A Novel Fast Computation without Divisions for MMSE Equalizer and Combiner

Khoirul Anwar, Masato Saito, Takao Hara, Minoru Okada and Heiichi Yamamoto

Graduate School of Information Science, Nara Institute of Science and Technology (NAIST)
8916-5 Takayama, Ikoma, Nara 630-0192 Japan Tel: +81-742-73-5348 Fax: +81-742-73-5348

***Abstract* — In this paper, we propose a novel fast and low complexity algorithm of computation for minimum-mean-square-error (MMSE) equalizer or combiner without divisions. Multiplicative effect of fading channel should be compensated by divisions at the receiver. Therefore, equalizer or combiner at the receiver is derived by inverting the channel impulse responses. Here, the number of divisions equals to the number of subcarriers. For the next generation with high bit rate applications, these divisions are necessary to be computed in a very short time and may impact to the increasing of hardware complexities. The main contribution of this paper is a proposed fast algorithm by replacing the large number of divisions with multiplications and subtraction due to its lower complexity. We improve the performance of Newton-Raphson Method by a *range extension* so that the Newton-Raphson Method is applicable for MMSE computation with small number of iterations. Our results in Carrier Interferometry Orthogonal Division Multiplexing (CI/OFDM) confirm that with only two iterations, performance of the proposed algorithm can achieve the similar performance as the normal computation with divisions.**

***Index Terms*— OFDM, MC-CDMA, MMSE, Complexity, Combiners, Fast Algorithm, Newton-Raphson Method, Carrier Interferometry**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is robust to the effect of frequency selective fading channel but weak to the Doppler spread effects [1]. One of the simple solutions is employing an *equalizer* at the receiver for recovering the corrupted signals. In the case of using spreading codes such as CDMA, MC-CDMA and other Spread OFDM, a *combiner* is playing a very important role at the receiver.

All equalizers and combiners need a division operation because the fading channel has a multiplicative effect. Therefore, for recovering the received data, a number of divisions are required at the receiver.

In comparison to other basic arithmetic operations, such as addition, subtraction and multiplication, the division is far more complex and expensive. The operation of division can not be computed directly by adding differently right-shifted terms of the input data, while the multiplication operation still can be done in a relatively straightforward way using combination of adders.

In this paper, we propose a fast computation of equalizers or combiners by improving the performance of Newton-Raphson Method with a *range extension* to obtain faster computation and low complexity (only 1 or 2 iterations). When the channel impulse response is obtained from the channel state information (CSI) module, we insert it to the mapping table (look-up table) of inversion to extract the initial values of combiner's weight without doing any divisions.

The result is still far from the expected value. Consequently, the Newton-Raphson Method is then employed to obtain the most nearest value. Unfortunately, this method needs more than 15 iterations to obtain the performance as the divisions. Therefore, we propose *a range extension* to improve the performance of computation so the required iteration can be reduced significantly. Our results proved that with only 2 iterations, we can obtain the same bit-error-rate (BER) performance as that of by computations with normal division.

II. SYSTEM MODEL OF A COMBINER

Because the weight's value of combiners and equalizers is same, for the reason of simplicity; in this paper we use the term "combiner" for representing both combiners and equalizers. The result is easily can be adapted to all kinds of combiner or equalizer for example as presented in [2]. In this paper, we consider a receiver model of Carrier Interferometry OFDM (CI/OFDM) which the complexity has been reduced significantly by [3]–[5].

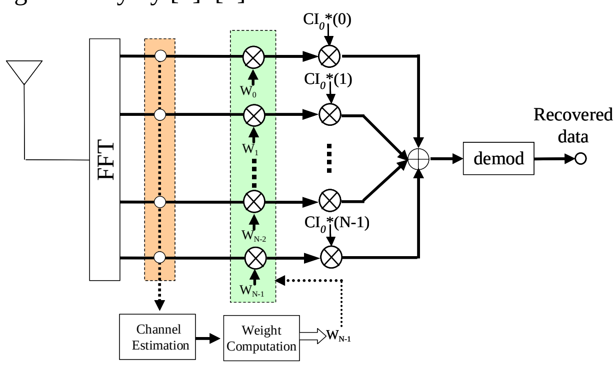

Fig. 1. Computation of weight combining and its combining process

In CI/OFDM receiver, despreading process using complex conjugate of CI Code (CI*) is performed. Based on the channel impulse response that has been obtained from the channel estimation module, the weight computation is performed. The output weight is then multiplied to the received signals in each subcarrier, combined and finally demodulated.

In additive white Gaussian noise (AWGN) channel, or flat fading channel, the optimal combiner is equal gain combining (EGC). However, EGC is not optimal strategy in frequency selective fading channel. Some equalizer were proposed such as orthogonal restoration combining (ORC), controlled equalization combining (CEC), threshold detection combining (TDC), and minimum mean square error (MMSE) combining [1-2].

MMSE is a sub-optimum solution that provides performance that is close to maximum likelihood (ML) method, but has lower complexity. The MMSE combiner (MMSEC) combines all components so that the minimum mean square error between received and desired signal is minimized. Weighting value which is derived from the MMSE criteria is given as

$$W(k) = \frac{H^*(k)}{|H(k)|^2 + \sigma} \quad (1)$$

where k is the number of subcarriers, $H(k)$ is channel impulse response (obtained from channel estimation method, we assume perfect channel estimation), $H^*(k)$ is the complex conjugate of $H(k)$ and σ is the variance of noise. Without loss of generality, in this paper, we select the MMSE combiner as an example of combiners for showing how our proposed algorithm works well.

From (1), it is clear that the combiner requires k computations of divisions that is proportional to the number of subcarriers. Fig. 2 shows the channel response of a fading channel model, weight of ORC and MMSE with high and low noise level.

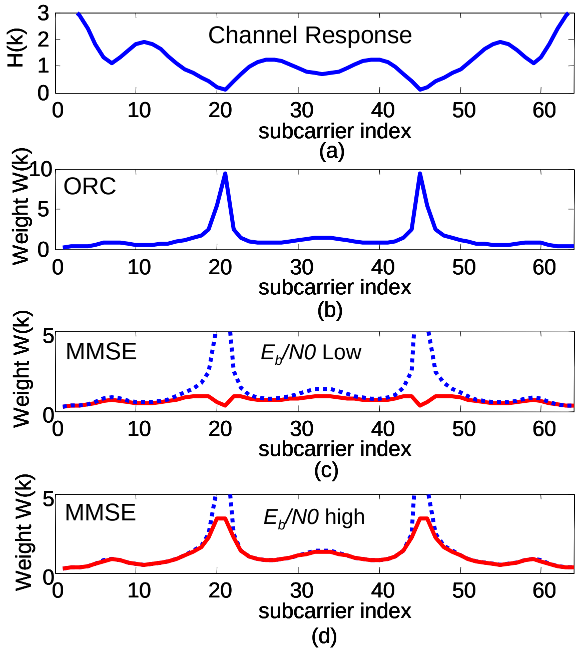

Fig. 2. (a). Channel impulse response (b). Combining weight of ORC (c). Combining weight of MMSEC with high noise level (d). Combining weight of MMSEC with low noise level

III. Proposed Computation Algorithm

Inspired from the idea of Joseph Raphson (1678-1715) [6] who proposed a method which avoided the substitutions in Newton's approach [7], we propose the algorithm of selecting the best value as an input in Newton-Raphson Method. The key point of our idea is starting Newton-Raphson Method with a better approximation of MMSE value by a range extension, so that the number of iterations can be reduced and suitable for high speed wireless communications system.

The proposed algorithm consists of two parts i.e. look-up table with range extension and the Newton-Raphson Method. The orientation of the proposed algorithm is on how simple the algorithm can be implemented in hardware.

A. Newton-Raphson Method

Isaac Newton (1643-1727) in [7] around year 1669 proposed a new algorithm to solve a polynomial equation (called Newton's approach). To obtain an accurate root he used an approximation and substitution. While in 1690, a new step improvement was made by Joseph Raphson [6] which avoided the substitutions in Newton's approach. Raphson's contribution then has shown a better approximation of Newton's approach, which this method is then called Newton-Raphson Method [8].

Newton-Raphson Method start to guess the value of x_{n+1} from the value of x_n as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (2)$$

Carefully observing (1), the inverse of channel response is a division of 1/h, where h is channel response. So, a function that has a root of 1/h can be easily obtained as

$$f(x) = \frac{1}{x} - h = 0 \quad (3)$$

$$f'(x) = -\frac{1}{x^2} \quad (4)$$

By inserting (3) and (4) to (2), we obtain

$$x_{n+1} = x_n - \frac{\left(\frac{1}{x_n} - h\right)}{\left(-\frac{1}{x_n^2}\right)} = x_n + (x_n - hx_n^2) \qquad (5)$$

$$x_{n+1} = x_n\left(2 - hx_n\right) \qquad (6)$$

Equation (6) is the final equation that we need for approximating the inverse channel response of channel *h*. It is clear that one division can be replaced by *two multiplications* and *one subtraction*, where *n* is the number of iteration.

Comparing (1) and (6), to obtain *W(k)*, first we set $h=|H(k)|^2+\sigma$, then the result of x_{n+1} is multiplied by *H*(k)*. When comparing to the ORC case, which is defined as *1/h*, the MMSE weight need more one addition and one multiplication.

B. Look-Up Table of the Inverse of CSI

The *look-up table* is very important to obtain the initial value of MMSE's weight without doing any divisions. Instead of this, we just do a mapping function from the channel response value in the register from the CSI value. The example of look-up table is shown in Table I.

The output value was obtained from the mean of division of the input. As an example, with input channel response of 5, the output should be 1/5 = 0.2. The value of 0.2 is then saved as the mapped value of CSI inversion.

C. Proposed Range Extension in MMSE Computation

From Table I, it can be observed that the maximum value of the inverse channel response (output) is always 1. Obviously, the mapping function in Table I is not good enough for obtaining better start approximation for the Newton-Raphson Method. The reason is that the range of inverse channel response is only from 0 (nearly 0) to 1, while the true inverse channel response can be more than 1 (>1) when the level of noise is very low as described in Figs. 2(b) or 2(d).

TABLE I
LOOK-UP TABLE WITHOUT RANGE EXTENSION

Input	Register	Expected value	Output
1	0000 0001	1	1
2 - 3	0000 001X	1/2 – 1/3	0.4
4 -7	0000 01XX	1/4 – 1/7	0.2
8 – 15	0000 1XXX	1/8 – 1/15	0.09
16 – 31	0001 XXXX	1/16 – 1/31	0.05
32 – 63	001X XXXX	1/32 – 1/63	0.02
…	…	…	…

TABLE II
LOOK-UP TABLE WITH RANGE EXTENSION

Input	Register	Expected value	Output
128x(1)	000 0000 0001	128	128
128x(2–3)	000 0000 001X	128/2–128/3	51
128x(4-7)	000 0000 01XX	128/4–128/7	26
128x(8–15)	000 0000 1XXX	128/8–128/15	12
128x(16–31)	000 0001 XXXX	128/16–128/31	6
128x(32–63)	000 001X XXXX	128/32–128/63	3
128x(64-127)	000 01XX XXXX	128/64-128/127	1.5
128x(128-255)	000 1XXX XXXX	128/128-128/255	0.5
128x(256-511)	001 XXXX XXXX	128/256-128/511	0.3
128x(512-1023)	01X XXXX XXXX	128/512-128/1023	0.18
128x(1024-2047)	1XX XXXX XXXX	128/1023-128/2047	0.09
…	…	…	…

Due to this reason, our proposal is to extent the range of mapping table for covering higher level of CSI inverse by adding one additional step before Newton-Raphson iteration, called *range extension*.

The reason of range extension can be describe below. Let us try by *h* as a channel response. Then weight value is

$$w = \frac{1}{h} \qquad (7)$$

then we multiply *h* with a constant *c*, we have *h'*= *h* x *c*. Suppose that

$$w' = \frac{1}{h'} \qquad (8)$$

we then get

$$w' = \frac{1}{h'} = \frac{1}{h \times c} = \left(\frac{1}{h}\right) \cdot \frac{1}{c} = \frac{w}{c} \qquad (9)$$

so that we obtain

$$w = w' \times c \qquad (10)$$

Equation (7)-(10) allow us to do a multiplication to the input (channel *h*), then (10) clarified that to obtain the same value we should multiply *w'* with the same constant *c*. The algorithm is then described as:

First, multiply the input with a constant value *c*. In Table II, we use constant value of c=128 (=2^7). With this value, it is easy to perform a multiplication only by bit shifting to the left.

Secondly, map the input with the conversion value as shown in Table II.

Third, perform Newton-Raphson Method and its iteration.

Finally, multiply the results with a constant *c* (as in the first step).

IV. Numerical Results

A. Simulation Conditions

For analyzing the performance of the proposed algorithm, we use the CI/OFDM system with the simulation parameter as in [3], [4] and shown in Table III. FFT point is 128 and MMSE combiner is performed for combining the spread data. The channel model is frequency selective model of bad urban (BU) COST-207 fading model [9]. *Ts* in Fig. 3 is time distance between two samples of channel impulse response.

TABLE III
SIMULATION CONDITIONS

	Parameter	Value
Transmitter	Modulation	QPSK
	Subcarriers	128
	Oversampling	4
	GI Length	32
	Channel Coding	Off
	Spreading Codes	Carrier Interferometry [3], [4], [5]
Channel	BU Cost 207 Fading Model	
Receiver	Channel Estimation	Perfect
	Combiner	MMSE

B. Accuracy of the Approximation

Fig. 4 shows the weighting value of MMSE when the noise level is high (*Eb/No*=0dB). The initial value is obtained from Table II. Because the noise level is high, the inverse channel response is not high (less then 2.0). Values with iteration 2 are close to the ideal one. The result with iteration more 3 are exactly same as the original results.

Fig. 5 describes the performance of Newton-Raphson Method when channel response has low noise level ($E_b/N0$ = 30dB). Here, our results confirm that the high value of MMSE's weight (when noise level is low) can be obtained if the number of iterations is more than 15. However, with the proposed range extension, iteration of 2 is enough to obtain the same performance as the original performance by divisions. It is clear that range extension is required for performing high value of MMSE weight. Therefore, only Table II should be used for these approximation, because Table I is limited to maximum value of 1.0. We can conclude that the proposed range extension is very efficient to reduce the number of iteration in Newton-Raphson Method.

C. BER Performance

The bit-error-rate performance (BER) of MMSE combiner with our proposed algorithm is shown in Fig. 6. BER performance of Newton-Raphson Method without range extension meet flat error rate at BER level of $2x10^{-2}$, while with the proposed range extension (iteration 1) is degraded by about only 2dB at BER level of 10^{-5}.

BER of the proposed algorithm with iteration of more than 2 is quietly similar to that of original MMSE weight computations with division. It can be concluded that the proposed method with iteration 1 or 2 is very efficient and faster for reducing the complexity of MMSE weight computations, especially for high bit rate applications.

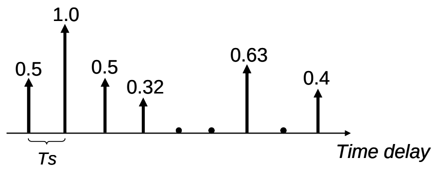

Fig. 3. Delay profile of Bad Urban COST-207 Fading Model

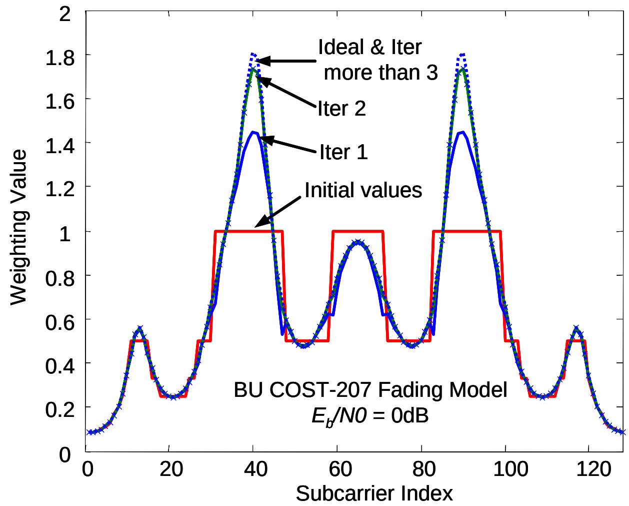

Fig. 4. Accuracy of Newton-Raphson Method in the computation of MMSE's weight when the noise level is high.

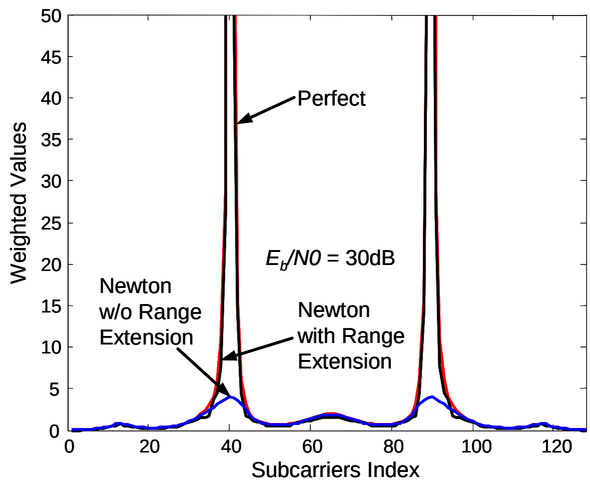

Fig. 5. Accuracy of Newton-Raphson Method in computation of MMSE's weight when the noise level is low.

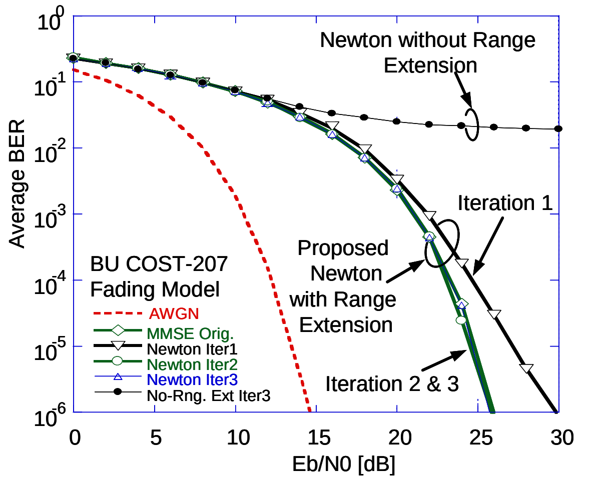

Fig. 6. BER performance of MMSEC using Newton-Raphson Method with and without range extension in CI/OFDM system

TABLE IV
COMPUTATIONAL COMPLEXITY REDUCTION OF MMSE

Arithmetic Operations	Original Computations	Proposed Computation	
		1 iteration	2 iterations
Div.	1	0	0
Mul.	2	4	6
Add/Sub.	1	2	3
Error	-	2dB at BER or 10^{-5}	0

Div. = Divisions, Mul. = Multiplication, Add/Sub.=Addition/Subtraction

D. Complexity Reduction

Table IV shows the computation of MMSE combiner with the proposed computation method. The increasing of iteration requires 2 additional multiplications and 1 subtraction, but this computation is not "heavy" compared with division and capable of supporting the computation process in a very short time. Multiplication for performing range extension can be ignored because it is the multiplication with a constant c = 128 (2^7) which can be performed simply by bit shifting to the right by 7 bits.

VII. CONCLUSIONS

A fast algorithm for computing MMSE combiner's weight without divisions has been proposed. The range extension is required for obtaining "smooth" values so that the results are very close to the expected values, so the process of Newton-Raphson Method requires only 2 iterations (without degradation) or one iteration with very small degradation (2dB on BER level of 10^{-5}). The results confirm that the proposed method is very efficient and faster for performing the combiner or equalizer's computation by replacing divisions with some multiplications and subtractions.

ACKNOWLEDGEMENT

The authors wish to acknowledge the JSAT Corp., Tokyo, ICF Scholarship and 21COE NAIST Project.

REFERENCES

[1] S. Hara and R. Prasad, "Overview of multicarrier CDMA," *IEEE Communications Mag.* Vol. 35, No. 12, pp. 126-133, Dec. 1997.

[2] T. Sao and F. Adachi, "Comparative study of various frequency equalization technique for downlink of a wireless OFDM-CDMA system", *IEICE Trans. On Comm.* Vol. E86-B, No.1, January 2003.

[3] K. Anwar, M. Saito, T. Hara, M. Okada and H. Yamamoto, "Simplified realization of carrier interferometry OFDM by FFT algorithm", *IEEE APWCS2005*, Hokkaido, Japan, pp. 199-203, August 2005.

[4] K. Anwar, M. Saito, T. Hara, M. Okada and H. Yamamoto, "Simplified realization of pseudo-orthogonal carrier interferometry OFDM by FFT algorithm", *IEEE MC-SS2005*, Oberpfaffenhofen, Germany, September 2005.

[5] K. Anwar and H. Yamamoto, "A New Design of Carrier Interferometry OFDM with FFT as Spreading Codes", *IEEE Radio & Wireless Symposium (RWS2006)*, January 2006.

[6] Joseph Raphson, "*Analysis Aequationum universalis*", London, 1690.

[7] Isaac Newton, "*Methodus fluxionum et serierum infinitarum*", 1664-1671.

[8] W. H. Press, W. T. Vetterling, S. A. Teukolsky and B. P. Flannery, "Numerical Recipes in C++", 2nd Edition, Cambridge University Press, 2002.

[9] M. Patzold, "Mobile Fading Channel", John Wiley & Sons, 2002.