# QUADRATURE-SPREAD CDMA ERROR PERFORMANCE ANALYSIS WITH DIVERSITY RECEPTION OVER MULTIPATH FADING CHANNELS

*Mohammad Adnan Andalusi*

King Fahd University of Petroleum & Minerals
Electrical Engineering Department
Dhahran 31261, Saudi Arabia
andalusi@kfupm.edu.sa

## ABSTRACT

The paper addresses the error performance analysis of selection diversity and maximum ratio combining over Rayleigh fading channels when the multiple access signalling schemes use OQPSK-modulated CDMA with random signature sequences and arbitrarily shaped chip waveform pulses. Closed-form bit error probability expressions are derived based on the use of the Standard Gaussian Approximation method, which is found to have excellent accuracy for OQPSK-type spreading. Various examples are then given to illustrate relative performance comparisons among different modulation types.

## 1. INTRODUCTION

Multipath fading arises in mobile radio communications when the signal reaches the receiver through several propagation paths, each with a different time delay and attenuation factor. The arriving signal replicas may add constructively or destructively depending on their time delays (and hence, relative phases). The resultant composite signal will then have a random fluctuating amplitude, giving rise to what's know as multipath fading, which constitutes a serious impairment to mobile radio communication channels [9].

The use of direct-sequence Code Division Multiple Access (CDMA) signalling is a well-known efficient technique particularly suited to combat distortion in multipath fading channels [2, 9]. This can be achieved either by suppressing late arriving signals as with other users' multiple-access interference or, more efficiently, by exploiting the inherent diversity provided by such delayed copies of the transmitted signal. There has been a lot of work on this subject, especially with the recent growing interest in wireless personal communication systems where multipath fading is a major challenge.

The work presented in this paper is an extension of the error performance analysis presented in [8] for single-path unfaded channels. Previous related work, for the case of BPSK spreading with deterministic signature sequences, can be found in [3, 4]. In this paper, we derive additional results for the case of arbitrary OQPSK-type modulation (i.e., with arbitrarily shaped chip waveform pulses), and we also include the effect of random spreading sequences. The latter forms of modulation and spreading are more relevant to the recently introduced 2G and 3G CDMA standards. The error analysis presented is based on the use of the Standard Gaussian Approximation (SGA) technique, which was shown in [8] to be very accurate for OQPSK-type signals (very much unlike the BPSK case [5]).

The paper is organized as follows. In Section 2, we discuss the signal and system models. In Section 3, several diversity receiver structures are described. Then, in Section 4, we study the error performance of these receivers using the standard Gaussian approximation technique. Numerical examples and comparative results are shown in Section 5, and final conclusions are given in Section 6.

## 2. SYSTEM AND SIGNAL MODELS

We consider the basic model for OQPSK-spread DS-CDMA signalling with $K$ multi-users. The binary random data symbols of the $k$-th user, $1 \le k \le K$, are split into two I & Q streams $b_k^I(t) = \sum_{i=-\infty}^{\infty} b_{k,i}^I \, p_T(t-iT)$ and $b_k^Q(t) = \sum_{i=-\infty}^{\infty} b_{k,i}^Q \, p_T(t-iT)$, where $p_T(t)$ is the unit pulse over the bit interval $[0, T]$. The data are spread by random sequences $a_k^I(t) = \sum_{m=-\infty}^{\infty} a_{k,m}^I \; \psi_c(t - jT_c)$ and $a_k^Q(t) = \sum_{m=-\infty}^{\infty} a_{k,m}^Q \; \psi_c(t-jT_c)$ using $N$ chips per bit, and a chip waveform pulse $\psi_c(t)$ defined over $[0, T_c]$, where $T_c$ is the chip interval. Offset quadrature modulation is obtained by modulating the spread data onto two in-phase and quadrature carriers with half a chip $T_c/2$ delay in the I-branch. Standard OQPSK is obtained with a rectangular chip pulse, but the general model and subsequent analysis applies to arbitrary pulse shapes as well. Denoting by $c(t)$ the product symbol stream $a(t)b(t)$, the resulting $k$-th user transmitted signal is

$$s_k(t) = c_k^I(t - \tfrac{T_c}{2})\cos(\omega_c t + \theta_k) + c_k^Q(t)\sin(\omega_c t + \theta_k)) \quad (1)$$

where $\theta_k$ is the carrier phase angle and $\omega_c$ its angular frequency.

The channel model assumes a slow-fading frequency-selective discrete multipath profile [9, 2], where the equivalent baseband impulse response (as seen by the $k$-th user, for example) is given by

$$h_k(\tau) \;=\; \sum_{l=1}^{L} \beta_{kl} e^{j\theta_{kl}} \, \delta(\tau - \tau_{kl}), \quad (2)$$

For simplicity, the number $L$ of received resolvable paths is assumed the same for all users, and the path gain $\beta_{kl}$, phase $\theta_k$, and delay $\tau_{kl}$ are modeled as time-independent random processes over each bit interval (a valid assumption for slowly fading, non-timeselective channels). The gains assume a Rayleigh distribution with the pdf $f_{\beta_{kl}}(x) = \frac{x}{\rho_0} e^{-x^2/2\rho_0}$, and the phases are modeled by uniform RVs over $[0, 2\pi]$. The path delays $\tau_{kl}$ are also assumed i.i.d with a uniform distribution over the bit interval $[0, T]$.

At the front end of the receiver, the signal present $r(t)$ is the sum of delayed, faded replicas of the transmitted signals from all active users, corrupted by an additive Gaussian noise process $n(t)$ with two-sided power spectral density $N_0/2$

$$r(t) = n(t) + \sum_{k=1}^{K} s_k(t) * h_k(t) \quad (3)$$

Without loss of generality and because of the symmetry of the model, we focus on the 1st user Q-signal, and assume a bit $B_1^Q = 1$ was sent, We also let $\tau_1 = \phi_1 = 0$. For the conventional coherent single-user correlation receiver, the decision statistic at the output of the correlator is given by

$$Z_1^Q = \int_0^T r(\tau) a_1^Q(\tau) \sin(\omega_c \tau) d\tau. \quad (4)$$

## 3. RECEIVER STRUCTURES

We consider coherent detection where the receiver is able to coherently lock onto each of the resolvable multipath echoes, with perfect acquisition of the path delays and phases. The paths strengths, however, are unknown to the receiver. It is understood that such a coherent receiver is difficult to implement because of the fast fluctuations in path phases due to Rayleigh fading. In practice, the transmission of a strong pilot signal is typically used to aid in the coherent demodulation at the receiver, as is the case with the 2G&3G CDMA digital cellular standards.

### 3.1. Correlation Receiver

As a baseline, we consider the simplest, non-diversity receiver structure made of a single correlator matched to a given path among the arriving ones. Because all paths strengths in the model are identically distributed, the receiver could use any one of them for demodulation. For the correlator matched to $j$-th path of the Q-branch of User 1, the decision statistic is found as

$$\begin{aligned} Z_{1j} &= \beta_{1j} T\sqrt{P/2} + \eta + \sqrt{P/2} \sum_{l=1, l\neq j}^{L} \beta_{1l} W_{1l} \\ &+ \sqrt{P/2} \sum_{k=2}^{K} \sum_{l=1}^{L} \beta_{kl} W_{kl} \end{aligned} \quad (5)$$

where the first term is due to the desired signal weighted by the $j$-th path gain $\beta_{1j}$, the second term is due to thermal noise, and is zero-mean Gaussian with variance $\frac{1}{4} N_0 T$. The sum in the third term represents the self-interference induced by multipath, and the sum in the last term is the other users' interference due to multiple-access and multipath combined. It is also found [1, 8] that

$$W_{kl} = U_{kl} \cos(\Phi_{kl} - \Phi_{1j}) - V_{kl} \sin(\Phi_{kl} - \Phi_{1j}). \quad (6)$$

where the phase variables $\Phi_{kl}$ include the original carrier phases as well as phase shifts due to user asynchronism and multipath delays, and are modeled as uniform over $[0, 2\pi]$. The terms $U_{kl}$ and $V_{kl}$ are given by

$$U_{kl} = b_{k,-1}^Q R_{k1}^{QQ}(\tau_{kl} - \tau_{1j}) + b_{k,0}^Q \hat{R}_{k1}^{QQ}(\tau_{kl} - \tau_{1j}), \quad (7)$$

$$V_{kl} = b_{k,-1}^I R_{k1}^{QI}(\tau_{kl} + \frac{T_c}{2} - \tau_{1j}) + b_{k,0}^I \hat{R}_{k1}^{QI}(\tau_{kl} + \frac{T_c}{2} - \tau_{1j}) \quad (8)$$

where the correlation functions $R_{k1}$ and $\hat{R}_{k1}$ are obtained in [1] as $R(\tau) = C(\gamma - N)\hat{R}_{\psi_c}(S) + C(\gamma + 1 - N)R_{\psi_c}(S)$ and $\hat{R}(\tau) = C(\gamma)\hat{R}_{\psi_c}(S) + C(\gamma + 1)R_{\psi_c}(S)$. The chip partial correlation functions are also specified in [1] as $R_{\psi_c}(s) \triangleq \int_0^s \psi_c(t)\psi_c(t + T_c - s)dt$ and $\hat{R}_{\psi_c}(s) \triangleq \int_s^{T_c} \psi_c(t)\psi_c(t - s)dt$, and $C_{(a_n),(a_m)}(.)$ is the discrete cross-correlation function between the sequences $((a_n), (a_m))$. The variable $\gamma$ denotes the integer $\lfloor \tau/T_c \rfloor$, while $S$ represents the nearest-chip delay given by $\tau - \gamma T_c$. We present next other receiver structures that exploit the inherent diversity of the transmitted DS-CDMA signals in order to gain back some of the performance loss caused by fading.

### 3.2. Selection Diversity Receiver

Since the receiver is assumed to be able to lock onto any of the received multipath echoes, it can use each of these paths to compute decision statistics similar to (5). The receiver can then choose the largest one for decoding. This scheme is known as selection diversity (SD) reception [10], and has a better performance than the correlation receiver because the different received paths are assumed to be independent, and hence, will have a small probability of undergoing simultaneous fading.

For simplicity of notation, we assume that the first $M$ arriving paths are used (a diversity of order $M$). The expression of the decision statistic used by the selection diversity receiver is the same as (5) with $\beta_{1j}$ replaced by $\beta_{\text{SD}}$, where

$$\beta_{\text{SD}} = \text{Max}\{\beta_{11}, \beta_{12}, \ldots, \beta_{1M}\}. \quad (9)$$

For the subsequent error performance analysis, the statistics of $\beta_{\text{SD}}^2$ will be needed, and it is easy to verify that its probability density function is given by [10]

$$f_{\beta_{\text{SD}}^2}(x) = \frac{M}{2\rho_0} \left(1 - e^{-x/2\rho_0}\right)^{M-1} e^{-x/2\rho_0} \quad \text{for } x \geq 0 \quad (10)$$

### 3.3. Maximum Ratio Combining Receiver

In slow fading, an ideal receiver can further be assumed to be able to estimate the path strengths $\beta_{1j}$'s in addition to their phases and time delays. Based on this additional information, it is known [2] that optimum demodulation (in the absence of intersymbol interference) is achieved by coherently combining the decision statistics of each path, weighted by the corresponding path strength. This maximum ratio combining (MRC) receiver uses the decision statistic (for diversity order $M$)

$$Z_{\text{MRC}} = \sum_{j=1}^{M} \beta_{1j} Z_{1j} \quad (11)$$

In this case, the error performance will depend on the statistic $\beta_{\text{MRC}}^2 = \sum_{m=1}^{M} \beta_{1m}^2$, which has a chi-square distribution with $2M$ degrees of freedom [10]

$$f_{\text{MRC}}^2(x) = \frac{1}{(M-1)!(2\rho_o)^M} x^{M-1} e^{-x/2\rho_0} \quad \text{for } x \geq 0 \quad (12)$$

## 4. ERROR PERFORMANCE ANALYSIS

The problem of evaluating the average SNR of DS-CDMA signals with multiple-access and multipath interference has been studied

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previously for deterministic sequences with BPSK [3, 4]. The case of random signature sequences with BPSK was analyzed in [7]. In the following, we pursue the extension to generalized OQPSK for the different receiver structures discussed in the previous section. We only use the standard Gaussian approximation technique which was found in [8] to have excellent accuracy with OQPSK-type modulation formats.

### 4.1. Correlation Receiver Error Performance

We begin by finding the average SNR at the output of the receiver assuming that the total interference (multipath plus multiple-access) is Gaussian with a resultant variance that adds on to the thermal noise variance. By first conditioning on the path strength $\beta_{1j}^2$, the conditional SNR is obtained, and averaging over the Rayleigh p.d.f of $\beta_{1j}^2$ gives the final unconditional probability of error. From (5), it follows that

$$\mathrm{E}[Z_{\mathrm{CR}}|\beta_{1j}] \;=\; \beta_{1j}\, T\sqrt{P/2} \tag{13}$$

The total variance of the decision statistic ius also given by

$$\mathrm{Var}[Z_{\mathrm{CR}}] = \frac{1}{4}N_0T + \frac{1}{2}P\,\mathrm{E}\left(\sum_{l=1,l\neq j}^{L}\beta_{1l}W_{1l} + \sum_{k=2}^{K}\sum_{l=1}^{L}\beta_{kl}W_{kl}\right)^2 \tag{14}$$

In the above equation, we note that the multipath terms in the first sum (due to User 1) are uncorrelated with the terms in the second sum (due to multiple-access and multipath from the remaining users). This can be seen by conditioning on the signature sequence $\underline{a}_1^Q = (a_{1,0}^Q, a_{1,1}^Q, \ldots, a_{1,N-1}^Q)$ and using the fact that, for $k \geq 2$, $\mathrm{E}[W_{1l}W_{kl}|\underline{a}_1^Q] = \mathrm{E}[W_{1l}|\underline{a}_1^Q]\mathrm{E}[W_{kl}|\underline{a}_1^Q] = 0$. Then, averaging over $\underline{a}_1^Q$ proves the result. By a similar argument, it is also clear that the $W_{1l}$'s are uncorrelated, and so are the $W_{kl}$'s for $k \neq l$. Therefore, we can add the second moments of the random variables in the last expectation of (14) to obtain the multipath and multiple-access interference variances

$$\begin{aligned}\sigma_{\mathrm{MPI}}^2 &= \sum_{l=1,l\neq j}^{L}\mathrm{E}[\beta_{1l}^2]\mathrm{E}[W_{1l}^2] \\ &= \rho_0 P\sum_{l=1,l\neq j}^{L}\mathrm{E}[W_{1l}^2],\end{aligned} \tag{15}$$

and

$$\begin{aligned}\sigma_{\mathrm{MAI}}^2 &= \sum_{k=2}^{K}\sum_{l=1}^{L}\mathrm{E}[\beta_{kl}^2]\mathrm{E}[W_{kl}^2] \\ &= \rho_0 P\sum_{k=2}^{K}\sum_{l=1}^{L}\mathrm{E}[W_{kl}^2].\end{aligned} \tag{16}$$

The expression of the multiple-access variance $\sigma_{\mathrm{MAI}}^2$ can be readily used from the results of [8], and is given by

$$\mathrm{E}[W_{kl}^2] \;=\; 2NT_c^2\,M_c,\quad k = 2,\ldots,K,\;\; l = 1,\ldots,L \tag{17}$$

where the the parameter $M_c \triangleq \frac{1}{T_c^3}\int_0^{T_c} R_{\psi_c}^2(s)ds$ represents a chip mean-squared correlation parameter that depends on the actual shape of the chip waveform $\psi_c(t)$. It then follows that

$$\sigma_{\mathrm{MAI}}^2 \;=\; (K-1)L\,E_b'\,T_c\,M_c, \tag{18}$$

where $E_b' = \mathrm{E}[\beta_{kl}^2]PT = 2\rho_0 E_b$ is the average faded energy per bit. On the other hand, the variance $\sigma_{\mathrm{MPI}}^2$ of the self-multipath term is derived differently because it involves the autocorrelation function $R_{11}^{QQ}(\tau_{1l})$, $l \neq j$. The details of the derivation of $\mathrm{E}[W_{1l}^2]$ are omitted here for space reasons. We finally obtain

$$\mathrm{E}[W_{1l}^2] \;=\; (3N-1)\,T_c^2\,M_c, \tag{19}$$

and therefore,

$$\begin{aligned}\sigma_{\mathrm{MPI}}^2 &= \frac{3N-1}{2N}(L-1)\,E_b'\,T_c\,M_c \qquad (20)\\ &\approx \frac{3}{2}(L-1)E_b'\,T_c\,M_c, \qquad (21)\end{aligned}$$

where the last approximation is obtained for large $N$. Considering now the average SNR for a fixed value of $\beta_{1j}^2$, this is given by

$$\begin{aligned}\mathrm{SNR} &= \frac{(\mathrm{E}[Z_{\mathrm{CR}}])^2}{\mathrm{Var}[Z_{\mathrm{CR}}]} \\ &= \frac{\frac{1}{2}\beta_{1j}^2\,T^2\,P}{\frac{1}{4}N_0T + \sigma_{\mathrm{MPI}}^2 + \sigma_{\mathrm{MAI}}^2}.\end{aligned} \tag{22}$$

Substituting (18) and (20) into (22) and regrouping terms, the expression of the SNR conditioned on $\beta_{1j}$ becomes

$$\mathrm{SNR} \;=\; \frac{(\beta_{1j}^2/\rho_0)\,E_b'}{N_0 + K_{eff}\,E_b'\,\frac{M_c}{N}} \;=\; \frac{\beta_{1j}^2}{2\rho_0}\,\mathrm{SNR}_0 \tag{23}$$

where $K_{eff} = 4KL + 2L - 6$ denotes the effective number of multiuser and multipath components, and $\mathrm{SNR}_0$ denotes the average SNR given by

$$\mathrm{SNR}_0 = \frac{2E_b'}{N_0 + K_{eff}\,E_b'\,\frac{M_c}{N}}. \tag{24}$$

If the thermal noise is negligible ($N_0 = 0$), the average SNR becomes independent of the faded bit energy and simplifies to

$$\mathrm{SNR}_0 \;=\; \frac{2N}{K_{eff}M_c} \tag{25}$$

From the expressions of $\mathrm{SNR}_0$ in (24) and (25), we point out the importance of the ratio $M_c/N$ (or equivalently, the normalized bandwidth-mean-squared-correlation product $BM_c$) as a single parameter for comparing the error performance of different chip waveforms. This will be further discussed next.

Finally, the bit error probability is approximated by integrating (23) over the p.d.f of $\beta_{1j}^2$ (which is exponential, with parameter $2\rho_0$). The evaluation of the integral is a classical result [9], giving

$$\begin{aligned}P_b &\approx \int_0^\infty Q\left(\sqrt{\mathrm{SNR}\,|\beta_{1j}^2}\right) f_{\beta_{1j}^2}(x)dx \qquad (26)\\ &= \frac{1}{2}\left[1 - \sqrt{\frac{\mathrm{SNR}_0}{\mathrm{SNR}_0 + 2}}\right] \qquad (27)\end{aligned}$$

### 4.2. Selection Diversity Error Performance

For the case of a selection diversity receiver, it was suggested in [3] that the bit error analysis is similar to the correlation receiver case, except that averaging of (23) should take place over the distribution of $\beta_{SD}^2$ as given in (10) instead of that of $\beta_{1j}^2$. However, we point out here that this is not exactly accurate because the path strengths of the remaining $M-1$ paths that are not selected are no longer independent and Rayleigh-distributed since it is know that they are smaller than the given $\beta_{SD}^2$. Therefore, in computing the self-multipath interference variance $\sigma_{sim}^2$, it is not accurate to replace the second moments $E[\beta_{1j}^2]$ by $2\rho_0$ (these second moments should be smaller). The correct approach is to characterize the joint statistics of $\beta_{SD}^2$ and the remaining $M-1$ paths, and use it to obtain a new expression for $\sigma_{sim}^2$ and SNR$_0$. But this joint characterization proved to be difficult. On the other hand, it is believed that the a posteriori independence and Rayleigh distribution assumptions for the non-selected paths after largest path selection will lead to an upper bound on the bit error probability that should be fairly tight, and this is primarily for two reasons. First, the contribution of the self-multipath from the intended user to the total interference variance (denominator of the SNR in (22)) is comparatively small, especially with a large number of other independent interferers, each with a multiple number of paths. And secondly, when the diversity order $M$ becomes large, the aforementioned assumptions will be more and more applicable since in this case, a large value for $\beta_{SD}^2$ will tend to restore the independence and Rayleigh statistics of the non-selected path strengths.

Therefore, in the following, we take a similar approach as in [3] and make use of the previous derivations for the error performance with correlation reception. To this effect, we note that the expansion of (10) gives $f_{\beta_{SD}^2}(x)$ as a weighted sum of exponential pdf's

$$f_{\beta_{SD}^2}(x) = M\sum_{m=0}^{M}\binom{M-1}{m}\frac{(-1)^m}{(m+1)/2\rho_{0m}}e^{-x/2\rho_{0m}} \quad (28)$$

where $\rho_{0m} \triangleq \rho_0/(m+1)$. Therefore, as shown in [6], the bit error probability is a sum of terms of the form (26) resulting in

$$\begin{aligned} P_b &\approx \int_0^\infty Q\left(\sqrt{\mathrm{SNR}|\beta_{SD}^2}\right) f_{\beta_{SD}^2}(x)dx \\ &\approx M\sum_{m=0}^{M-1}\binom{M-1}{m}\frac{(-1)^m}{2(m+1)}\cdot \\ &\quad \left[1-\sqrt{\frac{\mathrm{SNR}_0}{\mathrm{SNR}_0+2(m+1)}}\right] \end{aligned} \quad (29)$$

### 4.3. Maximum Ratio Combining Error Performance

The performance of this receiver is determined by considering the decision statistic in (11) conditioned on the $\{\beta_{1j}\}_{j=1}^M$. We have

$$\begin{aligned} \mathrm{E}[Z_{MRC}|\{\beta_{1j}\}] &= \sum_{j=1}^{M}\beta_{1j}\,\mathrm{E}[Z_{1j}|\beta_{1j}] \\ &= T\sqrt{P/2}\sum_{j=1}^{M}\beta_{1m}^2 \end{aligned} \quad (30)$$

and

$$\begin{aligned} \mathrm{Var}[Z_{MRC}|\{\beta_{1j}\}] &= \sum_{j=1}^{M}\beta_{1j}^2\,\mathrm{Var}[Z_{1j}|\beta_{1j}] \\ &= \frac{1}{4}\left(N_0T+K_{eff}E_b'\frac{M_c}{N}\right)\sum_{j=1}^{M}\beta_{1j}^2 \end{aligned} \quad (31)$$

Taking the ratio of the squared expectation to the variance, the conditional SNR is found to be the sum of the individual conditional SNR's per path

$$\begin{aligned} \mathrm{SNR} &= \frac{(1/\rho_0)E_b'}{N_0+K_{eff}E_b'\frac{M_c}{N}}\sum_{j=1}^{M}\beta_{1j}^2 \\ &= \frac{1}{\rho_0}\,\mathrm{SNR}_0\,\beta_{MRC}^2 \end{aligned} \quad (32)$$

Finally, averaging over the pdf of $\beta_{MRC}$ in (12) gives a closed form expression for the bit error probability [9]

$$\begin{aligned} P_b &\approx \int_0^\infty Q\left(\sqrt{\mathrm{SNR}|\beta_{MRC}^2}\right) f_{\beta_{MRC}^2}(x)dx \\ &= \frac{1}{2^M}\left[1-\sqrt{\frac{\mathrm{SNR}_0}{\mathrm{SNR}_0+2}}\right]^M\sum_{m=0}^{M-1}\frac{1}{2^m}\binom{M-1+m}{m}\cdot \\ &\quad \left[1-\sqrt{\frac{\mathrm{SNR}_0}{\mathrm{SNR}_0+2}}\right]^m \end{aligned} \quad (33)$$

### 4.4. Comparing Different Modulations

In order to have a fair comparison among different modulations on the basis of the average SNR, we also have to impose equal bit rate and equal bandwidth constraints. Considering two modulation schemes 1 and 2 with bit durations $T^{(1)}, T^{(2)}$, chip durations $T_c^{(1)}, T_c^{(2)}$, and having normalized[1] power bandwidth occupancies $B^{(1)}$ and $B^{(2)}$, we need

$$\frac{1}{T^{(1)}} = \frac{1}{T^{(2)}} \text{ and } \frac{B^{(1)}}{T_c^{(1)}} = \frac{B^{(2)}}{T_c^{(2)}} \quad (34)$$

which implies that

$$N^{(1)}B^{(1)} = N^{(2)}B^{(2)}. \quad (35)$$

To determine which system performs better, we compare the ratios $M_c^{(1)}/N^{(1)}$ and $M_c^{(2)}/N^{(2)}$. Recall that the normalized interference factor $M_c$ is only a function of the chip waveform shape, and is independent of its duration $T_c$. The number $N$ however would have to be changed with chip duration $T_c$ in order to satisfy (35). Considering for simplicity the asymptotic SNR ($N_0 = 0$), it then follows that the dB gain (or loss) of System 2 over System 1 is

$$G_{21} = 10\log_{10}\left[\frac{B^{(1)}M_c^{(1)}}{B^{(2)}M_c^{(2)}}\right] \text{ dB}. \quad (36)$$

[1] based on the in-band power bandwidth occupancies $W^{(1)}$ and $W^{(2)}$, normalized by the respective chip rates $1/T_c^{(1)}$ and $1/T_c^{(2)}$.

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The above equation demonstrates the importance of the normalized $BM_c$ parameter as a single figure for comparing the performance of different chip pulses in DS-CDMA. The larger this product the higher the SNR gain and the better the error performance, or equivalently, the higher the system capacity (as measured by the number $K$ of total active users) for a given error performance level.

## 5. NUMERICAL RESULTS

In this section, we present numerical results to illustrate the performance of the receiver structures discussed above, and we also give further comparisons among different modulations which include, in addition to standard OQPSK with a rectangular chip pulse, Minimum Shift Keying (MSK) with a half-sine chip pulse $\psi_c(t) = \sqrt{2}\sin(\frac{\pi t}{T_c})$, Sinusoidal Frequency Shift Keying (SFSK) with a $\psi_c(t) = \sqrt{2}\sin(\frac{\pi t}{T_c} - \frac{1}{4}\sin(\frac{4\pi t}{T_c}))$, and Time-Domain Raised-Cosine shaped OQPSK with $\psi_c(t) = \sqrt{2/3}(1 - \cos(\frac{2\pi t}{T_c}))$.

Fig.(1) shows the example of MSK for a channel with $L = 7$ paths. $N = 127$ chips/bit. The bit error probability is shown as a function of the number of users for all three receiver types with increasing diversity orders (a spreading factor $N = 127$ is used). Also shown on the same plot is the performance of the correlation receiver in the absence of fading. As expected, severe degradation is caused by multipath fading and the performance of the correlation receiver (with no diversity) is quite poor. With selection diversity, there is a modest improvement in performance as the diversity order $M$ increases, and the bit error level is only tolerable when the number of users is small. Notice also that the incremental improvement in performance tends to saturate with increasing $M$. On the other hand, maximum ratio combining performs substantially better and steadily improves with increasing $M$ (and would asymptotically approach the unfaded limiting case [10]).

Considering the relative error performance of OQPSK, MSK, SFSK and TDRC, the diversity order is fixed at $M = 4$ for a channel with $L = 7$ equal-strength paths. An equal bit rate and equal 99.9% bandwidth occupancy constraints are also maintained imposed. The number $N$ of chips/bit is set to 63 for OQPSK as a reference and adjusted according to (35) for the other modulations, thereby resulting in $N^{MSK} = 662$, $N^{SFSK} = 645$ and $N^{TDRC} = 1074$. As an example with MRC, Figure (2) shows the bit error probability versus the number of users $K$, and it is clearly seen that OQPSK is largely outperformed by the other modulations, mainly because of its large $BM_c$ product.

Finally, we give additional comparisons with increasing diversity orders in Figures (3) and (4). It is seen that, in the case of selection diversity, increasing the order $M$ quickly reaches a point of diminishing returns and the error probability starts saturating at a given floor. For maximum ratio combining however, this is not the case and the error probability continues decreasing with higher diversity. It is also noticeable that the relative gaps in performance between the different modulations get increasingly larger as the diversity order $M$ increases.

## 6. CONCLUSIONS

The paper addressed the error performance analysis of selection diversity and maximum ratio combining over Rayleigh fading channels when the multiple access signalling schemes use OQPSK-modulated CDMA with random signature sequences and arbitrarily shaped chip waveform pulses. Closed-form bit error probability expressions were derived based on the use of the Standard Gaussian Approximation method, which is found to have excellent accuracy for OQPSK-type spreading.

Selection diversity was found to yield modest improvements in error performance even with a high diversity order. Another disadvantage of selection diversity is that the bit error probability improved at a diminishing rate for increasing diversity orders. Maximum ratio combining, on the other hand, offered much better performance, without suffering from the diminishing returns problem of selection diversity.

Various examples that illustrated relative performance comparisons among different modulation types were also given, and it was the best error performance is achieved by chip waveforms that have a smallest normalized bandwidth-mean-squared-correlation product.

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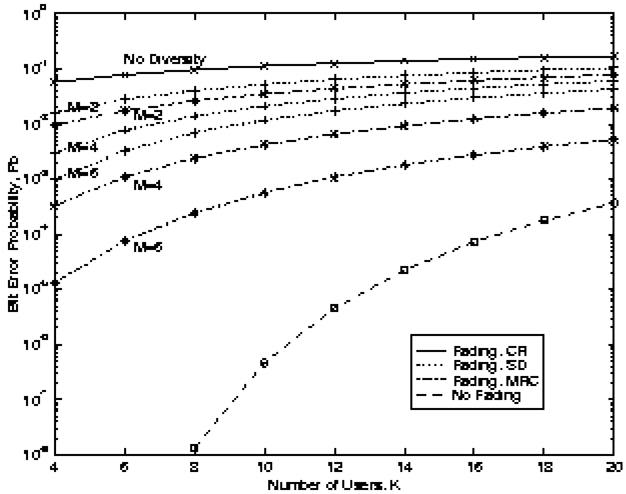


**Fig. 1.** Comparison of receiver structures: Correlation Receiver with no diversity (CR), Selection Diversity (SD), and Maximum Ratio Combining (MRC). Results for MSK with $N = 127$. No thermal noise ($N_0 = 0$). Number of paths $L = 7$.

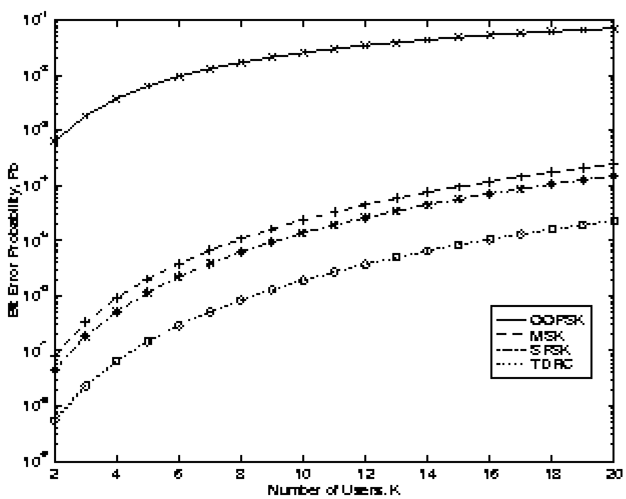


**Fig. 2.** Performance with maximum ratio combining. Bit error probability vs. number of users for different modulation schemes. Parameters: ($N_0 = 0$), number of paths $L = 7$, diversity order $M = 4$.

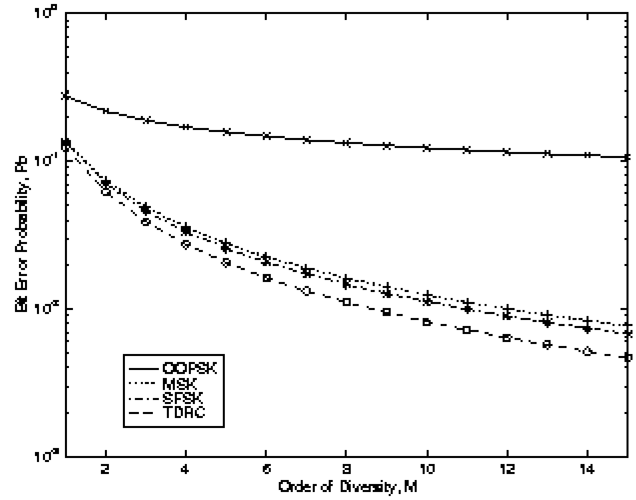


**Fig. 3.** Selection diversity $P_b$ vs. diversity order $M$. Parameters: $E_b/N_0 = 30$ dB, number of users $K = 10$, number of paths $L = 15$, mean path strength $2\rho_0 = 1/15$.

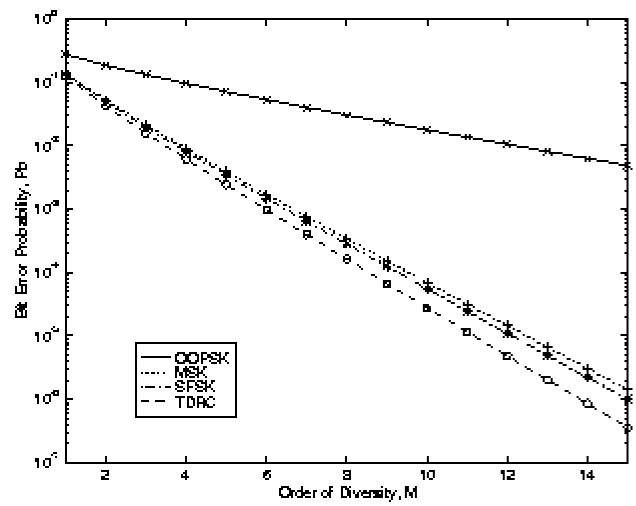


**Fig. 4.** Maximum ratio combining $P_b$ vs. diversity order $M$. Parameters: $E_b/N_0 = 30$ dB, number of users $K = 10$, number of paths $L = 15$, mean path strength $2\rho_0 = 1/15$.

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