Adaptive Uniform Fractional Guard Channel Algorithm: The Steady State Analysis

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Abstract - Recently, a subclass of fractional guard channel policy, which is called uniform fractional guard channel (UFG) policy, is introduced and shown that it performs better than guard channel (GC) policy under the low handoff/new traffic ratio. In order to find the optimal value for the UFG policy, a search algorithm is given, which assumes that input traffic is a stationary process with known parameters. Since the input traffic is not a stationary process and its parameters are unknown a priori, the adaptive version of UFG (AUFG) is given which uses a learning automaton. In this paper, we study the steady state behavior of the AUFG algorithm. It is shown that the AUFG algorithm converges to an equilibrium point, which is also optimal for UFG policy.

I. INTRODUCTION

Introduction of micro cellular networks leads to efficient use of channels but increases expected rate of handovers per call. As a consequence, some network performance parameters such as *blocking probability of new calls* (B_n) and *dropping probability of handoff calls* (B_h) are affected. In order to have these performance parameters at reasonable level, *call admission policies* are used. The call admission policy plays a very important role in the cellular networks because it directly controls B_n and B_h. Since the dropping probability of handoff calls is more important than the blocking probability of new calls, call admission policies usually give the higher priority to handoff calls. This priority is implemented through allocation of more resources (channels) to handoff calls. *Fractional guard channel policy* (FG), which is a general call admission policy, accepts new calls with a certain probability that depends on the current channel occupancy and accepts handoff calls as long as channels are available [1]. Suppose that the given cell has C full duplex channels. The FG policy uses a vector $\Pi = \{\pi_0, \ldots, \pi_{C-1}\}$ to accept the new calls, where $0 \leq \pi_i \leq 1$, $0 \leq i < C$. This policy accepts new calls with probability π_k when k ($0 \leq k < C$) channels are busy. Depending on the vector Π, we may have different call admission policies and some of which are reviewed below.

Guard channel policy (GC), which is a restricted version of FG, reserves a subset of channels allocated to a cell, called *guard channels*, for handoff calls (say $C - T$ channels) [2]. Whenever the channel occupancy exceeds a certain threshold T, the guard channel policy rejects new calls until the channel occupancy goes below the threshold. The guard channel policy accepts handoff calls as long as channels are available. Note that the GC policy can be obtained from FG policy by setting $\pi_k = 1$, $0 \leq k < T$, and $\pi_k = 0$, $T \leq k < C$. It has been shown that there is an optimal threshold T^* at which the blocking probability of new calls is minimized subject to the hard constraint on the dropping probability of handoff calls and an algorithm for finding such optimal threshold is given in [3]. The GC policy reserves an integral number of guard channels for handoff calls. If performance parameter B_h is considered, the guard channel policy gives very good performance, but performance parameter B_n is degraded to great extent. In order to have more control on blocking probability of new calls and dropping probability of handoff calls, *limited fractional guard channel policy* (LFG) is introduced [1]. The LFG can be obtained from FG policy by setting $\pi_k = 1$, $0 \leq k < T$, $\pi_T = \pi$ and $\pi_k = 0$, $T < k < C$. It has been shown that there is an optimal threshold T^* and an optimal value of π^* for which blocking probability of new calls is minimized subject to the hard constraint on dropping probability of handoff calls and an algorithm for finding these optimal parameters is given in [1]. In [4], a new version of FG policy called *uniform fractional guard channel policy* (UFG) is introduced. The UFG policy accepts new calls with probability of π independent of channel occupancy. The UFG can be obtained from FG by setting $\pi_k = \pi$, $0 \leq k < C$. It is shown that there is an optimal value for the parameter of UFG which minimizes blocking probability of new calls with the constraint on the upper bound on dropping probability of handoff calls and an algorithm for finding such optimal parameter is also given. Then conditions under which the UFG performs better than the GC is derived. It is concluded that, the UFG policy performs better than GC policy under the low handoff traffic conditions.

UFG and other call admission policies such as reported in [1, 2] are static and assume that all parameters of traffic are known in advance. These policies are useful when input traffic is a stationary process with known parameters. Since the parameters of input traffic are unknown and possibly time varying, adaptive version of these policies must be used. In [5], an adaptive algorithm is introduced, which uses a learning automata and accepts new calls as long as the dropping probability of handoff calls is below of a pre-specified threshold. The simulation results show that this algorithm cannot maintain the upper bound on the dropping probability of handoff calls. In order to maintain the upper bound on the dropping probability of new calls, in [6], *adaptive uniform fractional guard channel* (AUFG) algorithm is introduced. This algorithm uses a learning automaton to accept/reject new calls and a pre-specified level of dropping probability of handoff calls is used to penalize/reward the action selected by the learning automaton. This adaptive algorithm accepts new calls as long as the dropping probability of handoff calls is below the pre-specified threshold. The simulation results show that, the performance of the proposed algorithm is very close to the performance of the UFG policy, which needs to know all traffic parameters in high handoff traffic conditions and maintains the level of QoS in the system.

In this paper, we study the convergence of the AUFG algorithm in the steady state. It is shown that the AUFG algorithm converges to an equilibrium point, which is also optimal for the UFG policy.

The rest of this paper is organized as follows: The learning automata are given in section II. Section III reviews the UFG policy. The adaptive algorithm for finding the optimal value of parameter π is given in section IV and section V studies the behavior of the proposed algorithm. Section VI concludes the paper.

II. LEARNING AUTOMATA

The automata approach to learning involves determination of an optimal action from a set of allowable actions. An automaton can be regarded as an abstract object that has finite number of actions. It selects an action from its finite set of actions and applies to a random environment. The random environment evaluates the applied action and gives a grade to the selected action of automaton. The response from the environment (i.e. grade of action) is used by automaton to select its next action. By continuing this process, the automaton learns to select the action with the best grade. The learning algorithm used by automaton to determine the selection of next action from the response of environment. An automaton acting in an unknown random environment and improves its performance in some specified manner, is referred to as *learning automaton* (LA). Learning automata can be classified into two main families: *fixed structure learning automata* and *variable structure learning automata* [7]. Variable structure learning automata are represented by triple $<\beta, \alpha, T>$, where β is a set of inputs, α is a set of actions, and T is learning algorithm. The learning algorithm is a recurrence relation and is used to modify action probabilities (p) of the automaton. It is evident that the crucial factor affecting the performance of the variable structure learning automaton, is learning algorithm for updating the action probabilities. Various learning algorithms have been reported in the literature. Let α_i be the action chosen at time k as a sample realization from probability distribution $p(k)$. In what follows, two learning algorithms for updating the action probability vector are given. In linear reward-ϵpenalty algorithm ($L_{R-\epsilon P}$) scheme the recurrence equation for updating p is defined as

$$p_j(k+1) = \begin{cases} p_j(k) + a \times [1 - p_j(k)] & \text{if} \quad i = j \\ p_j(k) - a \times p_j(k) & \text{if} \quad i \neq j \end{cases} \tag{1}$$

when $\beta(k) = 0$ and

$$p_j(k+1) = \begin{cases} p_j(k) \times (1 - b) & \text{if} \quad i = j \\ \frac{b}{r-1} + p_j(k)(1-b) & \text{if} \quad i \neq j \end{cases} \tag{2}$$

when $\beta(k) = 1$. The parameters $0 < b \ll a < 1$ represent *step lengths* and r is the number of actions for learning automaton. The a and b determine the amount of increase and decreases of the action probabilities, respectively. If the a equals to b the recurrence equations (1) and (2) is called *linear reward penalty*(L_{R-P}) algorithm.

Learning automaton have been used successfully in many applications such as telephone and data network routing [8], solving NP-Complete problems [9], capacity assignment [10] and neural network engineering [11, 12, 13] to mention a few.

III. UNIFORM FRACTIONAL GUARD CHANNEL ALGORITHM

In this section, we review the UFG policy. We assume that the given cell has a limited number of full duplex channels, C, in its channel pool. We define the state of a particular cell at time t to be the number of busy channels in that cell and is represented by $c(t)$. The UFG policy uses admission probability π, which is independent of channel occupancy, to accept new calls and accepts handoff calls as long as channels are available. This policy can be obtained from FG policy by setting $\pi_k = \pi$ for $k = 0, 1, \ldots, C-1$. UFG policy reserves non-integral

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number of guard channels for handoff calls by rejecting new calls with some probability. The description of UFG policy is given algorithmically in figure 1

if (HANDOFF CALL) **then**
    if $c(t) < C$) **then**
        accept call
    else
        reject call
    end if
end if
if (NEW CALL) **then**
    if ($c(t) < C$ **and** rand (0,1) $< \pi$) **then**
        accept call
    else
        reject call
    end if
end if

Fig. 1. Unform fractional guard channel policy

In what follows, we study the blocking performance of the UFG policy. The blocking performance of the UFG policy is computed based on the following assumptions.

1. The arrival processes of new and handoff calls are poisson with rates λ_n and λ_h, respectively. Let $\lambda = \lambda_n + \lambda_h$ and $\alpha = \lambda_h/\lambda$.
2. The channel holding time for both types of calls are exponentially distributed with mean μ^{-1}. Let $\rho = \lambda/\mu$.
3. The time interval between two calls from a mobile host is much greater than the mean call holding time.
4. Only mobile to fixed calls are considered.
5. The network is homogenous.

The above first three assumptions have been found to be reasonable as long as the number of mobile hosts in a cell is much greater than the number of channels allocated to that cell. The fourth assumption makes our analysis easier and the fifth one lets us to examine the performance of a single network cell in isolation. $\{c(t)|t \geq 0\}$ is a continuous-time Markov chain (birth-death process) with states $0, 1, \ldots, C$. The state transition diagram of a cell with C full duplex channels and UFG call admission policy is shown in figure 2.

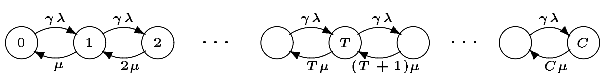

Fig. 2. Markov chain model of cell

At state $0 \leq n < C$, new calls are accepted with probability $0 \leq \pi \leq 1$ and handoff calls are accepted with probability 1. Both types of calls are blocked in state C. Thus, the state dependent arrival rate in the birth-death process is equal to $[a + (1-a)\pi]\lambda$. Because of the structure of the Markov chain, we can easily write down the steady-state balance equations. By solving the steady state balance equations, we can find the dropping probability of handoff calls, $B_h(C, \pi)$, by following expression.

$$B_h(C, \pi) = \frac{(\rho\gamma)^C}{C!} P_0. \tag{3}$$

Similarly, the blocking probability of new calls, $B_n(C, \pi)$ is given by the following expression.

$$B_n(C, \pi) = 1 - \pi \left[1 - B_h(C, \pi)\right]. \tag{4}$$

IV. ADAPTIVE UNIFORM FRACTIONAL GUARD CHANNEL ALGORITHM

In this section, we introduce a new adaptive version of UFG policy (figure 3). This algorithm is used to determine the admission probability, π, when the parameters a and ρ (or equivalently λ_h, λ_n and μ) are unknown or probably time varying. The proposed algorithm adjusts parameter π as network operates. This algorithm can be described as follows: The proposed algorithm uses one reward-penalty type learning automaton with two actions in each cell. The action set of this automaton corresponds to {ACCEPT,REJECT}. The automaton associated to each cell determines the probability of acceptance of new calls (π). Since initially the values of a and ρ are unknown, the probability of selecting these actions are set to 0.5. When a handoff call arrives, it is accepted as long as there is a free channel. If there is no free channel, the handoff call is dropped. When a new call arrives to a particular cell, the learning automaton associated to that cell chooses one of its actions. Let π be the probability of selecting the action ACCEPT. Thus, the learning automaton accepts new calls with probability π as long as there is a free channel and rejects new calls with probability $1 - \pi$. If action ACCEPT is selected by automaton and the cell has at least one free channel, the incoming call is accepted and the selected action is rewarded. If there is no free channel to be allocated to the arrived new call, the call is blocked and action ACCEPT is penalized. When the automaton selects action REJECT, the adaptive UFG computes an estimation of the dropping probability of handoff calls ($\hat{B}_h$) and uses it to decide whether or not to accept new calls. If the current estimate of dropping probability of handoff calls is less than the given threshold p_h and there is a free channel, then the new call is accepted and the action REJECT is penalized; otherwise, the new call is rejected and the action REJECT is rewarded.

```
if (NEW CALL) then
    if (LA.action () = ACCEPT) then
        if (c(t) < C )then
            accept call
            reward action ACCEPT
        else
            reject call
            penalize action ACCEPT
        end if
    else //LA selects action REJECT
        reject call
        if (B̂_h < p_h ) then
            penalize action REJECT
        else
            reward action REJECT
        end if
end if
```

Fig. 3. Adaptive uniform Fractional guard channel algorithm

V. STEADY STATE BEHAVIOR OF AUFG

In this section, we study the convergence of the adaptive UFG algorithm. We show that, when the adaptive UFG algorithm uses the L_{R-P} reinforcement scheme, a unique value for π is found by the learning automaton, which is also optimal for the UFG algorithm. In order to study the behavior of the adaptive UFG algorithm, we first model environment for the learning automaton in .

Lemma 1. *Let $p = (p_1, p_2)$ be the action probability vector of learning automata and $p_1 = \pi$ be the probability of accepting new calls. Then, the steady state behavior of the adaptive UFG algorithm can be shown by a triple $< \underline{\alpha}, \underline{\beta}, \underline{C} >$, where $\underline{\alpha} = \{\text{ACCEPT}, \text{REJECT}\}$ shows the set of actions of automaton, $\underline{\beta} = \{0, 1\}$ represents the set of inputs for automaton and $\underline{C(p)} = \{c_1(p), c_2(p)\}$ is the set of penalty probabilities, where $c_1(p)$ and $c_2(p)$ are given by the following expressions.*

$$c_1(p) = \frac{(\rho\gamma)^C}{C!} P_0$$
$$c_2(p) = \frac{1}{\sqrt{2\pi}\sigma_b} \int_{-\infty}^{p_h} e^{-\frac{1}{2}\left(\frac{x-\mu_b}{\sigma_b}\right)^2} dx \tag{5}$$

where μ_b and σ_b^2 are mean and variance of $\hat{B}_h$, respectively.

Proof. The proof of this lemma is given in [6].

The following lemma is concerned with the properties of the environment.

Lemma 2. *The environment corresponding to the adaptive UFG algorithm has the following characteristics when $\rho < C$. Let to write p for $p(n)$ and $c_i(p)$ for $c_i(n)$.*

1. *$c_i(p)$ (for $i = 1, 2$) is continuous function in p.*
2. *$c_i(p)$ (for $i = 1, 2$) are continuously differentiable in all their arguments.*
3. *$c_i(p)$ and $\frac{\partial c_i(p)}{\partial p_i}$ (for $i = 1, 2$) are Lipschitz function of all their arguments.*
4. *The derivative of $c_i(p)$ (for $i = 1, 2$) have the following features.*

$$\frac{\partial c_i(p)}{\partial p_i} > 0, \tag{6}$$
$$\frac{\partial c_1(p)}{\partial p_2} \ll \frac{\partial c_2(p)}{\partial p_2}, \tag{7}$$
$$\frac{\partial c_2(p)}{\partial p_1} \ll \frac{\partial c_1(p)}{\partial p_1}. \tag{8}$$

Proof. The proof of this lemma is given in [6].

The process $\{p(n)\}_{n \geq 0}$ defined by the adaptive UFG algorithm is a homogenous Markov process. The following theorem is concerned with its convergence behavior.

Theorem 1. *The Markov process $\{p(n)\}_{n \geq 0}$ is ergodic and converges in distribution as $n \to \infty$ to a unique stationary probability $\bar{p}$ independent of the initial distribution of $\bar{p}$.*

Proof. The proof of this theorem is given in [14].

In what follows, the steady state behavior of the adaptive UFG algorithm is analyzed. Define the average penalty rate of action α_i as $f_i(p(n)) = c_i(p(n)) p_i(n)$, $p^* = (p_1^*, p_2^*)$ and $p_1^* + p_2^* = 1$. In the following lemma, it is shown that there is a unique p^* for which the average penalty rates for both actions become equal.

Lemma 3. *For the adaptive UFG algorithm, there exists a unique p^* such that*

$$\begin{aligned} f(p^*) &= f_2(p^*) - f_1(p^*), \\ &= 0. \end{aligned} \tag{9}$$

Proof. The proof of this lemma is given in [6].

Since $\{p(n)\}_{n \geq 0}$ is ergodic and converges in distribution to a unique stationary probability $\bar{p}$, thus in steady state, we obtain $\text{E}[\Delta\bar{p}_i] = 0$ or $\text{E}[w(\bar{p})] = 0$. The zero of $\text{E}[w(\bar{p})]$ is p^* and, in general, $\text{E}[w(\bar{p})] = 0$ need not yield p^*. However, if the learning parameter a is chosen to be sufficiently small, then the difference between these two values may be made small, as indicated by the following theorem.

Theorem 2. *Let $p(0)$ be the initial action probability vector of the adaptive UFG algorithm with stationary measure $\bar{p}$, $z_i(n) = \frac{p_i(n) - p_i^*}{\sqrt{a}}$ and $z(n) = z_1(n)$, then $z_i(n)$ converges to a normal distribution with zero mean and known variance as $a \to 0$ and $na \to \infty$.*

Proof. The proof of this theorem is given in [6].

Theorem 3. *The equilibrium probability of learning automaton in the adaptive UFG algorithm, $p^* = (\pi^*, 1-\pi^*)$, minimizes the blocking probability of new calls subject to the hard constraint on the dropping probability of handoff calls ($B_h(C,\pi) \leq p_h$).*

Proof. In the equilibrium state, the average penalty rates for both actions are equal or $f_1(p^*) = f_2(p^*)$, which results $c_1\pi^* = c_2(1-\pi^*)$. Thus we have

$$\pi^* = \frac{\delta}{\delta + P_C}. \tag{10}$$

where $\delta = \text{Prob}\left[\hat{B}_h < p_h\right]$. Thus average number of blocked new calls, $\bar{N}_n$, is equal to

$$\begin{aligned}\bar{N}_n &= \lambda_n \left[1 - \pi^*(1-P_C)\right], \\ &= \lambda_n (1+\delta)\frac{P_C}{P_C + \delta}. \end{aligned} \tag{11}$$

Computing derivative of $\bar{N}_n$ with respect to δ results

$$\begin{aligned}\frac{\partial \bar{N}_n}{\partial \delta} &= -\lambda_n \frac{P_C(1-P_C)}{(P_C+\delta)^2}, \\ &< 0. \end{aligned} \tag{12}$$

Thus $\bar{N}_n$ is a strictly decreasing function of δ. Since the adaptive UFG algorithm gives the higher priority to the handoff calls, it attempts to minimize the dropping probability of handoff calls. Using this fact and equation (12), it is evident that $\bar{N}_n$ is minimized which results in minimization of the blocking probability of new calls and hence the theorem.

For simulation results, the reader may refer to [6].

VI. CONCLUSIONS

In this paper, we studied the steady state behavior of the adaptive uniform fractional guard channel algorithm. It is shown that the adaptive uniform fractional guard channel algorithm converges to an equilibrium point, which is also optimal for uniform fractional guard channel policy.

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