Stability Analysis of Periodically Perturbed Power Systems

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Abstract

Small signal stability refers to the response of power systems to perturbations arising from load switching, routine switching operations and similar day-to-day operations. In mathematical terms, as long as the real part of all eigenvalues of a linearised power system is negative, the system will be stable regardless of the nature of disturbances. Implicit in this approach is the assumption that there are no nonlinearities in the linearised model. When non-linearity is considered the dynamics of the power system exhibits a variety of oscillations some of which can induce instability. In this paper the stability and instability regimes for unloaded but energized power transformer will be discussed.

1. Introduction

Stability of a power system is related to the notion of disturbances. Steady state stability is concerned with small disturbances [1] and is assessed by eigenvalue analysis of the linearized system [2]. If the disturbance is large, the dynamics is assessed by numerical methods [2] or transient energy functions [3] since non-linearity is involved.

However, some disturbances need consideration of non-linearity such as magnetic characteristics of transformer cores or nonlinear controllers that are so extensively used today. The stability for such cases is assessed by perturbation methods [4,5,6].

In this paper the variational form of the dynamical equations is used to study the stability regimes of an energized but unloaded transformer. The variational equation of an unloaded but energized transformer is derived in Appendix-1. Equation 7 is the autonomous form of Mathieu equation. Such equations are characterized by several stable and unstable regimes due to the occurrence of parametric resonance. This will be discussed further in Section 2. The variation of the characteristic parameters a and q (see equation 1) occurs with changes in network topology or disturbances due to periodic load switching. These will be discussed in Sections 3. In Section 4 discussion and conclusions are presented. The effect of interactions between controllers is not being considered however.

2. The stability and instability regimes of Mathieu Equation [7, 8, 9]

One variant of the Mathieu equation is given in equation 1. This equation is not integrable for any arbitrary restoring force, which in this instance is both periodic and time dependent. The time-period corresponding to the periodic component of cos[2 τ] in equation 1 is p , is known as the pumping frequency [9].

$$\frac{d^2 y}{dt^2} + [a - 2q\cos(2t)]y = 0 \qquad (1)$$

When q=0 equation 1 has the periodic solution given by equation 2.

$$y(t) = A\cos\left(\sqrt{a}\,t\right) + B\sin\left(\sqrt{a}\,t\right) \qquad (2).$$

However, when $q \neq 0$ the excitation represented by the term $2q\cos[2t]$ has a period of $T = p$. This implies that the initial conditions y(0) and $\dot{y}(0)$ undergo at each period $T = p, 2p, 3p \ldots$ a linear transformation in the $(y, \dot{y})$ plane by a Floquet matrix M of the system. (Matrix M is conventionally known as the state transition matrix in linear systems theory). Based on this observation the Floquet theory of linear equations with periodic coefficients whose solution is given by equation 3.

$$\begin{aligned} y(t) &= c(t)\exp(m\,t) \\ where\quad c(t) &= c(t+p) \end{aligned} \qquad (3)$$

The parameter μ is known, as the Floquet exponent is the eigenvalue of the matrix M mentioned in the previous paragraph. Fourier series or perturbation methods can be used to derive the analytic expression for $c(t)$ (see [7, 8 or 9] for details). The solution y(τ) will be stable for $m^2 < 0$ and unstable when $\mu^2>0$. Thus $\mu^2 = 0$ is the boundary between the stable and unstable solutions. Clearly a combination of (a, q) gives rise to periodic, stable or unstable solutions. A few random examples of variations in (a, q) are shown in Figure 1. Here phase portraits $(y, \dot{y})$ have been used to present 12 cycles of the simulation. In this paper the information presented by phase portraits will be used to assess the stability of the system.

The second aspect is the interaction between two frequencies when $q \neq 0$: the natural frequency ω_0 (= $\sqrt{a}$) and the pumping frequency ω_p (=2). This is reflected by the multiplier $\exp(\mu\ \tau)$ in equation 3. If $\exp(\mu\ \tau)=1$ then the resonance condition $|n\ \omega_p + m\ \omega_0|$, where m and n are prime numbers, is satisfied. Subharmonics of the order m are obtained when n=1 and m>1, and harmonics of order n are obtained when m=1. However, if both n>1 and m>n ultra-subharmonics or subharmonics of the order (n, m) is obtained. When $\exp(\mu\ \tau) \neq 1$ its eigenvalues give the frequencies of oscillations.

The difference between ordinary resonance and parametric resonance is as follows. Ordinary resonance is the set of natural frequencies of the system at which the amplitude of the oscillation grows unbounded. In parametric resonance, the spectrum is a union of several unpredictable frequencies defined by the combination (a, q) of equation 1. As stated, it is defined by the strength of coupling q.

3. Induced Instability of Periodic oscillations

A power transformer is always maintained in a "hot" state. It is assumed that at its rated voltage, the core flux is sinusoidal. However, the equivalent source impedance at the bus is exposed to topological variations in power systems (see Figure 5). The equivalent system can then be represented by equation 4 (see Appendix-1 for its derivation). The pumping frequency will change as the frequency Ω of the line changes (see Table 1). Other circuit operations such as periodic load switching or nonlinear controllers will also change Ω.

$$\frac{d^2 \Delta \boldsymbol{l}}{d\ \boldsymbol{t}^2} + \left[1.573 - 2 \times 0.285 \times \cos\left(2\frac{\boldsymbol{w}_f}{\Omega}\boldsymbol{t}\right)\right]\Delta \boldsymbol{l} = 0 \qquad (4)$$

Under normal operating conditions the flux in the core is sinusoidal and as a consequence draws magnetizing current rich in harmonics.

The phase portraits for different pumping frequencies listed in Table 1 is shown in Figure 2. For q = 1.5253 (for example) $\mu^2 = -1$ and the stable subharmonic oscillations shown in Figure 2 can become unstable (see Figure 3). The flux buildup in the core now exceeds its rated value but changes very slowly in the first cycle of pumping frequency and then it changes rapidly. The change in flux is very rapid for the 100 km long line and falls as the line length increases. There is no dc offset in the cases studied but cannot be ruled out in practice and there is always some residual magnetization of the core. The increase in the core flux will increase the magnetizing current, which at some stage will exceed the rated full-load current by several multiples. The differential protection will operate provided that it is not prevented by the second and/or fifth harmonic restraints.

Some change in the pumping frequency is inevitable in this process. We avoid discussing its implications since the mathematical development is non-trivial.

Table 1: Pumping Frequency

Line length (km)	Ω (rad /s)	$\omega_p = 2 * \omega_f / \Omega$
100	4242.6	0.148
200	2121.32	0.296
400	1067.7	0.5882

The oscillations shown in Figure 4 (right) do not have a repeat frequency. Such oscillations are known as quasi-periodic. This form of "turbulence" has been previously experienced in the French grid [11].

4. Conclusions

Two types of parametric resonance known are: auto-parametric and hetero-parametric resonance. Auto-parametric resonance occurs as a result of natural oscillations of one mode working on other modes as internal periodic disturbance. Hetero-parametric resonance on the other hand implies that power swings diverge due to periodic external disturbance such as cyclic variations in load. These resonances can give rise instabilities such as subharmonic cascades, saddle-node bifurcations and Hopf bifurcations. The simulations for the selected values show the occurrence of Hopf bifurcations leading to unstable oscillations. In particular instances, short-time stability is also possible.

Dynamical systems theory has developed methods for determining the initial conditions for initiating different oscillation types. In this paper, this has been intentionally avoided as they involve tedious mathematical developments.

The effect of these subharmonic oscillations on the operation of protective relays has not been investigated.

5. References

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Appendix-1: Dynamics of a Power Transformer due to topological changes

A power transformer at no-load is connected at Bus-2 and is modeled by the core loss R_2 and a nonlinear magnetizing inductance $i_m = al + bl^3$ where λ is the flux in the core. The source impedance between buses 1 and 2 is represented by the Thevenin equivalent series inductance L_1 and an equivalent shunt capacitance C_1 that vary as the network topology changes. The system shown in Figure 5 is represented by equation 5 when R_1 is negligible and R_2 is very high. The value of $d = 2\times10^{-3}$ and for $e = 7.445 \times 10^{-7}$.

$$L_1 C_1 \frac{d^2 l}{d t^2} + (1+d)\, l + e l^3 = \frac{E_m}{w_f}\sin(w_f t) \qquad (5)$$

If the assumed periodic solution for the flux in the transformer core is $l_0 = l_m \sin(w_f t)$ where ω_f is the system frequency then the variational form of equation 5 is as given in Equation 6.

$$L_1 C_1 \frac{d^2 \Delta l}{d t^2} + \left[\left(1 + d + \frac{e}{2} l_m^2\right) - \frac{l_m^2}{2}\cos(2 w_f t) \right] \Delta l = 0 \qquad (6)$$

Defining a new time scale $t = \Omega t$ the equation 6 reduces to

$$\frac{d^2 \Delta l}{d t^2} + \left[a - 2q\cos\left(2\frac{w_f}{\Omega} t\right)\right]\Delta l = 0,$$
$$where\ \Omega^2 = \frac{1}{L_1 C_1},\ a = \left(1 + d + \frac{e}{2} l_m^2\right), \qquad (7)$$
$$q = \frac{3}{4} e l_m^2$$

For E (rated voltage) = 275 kV (line-to-line), ω_f = 314 rad/s one obtains a = 1.573 and q=0.285.

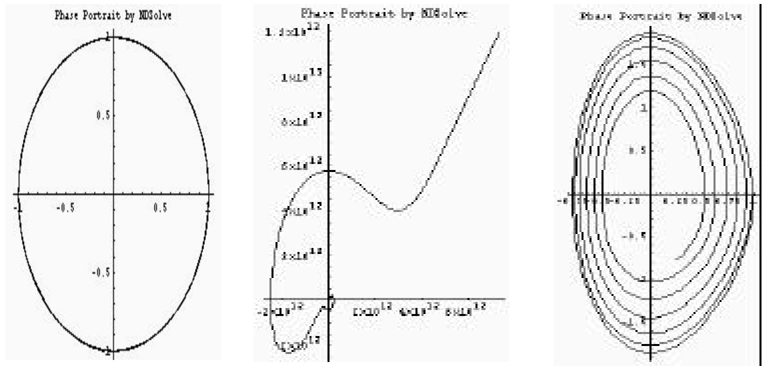

Figure 1: Phase-portraits present $(y, \dot{y})$ of the system for initial conditions $(y=1, \dot{y}=0)$. **Left** - is an example of periodic oscillation obtained for λ^2=0, a=1 and q=0; **Middle**- Loss of periodic stability (since both y and $\dot{y}$ are unbounded) obtained when λ^2=1, a=1, q= ± 2.236 and **Right** - Stable solution which is (slowly) decaying is obtained for λ^2=-1, a=4.5 and q=1.

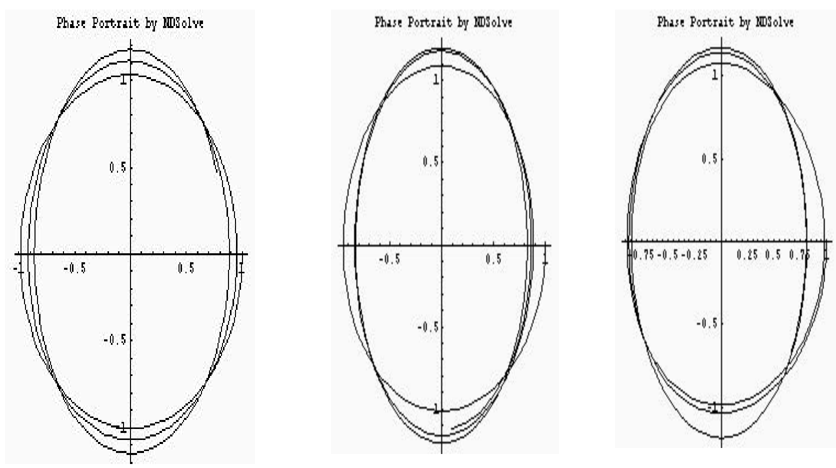

Figure 2: Stable oscillations of core flux for a = 1.573 and q=0.285. **Left**: 100 km long line, **Middle:** 200 km long line and **Right:** 400 km long line.

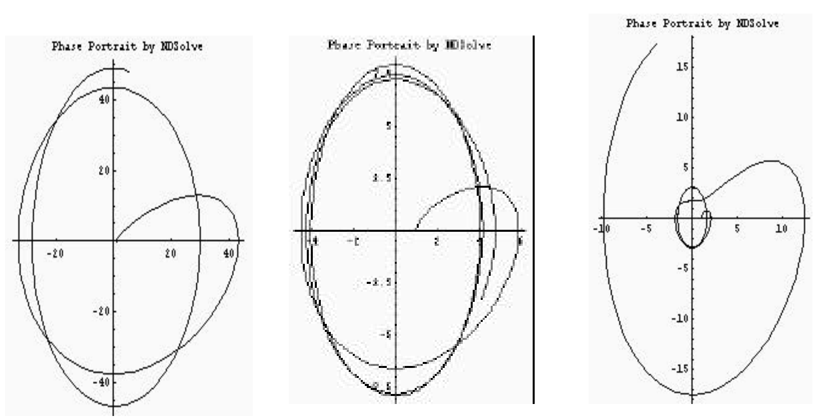

Figure 3: Unstable oscillations of core flux for a = 1.573 and q= 1.5253. **Left:** 100 km long line, **Middle:** 200 km long line and **Right:** 400 km long line.

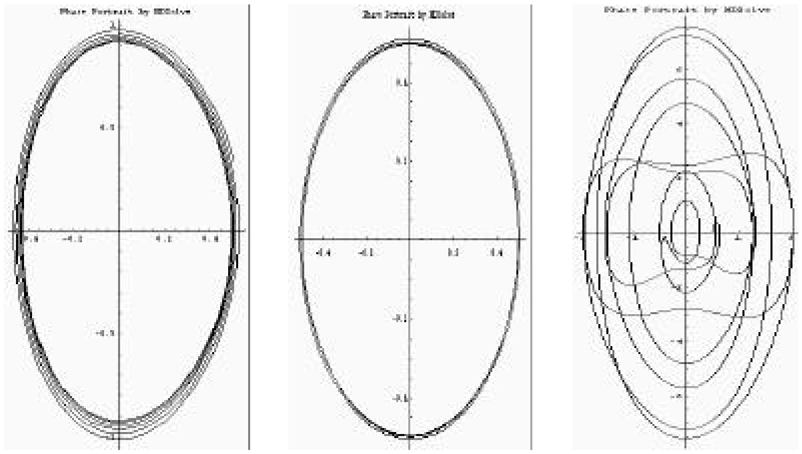

Figure 4: Stable solutions of equation 7. **Left and Middle**: Stable solution; **Right:** The system exhibits short-term stability.

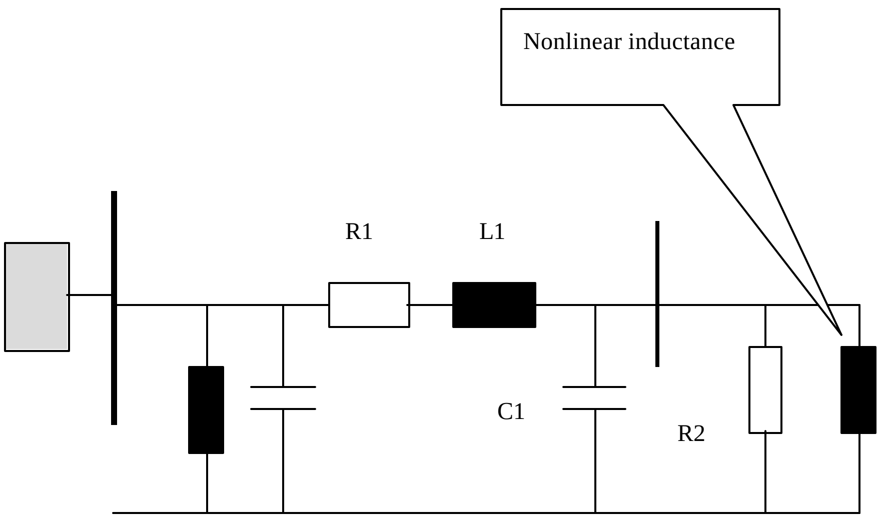

Figure 5: Transformer at no-load is energized from the grid. The equivalent circuit parameters resistance R1, inductance L1 and capacitance C1 represents the grid. The reactor shown near the source is used to cancel the effect the source capacitance. The resistance R2 is the core loss of the magnetic circuit while the nonlinear inductance represents the magnetizing characteristic of the transformer.