

A Simple and Accurate Formula for Calculating the Conduction Angle of Single Phase Rectifiers with RL Load

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Abstract

This paper presents simple and accurate formula for calculating a conduction angle of single-phase half-wave or full-wave rectifiers with RL load. The formula can also be used in case of a.c. voltage regulators under discontinuous current conduction mode. This study also compares the results with that obtained by the exact solution method. This method reduces the computational time in both the computer-aided design and manual classroom analysis.

Introduction

In power electronics applications it is often necessary to consider a load consisting of a resistive load in series with an inductive load. The inductor in this case will serve as a filter representing high series impedance to the alternating harmonic component in the rectifier output. The inductor may also be used as storage magnetic energy when the current is above the average value and releasing that energy to the load when the current tends to fall below the average, thus reducing the ripple.

Figure 1 shows a common power electronic circuit, a single phase half-wave controlled rectifier with RL load. The waveforms of the output voltage and current (v_o) and (i_o) are represented in Figure 2. The circuit differential equation for the conducting interval by neglecting the voltage drop across the thyristor can be written as;

$$v_{in} = V_m \sin \omega t$$

$$(1) \quad V_m \sin \omega t = i_o(t)R + L \frac{di_o(t)}{dt}$$

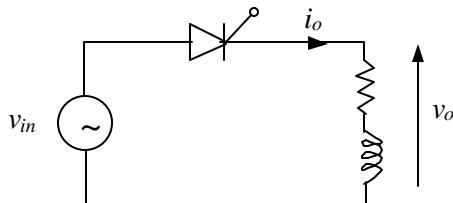


Figure 1, A single phase half-wave controlled rectifier with RL load

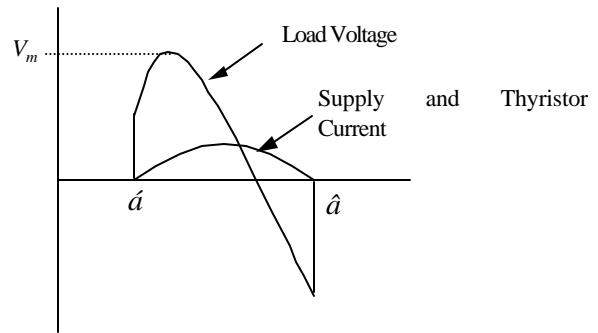


Figure 2: Wavefrms of output Voltage (v_o) and Current (i_o) of Figure 1.

Solving equation (1) using the differential equation;

$$i_o(t) = \frac{V_m}{|Z|} \sin(\omega t - \mathbf{f})$$

$$\frac{V_m}{|Z|} \sin(\mathbf{a} - \mathbf{f}) e^{-\frac{R}{\omega L}(\omega t - \mathbf{a})}$$

$$(2)$$

Where $|Z|$ is a magnitude of the load impedance and is equal to $(\sqrt{R^2 + (\omega L)^2})$, \mathbf{f} be a load angle $(\tan^{-1} \omega L / R)$, \mathbf{a} is a firing angle, and V_m is a supply peak voltage.

It is well known that equation (2) is also valid for the case of a single phase full-wave rectifier and a a.c. single phase voltage regulator with RL load under discontinuous current mode of operation ($\mathbf{a} > \mathbf{f}$) [1-5].

From equation (2) it can be seen that the current is existing between the firing angle (\mathbf{a}) and the

excitation angle (\mathbf{b}). This also can be seen in Figure 2. Nevertheless, in order to calculate the performance parameters of the circuits, i.e., output rms value, average output power, etc, it is necessary to know the value of the extinction angle (\mathbf{b}). Accordingly, (\mathbf{b}) can be found from the solution of equation (2) when $i_o(t) = 0$ at $\mathbf{w}t = \mathbf{b}$;

$$\sin(\mathbf{b} - \mathbf{f}) - \sin(\mathbf{a} - \mathbf{f})e^{-\frac{R}{\omega l}(\mathbf{b} - \mathbf{a})} = 0 \quad (3)$$

Hence, the conduction angle (\mathbf{d}) is defined as the period at which the power electronic device is conducting,

$$\mathbf{d} = \mathbf{b} - \mathbf{a} \quad (4)$$

Therefore, substituting (4) into (3) give,

$$\sin(\mathbf{d} + \mathbf{a} - \mathbf{f}) - \sin(\mathbf{a} - \mathbf{f})e^{-\frac{R}{\omega l}(\mathbf{d})} = 0 \quad (5)$$

It can be seen from (5) that it is non-linear and the iteration needed for solving for (\mathbf{d}) it is time-consuming. A family of curves for calculating the conduction angle (\mathbf{d}) can be used to overcome this difficulty [4-5]. However, these curves are not providing all possible values of (ϕ) and are not being easily usable in direct computations. Therefore, the objective of this paper is to develop a formula, which can be accurate in calculating the above conduction angle (\mathbf{d}) under discontinuous conduction mode.

Formula Development

In general, for a purely inductive load, $\mathbf{f} = 90^\circ$, the conduction angle (\mathbf{d}) can be found from [1-5].

$$\mathbf{d}_{\mathbf{f}=90^\circ} = 2(\mathbf{p} - \mathbf{a}) \quad (6)$$

Nevertheless, with resistive load, $\phi = 0$, the conduction angle (\mathbf{d}) can be found from [1-5].

$$\mathbf{d}_{\mathbf{f}=0^\circ} = \mathbf{p} - \mathbf{a} \quad (7)$$

Consequently, it can be seen from (6) and (7) that all the conduction characteristics lie in the (\mathbf{d}) versus (\mathbf{a}) plane, (between the two linear boundaries). It is proposed that an approximate value of the conduction angle (\mathbf{d}) can be found from the simple relationship [5]:

$$\mathbf{d} = \mathbf{p} + \mathbf{f} - \Delta - \mathbf{a} \quad (8)$$

Where, range of change of Δ for different load resistance is given in Table 1.

Table 1

Δ in degree		Load resistance (R)
From	To	
5	10	$> \omega l$
10	15	$\approx \omega l$
15	20	$< \omega l$

Using equation (8) with the possible values of Δ and by using tools provided by Matlab (curve fitting) [6,7], the following formula has been obtained:

$$\mathbf{d} = 3.342 + \mathbf{f}(0.8 - 0.08\mathbf{f}) - \mathbf{a}(0.9 - 0.09\mathbf{a}) \quad (9)$$

To verify the validity of the obtained formula, the exact values of the conduction angle are computed from the non-linear equation (5), by using a Newton-Rapason iteration method and compared with that obtained from equation (9). The values of the conduction angle are computed from both equation (8) and (9), for different values of load angles ($\phi = 30, 45, 60$ and 75 degree) as shown in Figures 3-6. It can be seen from figures that, the obtained formula simulates the conduction angle with a reasonable accuracy. Furthermore, detected that the percentage error is minimum ($< 3\%$) in most cases of load angle (\mathbf{f}). However, the maximum percentage of error can be obtained at highly inductive load, which is less than 8%. In other words, fine-tuning of the obtained formula can reduce this error.

Figure 3: Comparison Between Methods for $\phi=30$

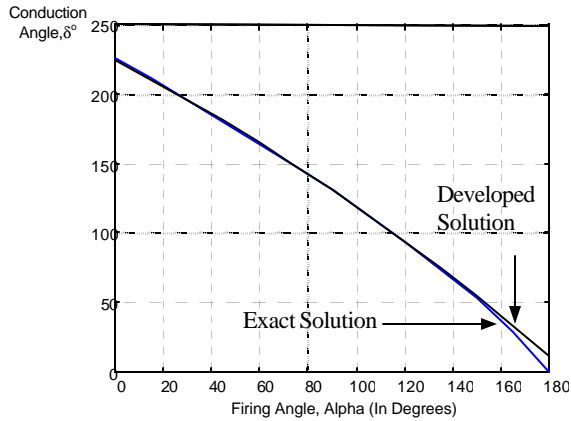


Figure 4: Comparison Between Methods for $\phi=45$

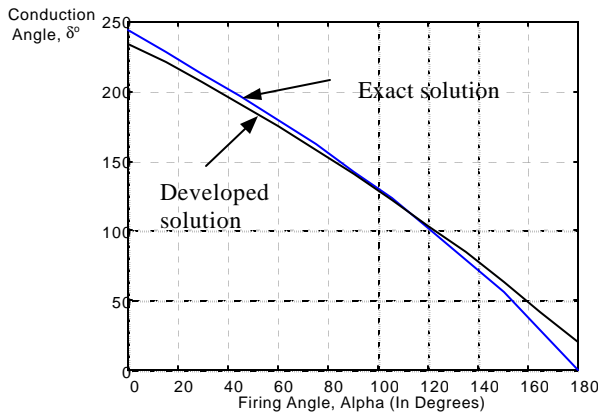


Figure 5: Comparison Between Methods for $\phi=60$

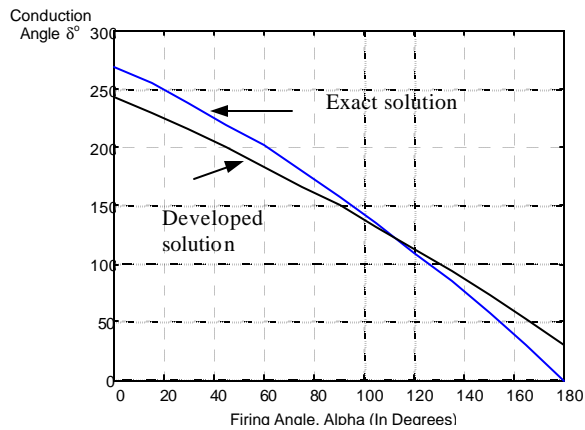


Figure 6: Comparison Between Methods for $\phi=75$

Conclusions

A simple and accurate formula has been obtained for calculating a conduction angle for a single-phase half-wave and full-wave rectifiers and a.c. voltage regulators under discontinuous current conduction mode with RL load. The accuracy of the developed formula has been verified by comparing the result obtained from the exact iteration solution with that obtained by the developed formula. This formula reduces the computational time and also reduces time for manual classroom calculation.

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