri} \in \{0,1\} , \ r = 1,2,\ldots, p ; \ i = 1,2,\ldots, n \qquad (1.6)$$

The function $H_{qr}(x)$ under the conditions 1-3 is easy to calculate using the elements of matrix x namely

$$H_{qr}(x) = \max_{1 \le j \le m(q)} x_{ri_j} , \quad x_{ri} \in \{0,1\}$$

It's easy to notice , that if we treat ar as the size of the segment set Sr , r =1,2,…,p , then left part inequality (1.5) is the size of working set R(q,x). Let's denote it by [R(q,x)] and write

$$[R(q,x)] = \sum_{r=1}^{p} a_r \cdot H_{qr}(x) , \quad q \in Q$$

In this way we can calculate the size (weight) of working set having assigned the matrix *x* and check program state *q*.

Constraint (1.3) implies, that the summed weight of the nodes belonging to any segment is no greater than the weight of this segment. The constraint (1.4) shows that each node belongs to the one segment only. The system constant N_{1q} , in particular $N_{1q} = N_1$ limits the size (weight) of the working set R(*q*,*x*).

Representation (1.2) of the functional F^0 containing non-linear terms, and generally unknown quantities E_{qi} ($q \in Q$, $i = 1,2, \ldots, n$). However, if all E_{qi} are known then the

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problem(0.1) with functional $\overset{\circ}{F}_{\tau}(x)$ and constraints (1.3) – (1.6) is the problem of non-linear Boolean programming. For it the method of implicit search, branch and bounds, modified method of pseudo-Boolean programming etc., are applicable , after some transformation.

The number of constraints (1.3)- (1.6) equal $p+n+|Q|$ and among them $|Q|$ is non-linear. The specific features of the problem with the functional $\overset{\circ}{F}_{\tau}(x)$ and constraints (1.3)–(1.6) allow its simplification, particularly the introduction of Boolean variable $\omega_{qi} \in \{0,1\}$ such that if the matrix x is fixed, then the variable ω_{qi} will have the same value as the function $\delta_{qi}(x)$.

Let ρ_0 is the radius of stability (following by Leontiev's paper[6]) of the problem(0.1). Thus under the conditions of lemma 1 the next theorem holds

Theorem 1. Let time is taken for any seance for the network with WS strategy of the forming the informational field is finite, then for the functional F° there exists Ω - representation $L_{a,\tau}(x,\omega)$

$$L_{a,\tau}(x,\omega) = \sum_{q\in Q} \sum_{i=1}^{n} E_{qi}\cdot\omega_{qi} + \sum_{i=1}^{n} E_{q_\circ i} \qquad (1.7)$$

such that for the radius of stability of the problem(0.1) the next formula holds

$$\rho_0 = \min_{j\notin\psi(A)} \max_{i\in\psi(A)} \frac{L_{a,\tau}(x^j,\omega^j)-L_{a,\tau}(x^i,\omega^i)}{\left\|\omega^j-\omega^i\right\|^*}$$

where $\omega = (\omega_{qi})$ is an aggregate of auxiliary variable corresponding to $x = (x_{ri})_{p\times n}$, $a = (E_{qi})_{|Q|\times n}$, $\psi(A)$ - is the set of numbers of optimal solutions of the main problem(0.1), $\|.\|^*$ - is a norm.

Eventually on the basis of the theorem 1 we write the problem(0.1) in the following form

$$\min_{(x,\omega)\in(G,\Omega)} \sum_{q\in Q} \sum_{i=1}^{n} E_{qi}\cdot\omega_{qi} + \sum_{i=1}^{n} E_{q_\circ i} \qquad (1.8)$$

and the problem(1.1) in the form :

$$\min_{(x,\omega)\in(G,\Omega)} \sum_{q\in Q} \sum_{i=1}^{n} E_{qi}^{(k)}\cdot\omega_{qi} + \sum_{i=1}^{n} E_{q_\circ i}^{(k)}$$

with the constraints which are determine the set (G,Ω)

$$\sum_{i=1}^{n} l_i\, x_{ri} \le a_r \qquad r = 1,2,\dots,p \qquad (1.9)$$

$$\sum_{r=1}^{p} x_{ri} = 1 \quad , \quad i = 1,2,\dots,n \qquad (1.10)$$

$$\sum_{j=1}^{m(q)} \sum_{\tau=1}^{p} x_{ri}\cdot x_{ri_j} + m(q)\cdot\omega_{qi} \ge 1 \qquad (1.11)$$

$$\sum_{j=1}^{m(q)} \sum_{\tau=1}^{p} x_{ri}\cdot x_{ri_j} + m(q)\cdot\omega_{qi} \le m(q) \ , \quad q\in Q \quad i = 1,2,\dots,n \qquad (1.12)$$

$$\sum_{r=1}^{p} a_r \cdot \max_{1\le j\le m(q)} x_{ri_j} \le N_{1q} \quad , \ q\in Q \qquad (1.13)$$

$$x_{ri} \in \{0,1\}, \omega_{qi} \in \{0,1\} \quad , r = 1,2,\dots,n\,;\ q\in Q \qquad (1.14)$$

The functional(1.8) contains $|Q|\cdot n$ of the Boolean variables ω_{qi} and the variable x_{ri} in (1.8) are already fictitious. The constraints(1.9)-(1.13) contain $n+p+2|Q|n+|Q|$ the correlations of which $2|Q|\cdot n$ are quadratic $|Q|$ are non-linear.

Mind that the form of the functional in(1.7) is convenient for investigation of the solution stability of the main problem (0.1) and for estimation quantity k. Here k is the number of the seances which allows acquire the necessary accuracy of solution of the problem(0.1) in terms of (1.15)

$$\Pr\left\{ \left| F^{\circ}(x^*) - F^{\circ}(x_k) \right| \le \varepsilon \right\} \ge 1-\eta \qquad (1.15)$$

where x^*- optimal solution of the main problem(0.1), x_k - an optimal solution of the auxiliary problem(1.1), $\varepsilon > 0, \eta \in (0,1)$ the quantity k is the function radius of

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stability ρ_0 of the main problem (0.1). Thus the next assertion holds.

Theorem 2. Let time is taken for any seance for the network with WS strategy of forming the informational field is finite. Let $\varepsilon > 0$, $\eta \in (0,1)$ and every sequence $\xi_{qi}^{(1)}, \xi_{qi}^{(2)}, \ldots, \xi_{qi}^{(k)}$, ($q \in Q$, $i = 1,2,\ldots, n$) satisfies the conditions (a), (b), (c) up to $k \geq \max b_{qi} / (\rho_0^2 \cdot \eta)$ then optimal solution x_k of auxiliary problem(1.1) with the probability no less than $1-\eta$ is the ε-optimal solution of the main problem(0.1) in the terms of (1.15).

In the talk we will discuss also about combinatorial particularities of the problems (0.1) and (1.1). We will show the way to avoid combinatorial difficulties of the problems (0.1) and (1.1) using particularities these problems.

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References

[1] A. E. Dusembaev, "Mathematical models of program segmentation",- M:Fizmatlit(Nauka),2001,monography.

[2] A. E. Dusembaev, "An operator approach to program segmentation," Short Presentation ICM'98.Berlin,pr290

[3] A.E. Dusembaev," One way to program segmentation". Report of Russian Academy of sciences.1993. V.329. N.6. pp712-714.

[4] P. Denning, "Working set: past and present," .IEEE Trans. Software Eng. V SE-6 ,N1 pp.64-84

[5] D. Ferrari , "Computer systems performance evaluation," Prentice-Hall Inc. New Jersey,1978.

[6] V.K Leontiev. Ustoichivost v lineinih discretnih zadachah. Problemi Cybernetiki, Vip. N35-M: Nauka 1979 c.169-184.