Home

Complexity Constrained Multi-Stream Detection Algorithms for High Data Rate MIMO-OFDM Wireless Systems

Anass Benjebbour and Susumu Yoshida

Graduate School of Informatics, Kyoto University
Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan
e-mail: {anass,yoshida}@hanase.kuee.kyoto-u.ac.jp

Abstract— **Multiple-input multiple-output (MIMO) wireless systems are expected to realize large spectral efficiency and high performance communication links. However, one of the major limitations is the complexity of the receiver that needs to separate several overlapped co-channel signals and delayed paths. In this paper, for wideband transmission, orthogonal frequency division multiplexing (OFDM) for MIMO channels (MIMO-OFDM) is considered in order to mitigate intersymbol interference. At the receiver, as a multi-stream detection algorithm for each subcarrier, several low complexity detection algorithms are evaluated and compared for both perfect and realistic channel estimations. Finally, an example of system parameters setup and performance evaluation of a near 1Gbps wireless system over a bandwidth of 40MHz is presented.**

I. Introduction

THE realization of high data rate wireless access is demanded by many applications. With conventional systems, in order to achieve higher data rate transmission more bandwidth needs to be allocated. However, increasing the bandwidth is often undesirable due to spectral or system complexity limitations. Therefore, in recent years, multiple-input multiple-output (MIMO) wireless systems with multiple antennas at both the transmit and receive sides, has attracted a lot of attention as an alternative solution. By using different transmit antennas to transmit different data streams in parallel at the same time and on the same frequency, compared with single-input single-output (SISO) systems, MIMO systems can theoretically achieve over frequency-flat Rayleigh fading (i.e., narrowband) channels, an improvement of the spectral efficiency by a factor of the minimum number of transmit and receive antennas [1]. However, wireless channels are wideband. Wideband transmission of MIMO systems has been investigated in [2][3]. In [2], the combination of orthogonal frequency division multiplexing (OFDM) with MIMO systems has been considered in order to mitigate intersymbol interference (ISI). Such a scheme is known as MIMO-OFDM. The main benefit of using a MIMO-OFDM scheme is that multi-stream separation and ISI mitigation can be performed disjointly. OFDM transforms the frequency-selective MIMO channel into several frequency-flat MIMO subchannels [2], where multi-stream detection is to be performed on each narrowband subchannel separately.

Multi-stream detection for MIMO systems over narrowband channels has been heavily investigated. Since the complexity of the optimum maximum likelihood detector (MLD) increases exponentially with the number of transmit antennas and the symbol alphabet size, complexity reduction of MLD in [5] as well as several suboptimum detectors in [6]-[9] have been considered. In [5], the complexity of MLD was reduced by employing sphere decoding. Also, it is shown that sphere decoding can reach the MLD performance with lower complexity that does not depend on the constellation size. On the other hand, a suboptimum detector well known as V-BLAST (Vertical Bell Laboratories LAyered Space-Time) was introduced in [6][7]. This scheme in its original form uses low complexity successive detection (SD) with nulling in the zero forcing (ZF) criterion, and at each stage of detection chooses among remaining streams, the stream of best signal to noise ratio (SNR) to be detected. An extension of the V-BLAST algorithm to the minimum mean square error (MMSE) criterion has been introduced in [10]-[12]. In the remaining, both the V-BLAST algorithm and its extended scheme are named as ordered successive detection and denoted as OSD.

The focus of our previous works was on the performance improvement of OSD in the ZF criterion (i.e., V-BLAST), by backward iterative detection (BID) in [9], and in the MMSE criterion for rapidly varying channels by noise variance estimation and backward iterative detection in [13]. In [12], the need of accurate weights generations and an appropriate choice of ordering metrics in order to maximize the effect of ordering was emphasized. Furthermore, complexity reduction by a semi-adaptive approach was also presented in [14].

In this paper, we investigate the performance of MIMO-OFDM systems. Our main focus here is the performance evaluation with MIMO-OFDM systems of several multi-stream detection algorithms including those in our previous works and sphere decoding. We will show that the complexity is compromised with the required performance, the antenna cost and the constellation size in use. Also, we show that for a certain level of performance, the required complexity increases with the channel variation rate. Finally, towards the realization of high spectral efficiency wireless access systems, an example of a MIMO-OFDM system that realizes a near 1Gbps data rate transmission over a bandwidth of 40MHz is evaluated.

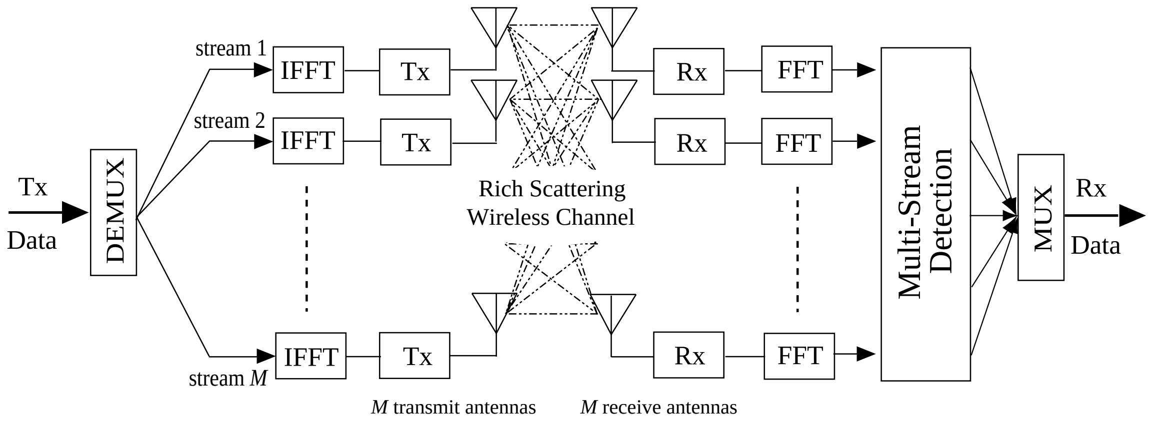

Fig. 1. An example of a parallel space-time transmission scenario using a MIMO-OFDM system.

II. The Signal Model for MIMO-OFDM Systems

Consider a MIMO-OFDM system using M transmit and M receive antennas as shown in Fig. 1. Such a system is denoted as a $M \times M$ MIMO-OFDM system. The main data stream is demultiplexed into M streams, and each of the M streams is broken into OFDM blocks with the n-th block for the i-th stream denoted by $a_i[n,k]$, $k = 0, \ldots, K-1$. Each OFDM block of constellation symbols is transformed using an inverse fast Fourier transform (IFFT) and transmitted by the antenna for its corresponding stream after inserting the guard interval (GI). Thus all M transmit antennas simultaneously transmit the transformed symbols. The received signals at each antenna, after removing the GI, are similarly broken into blocks and processed using FFT. After FFT processing, the n-th block at receive antenna j is denoted by $y_j[n,k]$, $k = 0, \ldots, K-1$. For reasons of simplicity, the guard interval insertion and removal parts are not shown in Fig. 1.

Assuming perfect sample timing and a guard interval long enough to mitigate the ISI effect and preserve the orthogonality between the subcarriers, the receive signal after FFT at the n-th block and on the k-th subcarrier can be expressed as

$$\boldsymbol{y}[n,k] = \boldsymbol{H}[n,k]\boldsymbol{a}[n,k] + \boldsymbol{n}[n,k] \tag{1}$$

where $\boldsymbol{H}[n,k]$ denotes the normalized channel frequency response for the k-th subcarrier and the n-th OFDM block. Each component $H_{ij}[n,k] (1 \leq i,j \leq M)$ of $\boldsymbol{H}[n,k]$ is normalized to satisfy $E[\|H_{ij}[n,k]\|^2] = 1$. $E[\cdot]$ is the expectation function. The noise vector $\boldsymbol{n}[n,k]$ is the additive zero-mean Gaussian noise vector observed for the k-th subcarrier of the n-th OFDM block, and satisfy $E[\boldsymbol{n}\boldsymbol{n}^\dagger] = \sigma^2 \boldsymbol{I}_M$. $\boldsymbol{X}^\dagger$ stands for the Hermitian of the matrix $\boldsymbol{X}$ and $\boldsymbol{I}_M$ is the $M \times M$ unit matrix. Without loss of generality, we consider that the signal vector $\boldsymbol{a}$ satisfies $E[\boldsymbol{a}\boldsymbol{a}^\dagger] = \frac{1}{M}\boldsymbol{I}_M$ as the total transmitted power is kept equal to unity independently of the number of transmit antennas, and distributed uniformly on the transmit antennas.

Using the time channel impulse response, the frequency response at the k-th subcarrier of the n-th OFDM block corresponding to the i-th transmit j-th receive antenna $H_{ij}[n,k]$ can be expressed as

$$H_{ij}[n,k] = \sum_{l=0}^{L-1} h_{ij}[n,l] \exp(-\jmath(2\pi kl/L)) \tag{2}$$

We denote the OFDM block length, the OFDM symbol duration, and the subcarrier spacing as T_f, T_s and Δf respectively. $T_s = 1/\Delta f$ and $T_f = T_g + T_s$, where T_g is the duration of the guard interval. In (2), $(h_{ij}[n,0], \ldots, h_{ij}[n,L-1])$ corresponds to the time channel impulse response of the n-th block between the i-th transmit and j-th receive antenna. L is the number of nonzero taps of the channel impulse response sampled at a rate of $K\Delta f$.

When the channel is time varying, it is assumed that its impulse response varies from an OFDM block to another, but stays invariant within each single OFDM block length. The channel variation rate from an OFDM block to another is measured by $f_{\mathrm{D}}T_f$ which stands for the maximum Doppler frequency f_{D} normalized by the OFDM symbol rate $1/T_f$.

III. Multi-Stream Detection Algorithms for MIMO-OFDM Systems

In order to separate the overlapped co-channel signals over each subcarrier, MLD, OSD and BID algorithms are considered for multi-stream detection.

In the following, over each subcarrier, (1) is rewritten for simplicity as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{a} + \boldsymbol{n} \tag{3}$$

A. Maximum Likelihood Detection and Sphere Decoding

MLD requires the minimization of the metric

$$\|\boldsymbol{y} - \boldsymbol{H}\bar{\boldsymbol{a}}\|^2 \tag{4}$$

over all possible transmit symbol vectors $\bar{\boldsymbol{a}}$, giving MLD a complexity exponential in M, with a base equal to the size of the complex signal constellation.

As for sphere decoding, this one uses the lattice representation of (3) in the attempt to minimize the metric of (4). A lattice representation of the MIMO system described by (3) can be obtained by transforming the complex equation of (2) into a real matrix equation. Sphere decoding reduces the complexity of MLD by restricting the search only to the points of the lattice found within a sphere of radius $\sqrt{C}$ centered around the received point $\boldsymbol{y}$. And each time a valid lattice point is found, the search is restricted further by reducing the radius so that the newly discovered lattice point lies on the surface of the sphere. The derivation of the algorithm is described in [4] and its extension to the MIMO system in [5].

B. Ordered Successive MMSE Detection (OSD)

OSD is based on three operations nulling, cancellation and ordering. The streams are detected in a successive manner over multiple stages. At each stage of detection, nulling weights are generated for all remaining streams, then we choose to first detect the stream of the best post-combining SINR (e.g., best quality), and then we remove it from the received signal for detection of other streams. The detection process of OSD is described in details in [10][12]. OSD differs from SD in its ordering feature employed to choose the stream to be detected at each stage of successive detection. And unlike SD, OSD can extract stream selection diversity [12].

The weights matrix generated at the s-th stage are expressed as

$$\boldsymbol{D}_s{}^{\dagger} = \boldsymbol{H}_s{}^{\dagger}(\boldsymbol{H}_s\boldsymbol{H}_s{}^{\dagger} + M\ \sigma^2\boldsymbol{I}_M)^{+} \quad (5)$$

where $\boldsymbol{X}^{+}$ stands for the Moore-Penrose generalized inverse matrix of matrix $\boldsymbol{X}$ and it is obtained through a singular value decomposition (SVD) [17]. $\boldsymbol{H}_s$ in (5) denotes the deflated version of $\boldsymbol{H}$ obtained by zeroing the k_1,k_2,. . .,k_{s-1}-th columns of $\boldsymbol{H}$. k_i $(i = 1, \ldots, s-1)$ is the index of the stream detected at the i-th stage.

With realistic channel estimations, the nulling weights for OSD are affected by channel estimation errors. The following three approaches for weights generation were considered in [14].

- Direct approach : It generates its weights by the direct substitution of the deflated version $\bar{\boldsymbol{H}}_s$ $(s = 1, \ldots, M)$ of the estimated channel matrix $\bar{\boldsymbol{H}}$ in (5).
- Adaptive approach : This stands for the determination of the Weiner-Hopf solution to the weights generation corresponding to all detection stages in a recursive manner using the RLS (Recursive Least Squares) algorithm.
- Semi-adaptive approach : This consists of a combination of both direct and adaptive approaches in a way that the weights are mainly generated by the adaptive approach but occasionally renewed using the direct approach. Such approach was proposed in order to both compromise performance and complexity [14].

C. Backward Iterative Successive MMSE Detection (BID)

However the performance of OSD is improved by ordering, the performance of all streams is affected by the accuracy of the detection of the first stream. In order to alleviate the performance degradation of the first stream, a backward iterative detection that is initialized by OSD was firstly proposed in [9] and evaluated with nulling in the MMSE criterion using the estimated channel in [13]. The backward iterative detection aims to improve the performance of early detected streams (especially the first one) using the available decision on lately detected streams in a way that at the i-th iteration $(i = 1, \ldots, M-1)$, only replicas from the $k_M, \ldots, k_{(M-i+1)}$-th streams are subtracted.

IV. Computational Complexity

The complexity associated with MLD is q^M, where q is the symbol alphabet size (i.e., constellation size). Its complexity reduction by sphere decoding is polynomial in M and it is of a complexity order of $O(2^6M^6)$.

On the other hand, the main term of complexity in OSD is due to the need of computing pseudoinverses of matrices of rank values going from M to 1. This requires $O(M^4)$ arithmetical operations [17]. This complexity can be reduced by one order to $O(M^3)$ using the semi-adaptive approach in above or the unitary transformation of [11].

Furthermore, the BID algorithm as it consists of repeating a reduced-order version of OSD for $M-1$ iterations, its complexity is approximately M times that of OSD. Its complexity can be reduced by one order in the same manner as for OSD.

The complexity orders of MLD, OSD and BID algorithms are summarized in Table I with and without complexity reductions.

TABLE I
Computational complexity evaluations.

	Complexity Orders	
	w/o reduction	w/reduction
MLD	$O(q^M)$	$O(2^6M^6)$
OSD	$O(M^4)$	$O(M^3)$
BID	$O(M^5)$	$O(M^4)$

V. Performance Evaluation with Computer Simulations

A. System Parameters

In our simulations, both narrowband and broadband cases are considered. In Fig. 2, the performance of both OSD and sphere decoding is compared. As the channel estimation is assumed to be perfect, the performances for MIMO and MIMO-OFDM systems are the same.

For the MIMO system (narrowband case), it is evaluated with realistic channel estimation in Figs. 3 and 4. In this case, the channel estimation and update are performed using the RLS (Recursive Least Squares) algorithm in its matrix form. For the semi-adaptive receiver, this one has P direct receiver blocks over a set S [14]. The forgetting factor of the RLS algorithm is $\lambda = 0.875$ for $f_{\mathrm{D}}T_s$=1/5000, and $\lambda = 0.775$ for $f_{\mathrm{D}}T_s$=1/1000. In addition, we denote a transmission burst that consists of a training period length TL and data transmission period length IL as $B(TL, IL)$.

For the MIMO-OFDM system (broadband case), it is evaluated with realistic channel estimation in Fig. 5. In this case, the MIMO wireless channel is modeled for each pair of transmit and receive antennas with 5 equal independent Rayleigh fading paths. The five paths are assumed to

Top

be uncorrelated spatially and temporally. To construct an OFDM signal, we assume an entire channel bandwidth of 40MHz that we divide to 128 subchannels. The four subchannels on each end are used as guard tones, and the rest (120 tones) are used to transmit data. To make the tones orthogonal to each other, the symbol duration is about 3.2 μ/s. An additional 0.8 μ/s guard interval is used to provide protection from ISI due to the channel multipath delay spread. This results in a total block length $T_f = 4\mu/s$. At each transmit antenna, each burst consists of 51 OFDM blocks, with the first block used for training and the following 50 blocks used for data transmission. Using 16QAM and 10 antennas, for each 4 μ/s, $4 \times 10 \times 120 \times (50/51)$ effective bits are transmitted. Consequently, the described system can transmit at 1.17Gbps over a 40MHz channel.

The channel estimation for the MIMO-OFDM system is performed in the time domain and the optimum training sequences of [15] are employed. For the channel update, it is performed by the LMS algorithm over each subcarrier. For the forgetting factor in the semi-adaptive receiver, it is set to $\lambda = 1.0$ for $f_{\mathrm{D}}T_f$=0.0 and $\lambda = 0.98$ for $f_{\mathrm{D}}T_f$=1/5000. This semi-adaptive receiver has one direct block which is the first OFDM block of the 51 OFDM blocks ($P = 1, S = \{1\}$). The inverse autocorrelation matrices produced in the direct block are multiplied by a reliability factor of 1/5.0 and 1/40.0 for $f_{\mathrm{D}}T_f$ equals 0.0 and 1/5000 respectively.

In our graphs, the bit error rate (BER) is plotted as a function of the average bit energy to noise power ratio E_{b}/N_0 per receiver and not the one per branch. The plotted BER values are calculated by averaging over all streams. For both the narrowband and wideband cases, the desired E_{b}/N_0 value is obtained by adjusting the noise variance σ^2 using the following formula, $\sigma^2 = \frac{E[\sum_{i=1}^{M}\sum_{j=1}^{M}\|H_{ij}a_i\|^2]}{q\times M}10^{\frac{-E_{\mathrm{b}}/N_0}{10}}$.

B. Results

In Fig. 2, sphere decoding always outperforms OSD for mid-to-high E_{b}/N_0 values. Both receivers reveal a diversity order that increases with the number of antennas. The diversity order extracted by sphere decoding is similar to that of MLD. On the other hand, OSD extracts stream selection diversity. However, the order of this stream selection diversity decreases when the constellation size in use becomes larger. In fact, the higher the constellation size and the fewer the number of antennas in use are, the larger the performance gap between sphere decoding and OSD becomes. Obviously, such a gap can be much reduced if more receive than transmit antennas are employed. Hence, compared to sphere decoding, OSD is much attractive for MIMO systems where the number of antennas employed in the system is large enough to resolve the constellation size in use (e.g., QPSK/5×5, 16QAM/10×10).

In Fig. 3 ($f_{\mathrm{D}}T_s = 1/5000$), the adaptive form of OSD suffers from higher performance degradation than the direct form. The semi-adaptive approach improves the performance of the adaptive receiver significantly to the performance of the direct receiver. Using the semi-adaptive approach, a reduction of the complexity by one order is possible.

In Fig. 4 ($f_{\mathrm{D}}T_s = 1/1000$), the performance of OSD is improved by noise variance estimation (i.e., improved OSD) and furthermore by backward iterative detection (BID). The diversity loss of the conventional OSD can be compensated using BID. BID requires an increase of the complexity by one order.

In Fig. 5, the performance evaluation of the MIMO-OFDM system described above is presented for ($f_{\mathrm{D}}T_f = 0.0, f_{\mathrm{D}}T_f = 1/5000$). The idea of semi-adaptive is introduced for the MIMO-OFDM case with channel estimation over the first OFDM block. It is shown that the semi-adaptive approach can enhance the performance as well as reduce the complexity of OSD for MIMO-OFDM systems when compared to the direct approach. Actually, for a complexity order of $O(M^3)$ using the semi-adaptive approach, the BER of 10^{-4} and 10^{-3} can be achieved for the channel variation rates of $f_{\mathrm{D}}T_f = 0.0$ and $f_{\mathrm{D}}T_f = 1/5000$ for E_{b}/N_0 per branch equals to 14dB=$(24 - 10\log10)$dB and 16dB=$(26 - 10\log10)$dB respectively.

VI. Summary

In this paper, several low complexity multi-stream detection algorithms for MIMO-OFDM systems were investigated for different numbers of antennas, constellation sizes and channel variation rates. Actually, there is a tradeoff between the receiver complexity from a side and the required performance, the antenna cost and the constellation size in use from another side. By employing high performance low complexity multi-stream detection algorithms, the MIMO-OFDM technology can be expected to effectively profit from its large spectral efficiencies as larger number of signals can be overlapped and therefore accommodated without increase of the bandwidth within reasonable complexity, performance and antenna cost.

Furthermore, an example of parameters set up of a MIMO-OFDM system that transmits at near 1Gbps over a 40MHz channel was presented. It is shown that the spectral efficiency of almost 40bit/s/Hz can be achieved with a complexity order of $O(M^3)$ with 16QAM, 10×10 system, where the required SNR per bit is 14dB for the BER of 10^{-4} and a channel variation rate of $f_{\mathrm{D}}T_f = 0.0$.

References

[1] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, pp. 41–59, Autumn 1996.

[2] P. Vandenameele, L. V. D. Perre et al., "A combined OFDM/SDMA approach," *IEEE J. Sel. Areas in Commun.*, vol. 18, no. 11, pp. 2312–2321, Nov. 2000.

[3] A. Lozano et al., "Layered space-time receivers for frequency-selective wireless channels," *IEEE Transactions on Communications*, vol. 50, no. 1, pp. 65–73, Jan. 2002.

[4] E. Viterbo et J. Boutros, "A universal lattice code decoder for fading channels," *IEEE Transactions on information theory*, vol. 45, no. 5, pp. 1639–1642, Jul. 1999.

[5] O. Damen, A. Chkeif and J. C. Belfiore, "Lattice code decoder for space-time codes," *IEEE Communications Letters*, vol. 4, no. 5, pp. 161–163, May 2000.

[6] P. W. Wolniansky, G. J. Foschini, G. D. Golden, R. A. Valenzuela, "V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel," *in Proc. ISSSE*, Pisa, Italy, Sept. 1998.

[7] G. Foschini et al., "Simplified processing for high spectral efficiency wireless communication employing multi-element arrays," *IEEE J. Sel. Areas in Commun.*, vol. 17, pp. 1841–1852, Nov. 1999.

Top

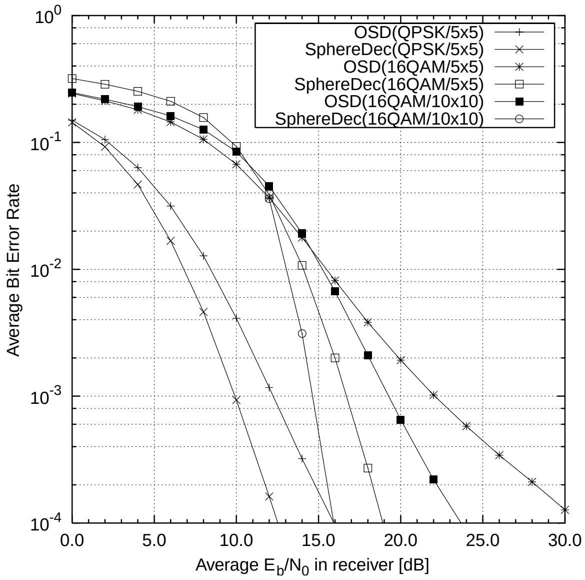

Fig. 2. Performance comparison of OSD and sphere decoding with various constellation sizes and number of antennas (narrowband MIMO/broadband MIMO-OFDM, w/perfect channel estimation).

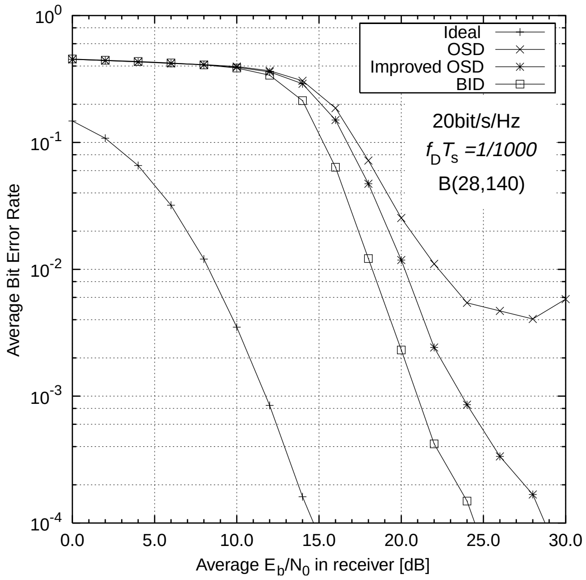

Fig. 4. Performance improvement of OSD using noise variance estimation and BID for increased channel variation rate ($f_D T_s = 1/1000$)(10×10/QPSK) (narrowband MIMO).

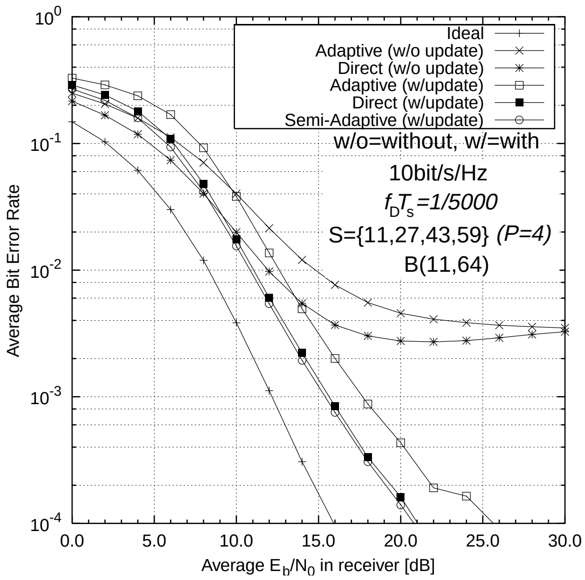

Fig. 3. Performance of OSD using the complexity reduced semi-adaptive approach ($f_D T_s = 1/5000$)(5×5/QPSK)(narrowband MIMO).

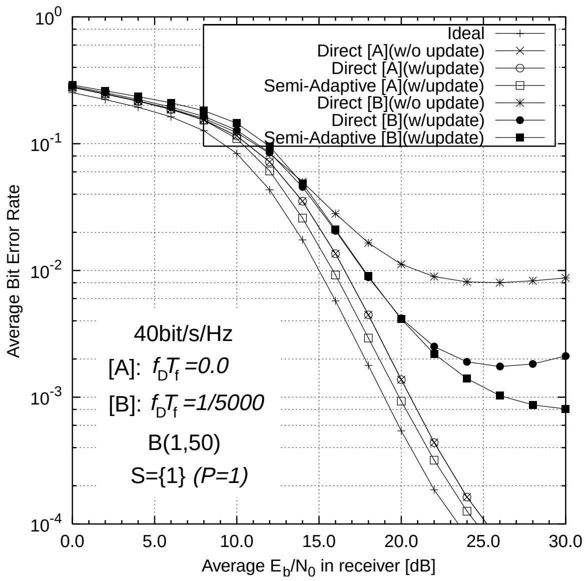

Fig. 5. Performance evaluation of a near 1Gbps MIMO-OFDM wireless system using OSD ($f_D T_f = 0.0, f_D T_f = 1/5000$)($10 \times 10$/16QAM)(broadband MIMO-OFDM).

[8] W. J. Choi, R. Negi, J. M. Cioffi, "Combined ML and DFE decoding for the V-BLAST System," *Proc. IEEE ICC*, vol. 3, pp. 1243–1248, New Orleans, June 2000.

[9] A. Benjebbour, H. Murata and S. Yoshida, "Performance of iterative successive detection algorithm with space-time transmission," *Proc. IEEE VTC Spring*, vol. 2, pp. 1287–1291, Rhodes, Greece, May 2001.

[10] "M. Debbah et al., "A MMSE successive interference cancellation scheme for a new adjustable hybrid spread OFDM system," *Proc. IEEE VTC Spring*, vol. 2, pp. 745–749, Tokyo, Japan, May 2000.

[11] B. Hassibi, "An efficient square-root algorithm for BLAST," *Proc. IEEE ICASSP*, pp. 737–740, June 2000.

[12] A. Benjebbour, H. Murata and S. Yoshida, "Performance comparison of ordered successive receivers for space-time transmission," *Proc. IEEE VTC Fall*, vol. 4, pp. 2053–2057, Atlantic, USA, Sept. 2001.

[13] A. Benjebbour, S. Yoshida, "Improved ordered successive MMSE detection over rapidly varying MIMO channels," *submitted to IEEE ICC 2003*.

[14] A. Benjebbour and S. Yoshida, "Novel semi-adaptive ordered successive receivers for MIMO wireless systems," *in Proc. of the 13th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'02)*, pp. 582-586, Lisbon, Portugal, Sept. 14–18, 2002.

[15] Y. Li, "Simplified channel estimation for OFDM systems with multiple transmit antennas," *IEEE Transactions on Wireless Communications*, vol. 1, no. 1, pp. 67–75, Jan. 2002.

[16] C. J. Jakes Jr. et al., *Microwave Mobile Communications*, New York: Wiley, 1974.

[17] G. H. Golub et al., *Matrix Computations*, Johns Hopkins, 1991.

[18] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, 1991.

Top