Signal Characterization Using Phase Polar Plots

A. Elkhouly* (student member IEEE) and A. S. Elwakil* (senior member IEEE)
**Department of Electrical Engineering, University of Sharjah, P.O. Box 27272, United Arab Emirates* (email: elwakil@ieee.org)

***Abstract*— We demonstrate here the possibility of using the amplitude-phase polar plot, constructed for a scalar signal as a characterization tool. In particular, we show how phase noise and total harmonic distortion can be measured using this polar plot. In addition, we exploit using the technique for ECG identification.**

I. Introduction

Traditionally, the power spectrum is the widely used technique for signal characterization. This spectrum plots the power content of the signal harmonics, obtained via an FFT, versus their frequency. Important quantities such as Total Harmonic Distortion (THD) and Phase Noise are measured using this power spectrum. However, an FFT signal decomposition of the form $f(t) = a_0 + \sum_{n=1}^{N} A_n \cos(n\omega_0 t + \phi_n)$ generates a phase component ϕ_n. This component is ignored in the power spectrum and is seldom considered useful although few researchers have indicated that nonlinear dynamical operations performed on the phase of a signal are the main cause of behaviors such as chaos and phase noise [1]-[3].

In this work we investigate the amplitude-phase polar plot and show that THD and phase noise can be measured using this plot. In addition, we construct the plot from measured ECG signals for normal and sick individuals (diagnosed with Congestive Heart Failure) and show that clear differences can be observed.

II. Phase Polar Plots

To demonstrate the phase polar plot, consider for example a periodic square wave $x(t) = 3\text{sgn}(\sin(2000\pi t + \frac{\pi}{3}))$. The phase polar plot for this signal is shown in Fig. 1(a) where it is seen that the fundamental component is located at $-\pi/6$, the second harmonic is located at $\pi/2$ and the third harmonic is located at $7\pi/6$ respectively. Another example is shown in Fig. 1(b) where a white Gaussian noise of zero mean and 2.25 variance is added to the amplitude of $x(t) = 2\cos(2\pi 10^8 t + \frac{\pi}{6})$. The fundamental component is clear at $\pi/6$ and the noise content is magnified in the lower subplot. Figure 1(c) is the phase polar plot of $x(t) = 2\cos(2\pi 10^8 t + \frac{\pi}{6})$ when noise with variance 0.02 is added to the phase instead of the amplitude. This case reflects a typical sinusoidal oscillator output operating at 100MHz.

A. Phase noise

From Fig. 1(c) we suggest a new method for measuring the phase noise. The method is based on finding the best fit noise circle (shown in gray in the subplots of Figs. 1(b) and 1(c)), measuring its radius R_0 and applying the formula

$$\text{phase noise } (dB) = 20\log(R_o/R) \quad (1)$$

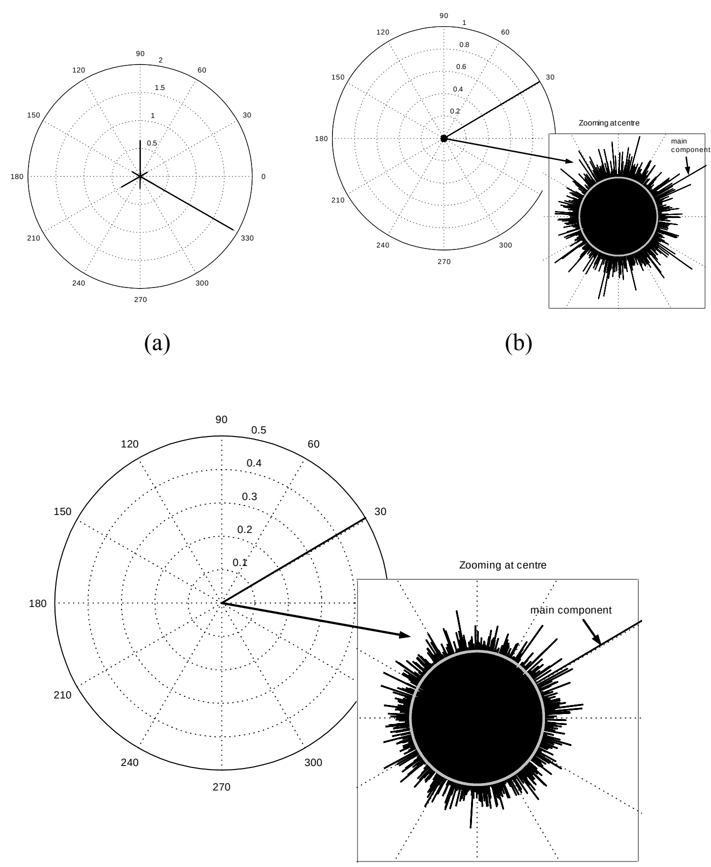


Figure 1: Phase polar plots of (a) a square wave with amplitude 3 and frequency 1kHz, (b) a 100MHz sinusoid with amplitude noise and (c) a 100MHz sinusoid with phase noise.

where R is the amplitude of the fundamental component.

For example, the best fit circle in Fig. 1(c) has a radius $R_0 = 0.000025$ and hence using (1) the phase noise $\approx -86dB$. It is not straightforward to compare this value with that obtained from the power spectrum, shown in Fig. 2, since the measurement from the power spectrum is based on a specific offset from the carrier (fundamental component) [4]. For example, at $100KHz$ offset from carrier the measured phase noise is $-95.2405dBc/Hz$. The advantage of measuring phase noise from the phase polar plot is thus obvious as it is independent of any frequency offset.

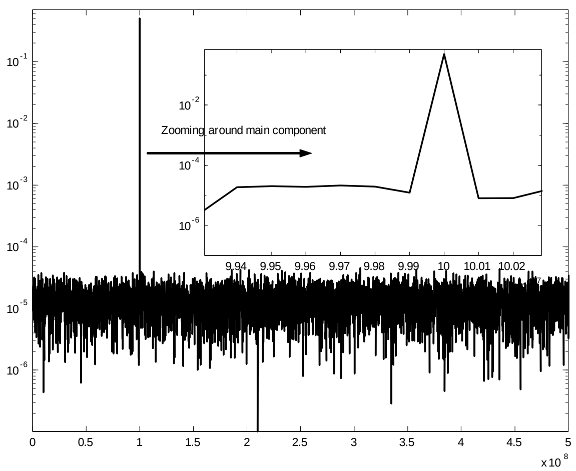
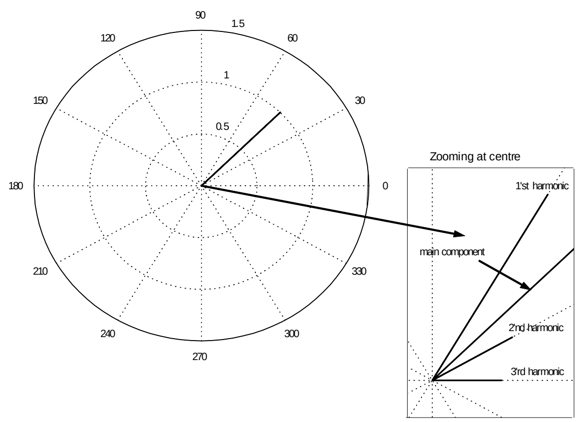
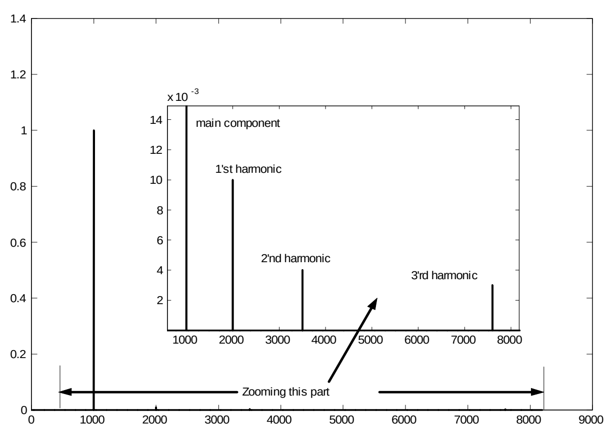


Figure 2: Phase noise measurement from the power spectrum.



(a)



. (b)

Figure 3: Measuring THD (a) from phase polar plot and (b) from ampltude spectrum.

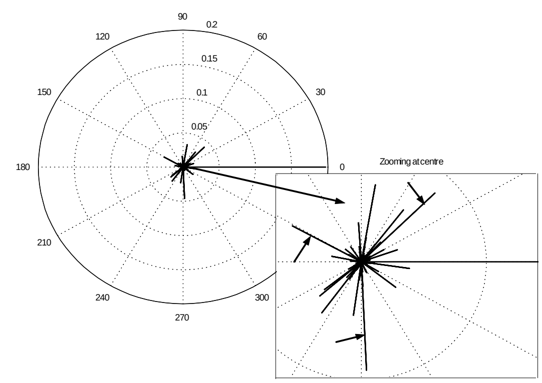
B. Harmonic distortion

The phase polar plot may also be used to measure THD. Consider for example the polar plot shown in Fig. 3(a) which has a main component of amplitude one and three harmonics of amplitudes 0.01, 0.003, and 0.004 respectively. The calculated THD is then 1.1%. Exactly the same value can be obtained from the amplitude spectrum shown in Fig. 3(b), which is the traditional method for measuring THD.

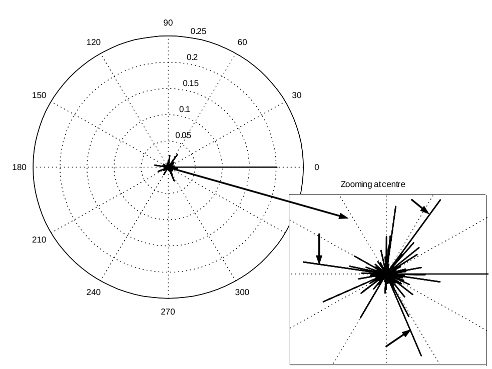
C. Signal identification

1) ECG signals: As an application for the phase polar plot in signal identification, we consider several recorded ECG signals [5]. The aim is to be able to detect cardiac abnormalities. Figures 4(a) and 4(b) are ECG phase polar plots for two normal individuals. From these plots among many others, it was observed that the fundamental component with maximum amplitude always has zero phase and that the largest component in the first quadrant always lies between 40° and 60°. In the second quadrant, the largest component always lies between 150° and 180° and in the fourth quadrant it lies between -90° and -60°.

Next we consider abnormal ECG signals for patients diagnosed with Congestive Heart Failure [5]. Samples of the constructed phase polar plots for two patients are shown in Figs. 5(a) and 5(b). It was observed for all patients of this disease that the fundamental component has a phase of 180°; i.e. it is shifted 180^0 with respect to a normal person. Further, new large components appear in the third quadrant.

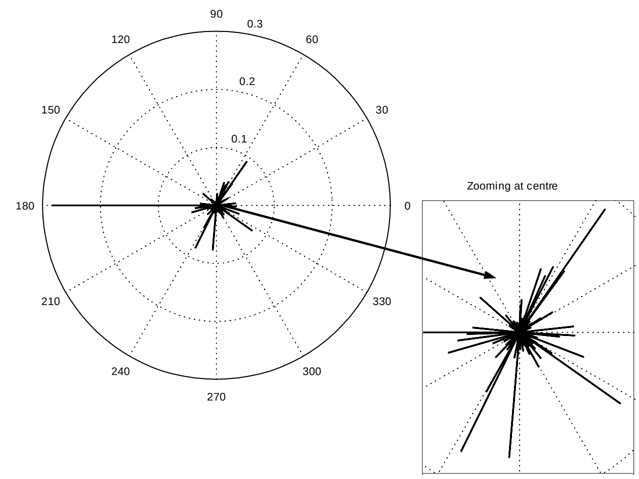


(a)

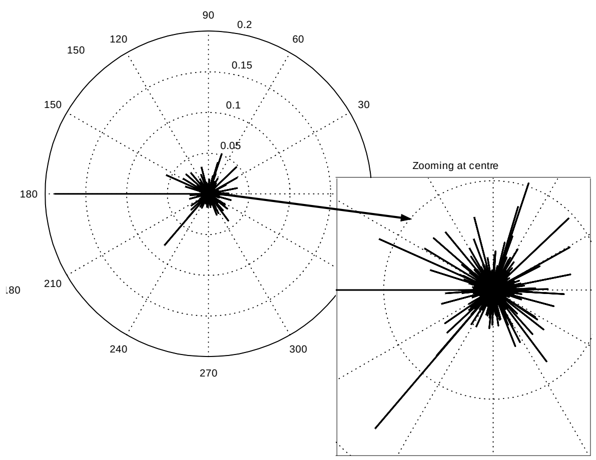


. (b)

Figure 4: Phase polar plots constructed from ECG signals of two normal individuals



(a)



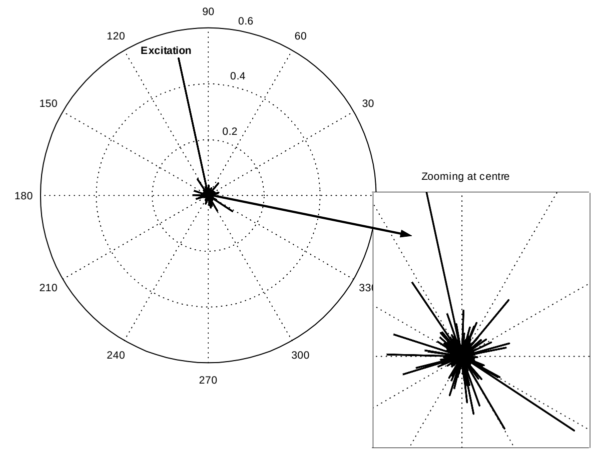
(b)

Figure 5: Phase polar plots of two patients diagnosed with Congestive Heart Failure.

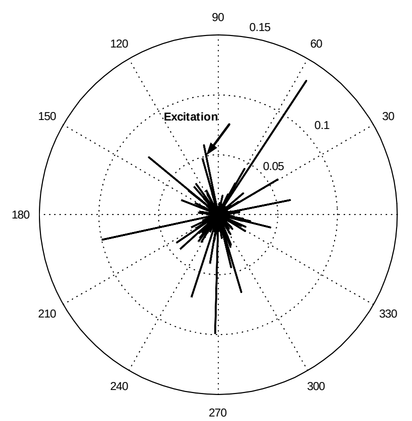
2) Chaotic signals: Chaotic signals are produced by fully deterministic systems yet they appear to be random and share some main features with noise. In [1], it was postulated that chaos is generated because of a nonlinear stretch and fold mechanism which amplifies phase-noise to the extent that the noise may eventually be larger than the signal itself. Chaos is generally produced by two types of systems: autonomous free running systems and non-autonomous externally excited systems. The Duffing oscillator is one of the famous nonautonomous chaos generators and is described by the set of equations

$$\ddot{x} + k(x^2 - 1)\dot{x} + x = A\sin(\omega t + \phi) \tag{2}$$

Figures 6(a) and 6(b) show the phase polar plots constructed from the two signals $x(t)$ and $\dot{x}(t)$ obtained via numerical simulations of the Duffing oscillator in chaotic mode. While the sinusoidal excitation $0.5\sin(\omega t + 105^0)$ is clearly visible in the phase plot of $x(t)$ (Fig. 6(a)), it cannot be distinguished from other components in the phase plot of $\dot{x}(t)$ (Fig. 6(b)). In fact some components in Fig. 6(b) are larger than the excitation itself which indicates that for such a chaotic system, mixing and noise amplification is performed much better in the $\dot{x}(t)$ space direction. It is possible to use the same best-fit circle phase noise measurement procedure explained above to quantify the efficiency of this system in generating chaos; i.e. the ratio of the generated chaos noise to the necessary excitation amplitude $A = 0.5$.



(a)



(b)

Figure 6: Phase polar plots of the two states of a forced Van der Pol equation in chaotic mode.

On the otherhand, Chua's circuit equations form a famous set of autonomous chaos generating equations. They are given by

$$\omega_r \dot{x} = (1-k)x - y + kz \tag{3a}$$
$$\dot{y} = \omega_r (x - \Delta y) \tag{3b}$$
$$\omega_r \dot{z} = \frac{1}{\varepsilon_c}[k(x-z) - f(z)] \tag{3c}$$
$$f(z) = \begin{cases} A & z \le -1 \\ -Az & -1 < z < 1 \\ -A & z \ge 1 \end{cases} \tag{3d}$$

Numrical simulations of the above system produce the well-known double-scroll attractor, which is a 3-dimensional fractal structure. The phase polar plots for $x(t)$, $y(t)$ and $z(t)$ are shown in Fig. 7. Note that the amplitudes of the $y(t)$ phase components are very low compared to $x(t)$ and $z(t)$ and that the best fit circle with larger radius is that for Fig. 6(a).

Also note that the component with largest amplitude (=0.3) in Fig. 6(a) is located at 335^0 while in Fig. 6(c) it is located 180^0 earlier i.e. at 155^0 and with the same amplitude.

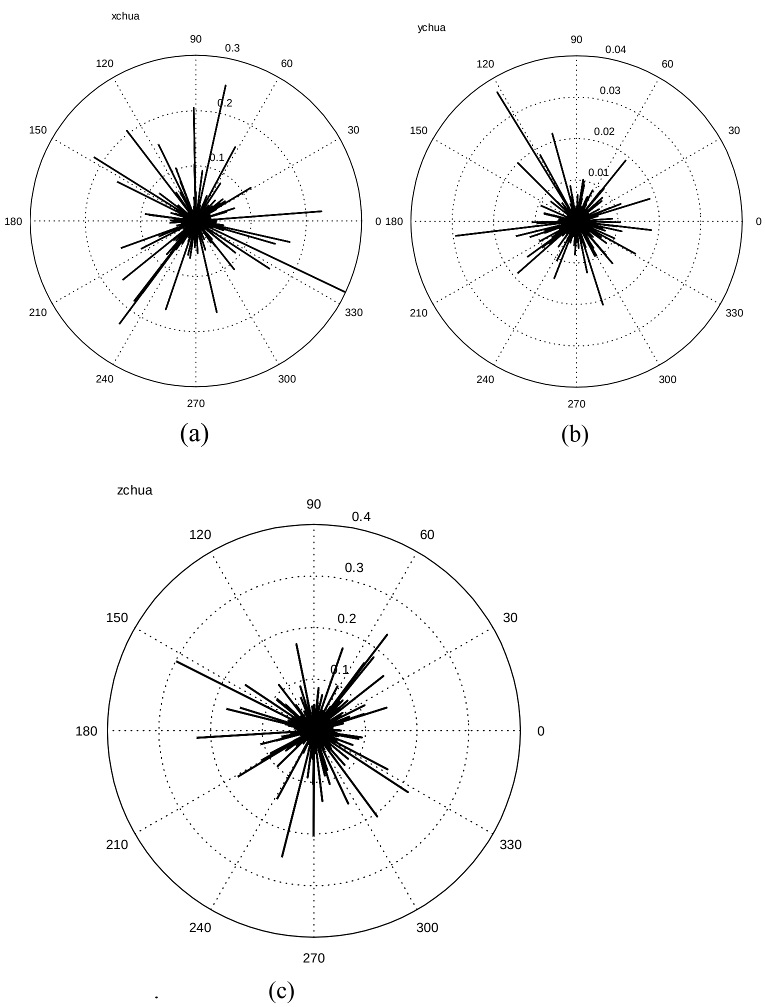


Figure 7: Phase polar plots for Chua's system

III. Conclusion

We have introduced the phase polar as a useful signal analysis and identification tool. Measuring phase noise and THD from this plot is particularly more relevant than using traditional power spectrum.

References

[1] T. Yalcinkaya and Y. Lai, "Phase characterization of chaos," *Physical Review Letters*, vol. 79, pp. 3885-3888, 1997.

[2] L. Trulla, A. Giuliani, J. Zbilut and C. Webber, "Recurrence quantification analysis of the logistic equation with transients," *Physics Letters A*, vol. 223, pp. 255-258, 1996.

[3] A. Hajimiri and T. H. Lee, "A general theory of phase noise in electrical oscillators," *IEEE J. Solid-State Circuits*, vol. 33, pp. 179–194, 1998.

[4] J.Tang, D. Kasperkovitz and A.Roermund, "Fast Oscillator Phase Noise Estimation Based on AC Analysis," *Integrated Circuits and Signal Processing*, vol. 45, pp. 27-36, 2005.

[5] Physiologic Signal Archives for Biomedical Research (http://www.physionet.org/physiobank)