A New Eigen Decomposition Based Algorithm for Beamforming in CDMA Communication Systems

Farhad Tavassoli, Bahman Abolhassani, and Homayoon Oraizi

College of Electrical Engineering, Iran University of Science and Technoligy, Tehran 16188, Iran

***Abstract* — A new adaptive beamforming method for direct sequence code division multiple access (DS-CDMA) communication systems is proposed, which has low computational load. The proposed method needs only 2*N*+1 complex multiplications and 2*N*-1 complex additions, where *N* is the number of antennas in the array. We evaluate the performance of the proposed method in estimating the angle-of-arrival (AOA) of the desired signal and in producing the array beam pattern. Moreover, the performance of the proposed method in alleviating the probability of bit error in both fading and non-fading channel is verified. It is observed that although the proposed method has low computational load, nevertheless it has desirable performance.**

***Index Terms* — Beamforming, CDMA, Eigenvector Decomposition, Smart Antenna, Adaptive Antenna.**

I. Introduction

The effects of noise, interference, and fading in communication systems can be reduced by using beamforming [1-4]. Several methods have been presented for beamforming by smart antennas in different communication systems [5-10]. Although these methods lead to satisfactory results, they cannot be employed in real-time communication systems, because of their computational complexity. Another main drawback of these methods is the large number of antennas employed by these methods [11-12]. To have satisfactory results, the number of antennas in the array should be more than the number of signal sources [11-12]. Considering actual networks, which have a large number of signal sources (active users) and therefore require a large number of antennas in the base stations, it is impractical to use these methods.

Extensive researches have been done to alleviate these two problems and to propose beamforming methods, which can be used in real-time communication systems [11-13]. In [11], a non-iterative beamforming method is presented for CDMA-based IS-95 communication systems. This method maximizes the ratio of the signal to interference plus noise. Although the method presented in [11] is less time consuming than conventional beamforming methods, it is still too time consuming to be used in real-time systems.

It is shown, in [12], that if the signal of the desired user is much stronger than the interference signals, the process of beamforming would be equivalent to the process of finding the largest eigenvector of the covariance matrix of the received signal. The largest eigenvector of a matrix is the eigenvector corresponding to the largest eigenvalue of that matrix. In this case, if we use a uniform linear array (ULA) in the base station (BS), the desired weight vector for maximizing the SNIR would be equivalent to the largest eigenvector of the covariance matrix. Therefore, the procedure of finding the desired weight vector (beamforming procedure) could be considered as a procedure of finding the largest eigenvector of the covariance matrix.

Moreover, it is shown in [12] that for a direct sequence code division multiple access (DS-CDMA) system, the desired signal power after despreading is much larger than each interference signal power. Consequently, in DS-CDMA systems, the process of beamforming can be considered as a process of finding the largest eigenvector of the covariance matrix of the despread signal. Reference [12] uses conjugate gradient method (CGM) to find the largest eigenvector of the covariance matrix. In [13], for the above problem, the Lagrange multiplier method, instead of CGM, is used to find the largest eigenvector. In [14], a time-efficient but effective method for beamforming in CDMA is proposed, which is based on the mathematical work of Oja [15].

In this paper, a new adaptive beamforming algorithm for DS-CDMA communication systems is proposed. This method is based on the work by Xu, presented in reference [16], for finding eigenvectors of a matrix. It is shown that the proposed method needs only 2*N*+1 complex multiplications and 2*N*-1 complex additions for a single run, where *N* is the number of antennas in the array. Therefore, our proposed method has lower complexity than the methods presented in [11-13].

In the next section, we present a mathematical model for DS-CDMA communication systems. Based on this model, we propose our new algorithm in section III. Finally, in section IV, we evaluate the proposed algorithm by computer simulations.

II. Mathematical Model

It is considered that there are *M* active users in the system and a ULA consisting of *N* omni-directional antennas employed by the base station. In a DS-CDMA system, the signal transmitted by the i^{th} user ($s_i(t)$) can be written as

$$s_i(t) = P_i b_i(t) c_i(t)\cos(\omega_c t), \quad (1)$$

where, $b_i(t)$ is the data signal of the i^{th} user with pulse width of T_b, $c_i(t)$ is the spreading code for the i^{th} user with pulse width of T_c, ω_c is the angular frequency of carrier, and P_i is the transmitted power of the i^{th} user. In this paper, P_i's are assumed to be 1, for simplicity. In such a system, the processing gain (G) is defined as

$$G = \frac{T_b}{T_c}. \quad (2)$$

Assuming a channel having one path for each user, then the base-band received signal vector from the i^{th} user at the antenna array ($\mathbf{x}_i(t)$) is

$$\mathbf{x}_i(t) = b_i(t-\tau_i)c_i(t-\tau_i)e^{j\varphi_i}\mathbf{a}_i, \quad (3)$$

where, $\mathbf{a}_i$ is the channel vector of the i^{th} user, (obtained in [18]), τ_i is the delay of the i^{th} user's path, and $\varphi_i = \omega_c \tau_i$. Considering M active users in the environment, the total received signal vector (from all users) is

$$\mathbf{x}(t) = \sum_{i=1}^{M} \mathbf{x}_i(t) + \mathbf{n}(t), \quad (4)$$

where, $\mathbf{n}(t)$ is the additive noise vector at the antenna array. The first user (i=1) is considered to be the desired user. In this case, the total received signal vector can be written as

$$\mathbf{x}(t) = b_1(t-\tau_1)c_1(t-\tau_1)e^{j\varphi_1}\mathbf{a}_1 + \mathbf{i}(t) + \mathbf{n}(t), \quad (5)$$

where $\mathbf{i}(t)$ is the interference vector as follows:

$$\mathbf{i}(t) = \sum_{i=2}^{M} b_i(t-\tau_i)c_i(t-\tau_i)e^{j\varphi_i}\mathbf{a}_i. \quad (6)$$

At the receiver, the output signal of each antenna is multiplied by the desired user's code ($c_1(t-\tau_1)$) and is integrated in a time interval equal to one data pulse width (T_b), in order to extract the transmitted data. We assume that the delay of the desired signal (τ_1) is known at the receiver. In this case, the integral relation to extract the n^{th} transmitted bit is

$$\mathbf{y}_1(n) = \frac{1}{\sqrt{T_b}} \int_{(n-1)T_b+\tau_1}^{nT_b+\tau_1} \mathbf{x}(t) c_1(t-\tau_1)dt, \quad (7)$$

where, $\mathbf{y}_1(n)$ is the extracted signal vector after despreading. Computing the result of the integral, we can rewrite (7) as

$$\mathbf{y}_1(n) = 2\sqrt{T_b}\, b_1(n)e^{j\varphi_1}\mathbf{a}_1 + \mathbf{i}_1 + \mathbf{n}_1, \quad (8)$$

where, $\mathbf{n}_1$ and $\mathbf{i}_1$ are noise and interference vectors after despreading, respectively and can be written as

$$\mathbf{n}_1 = \frac{1}{\sqrt{T_b}} \int_{(n-1)T_b+\tau_1}^{nT_b+\tau_1} \mathbf{n}(t) c_1(t-\tau_1)dt, \quad (9)$$

$$\mathbf{i}_1 = \sum_{i=2}^{M} I_{1,i} e^{j\varphi_i}\mathbf{a}_i. \quad (10)$$

In (10), $I_{1,i}$ is the interference signal from the i^{th} user ($i = 2,3,\ldots,M$), obtained as follow

$$I_{1,i} = \frac{1}{\sqrt{T_b}} \int_{(n-1)T_b+\tau_1}^{nT_b+\tau_1} b_i(t-\tau_i)c_i(t-\tau_i)c_1(t-\tau_1)dt \quad (11)$$

Considering that in a DS-CDMA system, the correlation between two different users' spreading codes is much lower than one code's autocorrelation, the desired signal would be much stronger than each of $I_{1,i}$'s. So, as mentioned before, the process of finding the desired weight vector for array elements (beamfoming) could be considered as a process of finding the largest eigenvector of the despread signal's covariance matrix ($\mathbf{R}_{yy,1} = E\left[\mathbf{y}_1(n)\mathbf{y}_1^H(n)\right]$).

III. Proposed Method

As mentioned before, the process of beamforming in a DS-CDMA system is equivalent to the process of finding the eigenvector corresponding to the largest eigenvalue (i.e. the largest eigenvector) of the covariance matrix of the despread signal ($\mathbf{R}_{yy,1}$). Our proposed beamforming method is based on the algorithm proposed by Xu for finding the eigenvectors of a matrix [16]. So, we define the cost function (for winding the weight vector, $\mathbf{w}$) as

$$J(\mathbf{w}) = -\mathbf{w}^H\mathbf{R}_{yy,1}\mathbf{w} + \mathbf{w}^H\mathbf{R}_{yy,1}\mathbf{w}(\mathbf{w}^H\mathbf{w}-1), \quad (12)$$

where, $\mathbf{R}_{yy,1}$ is the covariance matrix of the despread signal, $\mathbf{w}$ is the weight vector, and $(\,)^H$ denotes the Hermitian transpose. We can rewrite (12) as a simpler form of

$$J(\mathbf{w}) = -2\mathbf{w}^H\mathbf{R}_{yy,1}\mathbf{w} + \mathbf{w}^H\mathbf{R}_{yy,1}\mathbf{w}\mathbf{w}^H\mathbf{w}. \quad (13)$$

By using the steepest descent algorithm for updating the weight vector, we have

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu\nabla J(\mathbf{w}), \quad (14)$$

where $\mathbf{w}(k)$ denotes the weight vector at the k^{th} sampling time, μ is a real number which is determined for the convergence of the adaptive procedure, and $\nabla J(\mathbf{w})$ is the gradient vector of $J(\mathbf{w})$ with respect to $\mathbf{w}$. Considering (13), the gradient vector ($\nabla J(\mathbf{w})$) is

$$\nabla J(\mathbf{w}) = -(2\mathbf{R}_{yy,1}\mathbf{w} - \mathbf{w}\mathbf{w}^H\mathbf{R}_{yy,1}\mathbf{w} - \mathbf{R}_{yy,1}\mathbf{w}\mathbf{w}^H\mathbf{w}). \quad (15)$$

Therefore, the updating relation for the weight vector would be

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu(2\mathbf{R}_{yy,1}\mathbf{w} - \mathbf{w}\mathbf{w}^H\mathbf{R}_{yy,1}\mathbf{w} - \mathbf{R}_{yy,1}\mathbf{w}\mathbf{w}^H\mathbf{w}). \quad (16)$$

If we impose the criterion $\mathbf{w}^H\mathbf{w} = 1$ (for normalizing the array gain) to (16), we would have

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu(\mathbf{R}_{yy,1}\mathbf{w} - \mathbf{w}\mathbf{w}^H\mathbf{R}_{yy,1}\mathbf{w}). \quad (17)$$

Now, if we replace $\mathbf{R}_{yy,1}$ by $\mathbf{y}_1\mathbf{y}_1^H$, the updated weight vector can be written as

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\left(\mathbf{y}_1\mathbf{y}_1^H\mathbf{w}(k) - \mathbf{w}(k)\mathbf{w}^H(k)\mathbf{y}_1\mathbf{y}_1^H\mathbf{w}(k)\right) \quad (18)$$

Here, we define the complex parameter of α as follow

$$\alpha = \mathbf{y}_1^H\mathbf{w}. \quad (19)$$

Considering the definition of α in (19), we could rewrite (18) as

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\left(\alpha\mathbf{y}_1 - |\alpha|^2\mathbf{w}(k)\right) \quad 20)$$

(20) is the updating equation for finding the new weight vector ($\mathbf{w}(k+1)$).

At last, the beamformer output, y, is obtained from multiplying the despread signal vector by the weight vector, i.e.

$$y(k+1) = \mathbf{w}^H(k+1)\mathbf{y}_1(k+1). \quad (21)$$

For starting the beamforming process, one proper initial value is for the weight vector, $\mathbf{w}(0)$, is

$$\mathbf{w}(0) = \frac{1}{\sqrt{N}}\frac{\mathbf{y}_1(0)}{\|\mathbf{y}_1(0)\|}. \quad (22)$$

where, $\mathbf{y}_1(0)$ is the value of the despread signal vector at the first sampling time, N is the number of antennas in the array, and $\| \ \|$ denotes the norm of the vector. The term $\frac{1}{\sqrt{N}}$ and the normalization are used to have a unit gain in the direction of the desired signal.

The flowchart of the proposed method is illustrated in Fig. 1.

The number of complex multiplications and complex additions needed for a single run of the proposed method is shown in Table I. We can observe from Table I that each run of the proposed method needs only 2N+1 complex multiplications and 2N-1 complex additions. It means that our proposed method has lower computational load than the methods presented in [11-13].

TABLE I
NUMBER OF COMPLEX OPERATIONS NEEDED FOR A SINGLE RUN OF THR PROPOSED METHOD

	Number of complex multiplications	Number of complex additions
$\alpha = \mathbf{y}_1^H\mathbf{w}$	N	N-1
$\beta = \alpha \times \alpha^*$	1	0
$\mathbf{w} = (1-\mu\beta)\mathbf{w} + \mu\alpha\mathbf{y}_1$	N	N
Total	2N+1	2N-1

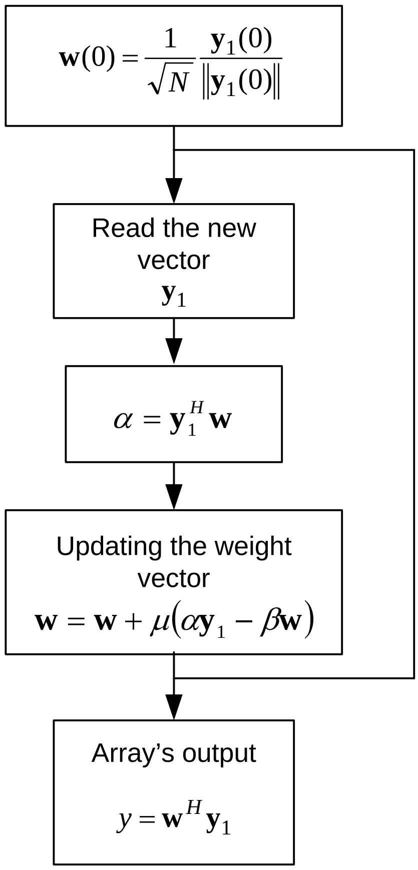

Fig. 1. Flowchart of the proposed method

IV. Simulation Results

In this section, we evaluate the performance of the proposed method by computer simulations. In our simulations, m-sequences [17] are used as the users' spreading codes. Moreover, we use binary shift keying (BPSK) modulation as the data modulation. The array used in the base station (BS) is uniform linear array (ULA) with half a wavelength distance between elements. Furthermore, we assume that the channel noise is additive white Gaussian and, for a fading channel, we assume the Rayleigh fading.

First, we verify the AOA estimation quality and the produced array beam pattern. We assume that there are 20 undesired users uniformly distributed in the cell. The SNR and the processing gain are assumed to be 4 dB and 31, respectively. A four antenna array is used at BS.

Fig. 2 shows the array beam pattern, when the AOA of the desired signal is 10 degrees. We can see from Fig. 2 that the estimated AOA for the desired user is 10.1 degrees. It means that the difference between the exact AOA and the estimated one is only 0.1 degrees. In this case, the ratio of the main lobe magnitude to that of the largest side lobe (SLL) is 13.5.

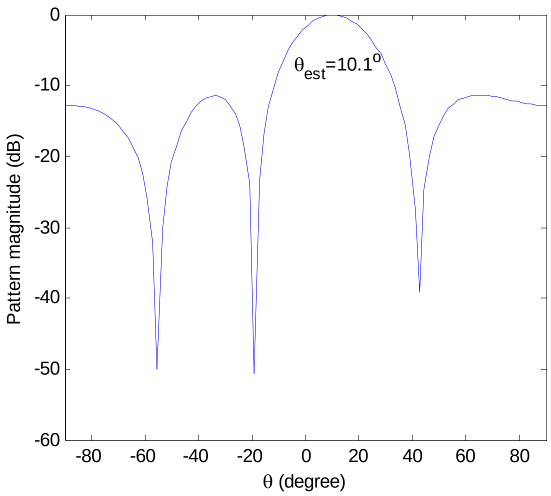

Fig. 2. Pattern of array after beamforming for AOA=10 degree. 21 active users, SNR=4dB, *G*=31, and 4 antennas in the array

The estimated AOA, standard deviation of the estimation and the SLL of the produced array pattern, for different user AOAs, have been illustrated in Table II. It is observed from Table II that the difference between the exact AOA and the estimated one does not change significantly by increasing the AOA of the desired signal. This difference is less than 0.1 degrees for AOA=0 degree and is 0.1 degrees for other AOAs. Considering Table II, we can see that the standard deviation of the estimation increases by increasing the AOA of the desired user. The standard deviation reaches from 1.0 for AOA=0 degree to 2.8 for AOA of 60 degrees. It means that the quality of the estimation is reduced by increasing the AOA of desired user.

Moreover, we can observe from Table II that the SLL of the array beam pattern decreases by increasing the AOA. The SLLs for AOAs below 30 degrees is more than 13, which is a satisfactory value. For AOA of 40 degrees, this ratio (SLL) is reduced to 7.3 and for AOAs of 50 and 60 degrees, the SLL reaches to small values of 2.7 and 1.25, respectively.

TABLE II

THE VALUES OF ESTIMATED AOA, STANDARD DEVIATION, AND SLL

AOA (degree)	Estimated AOA (degree)	Standard deviation (degree)	SLL
0	0.05	1.0	13.5
10	10.1	1.6	13.5
20	20.1	2.2	13.3
30	29.9	2.4	13.2
40	40.1	2.5	7.3
50	50.1	2.8	2.7
60	59.9	2.8	1.25

Now we investigate the convergence rate of the proposed method. For this purpose, we consider the number of sampling times needed for the difference between the estimated AOA and the exact AOA to become less than 1 degree, as the number of sampling times needed for the convergence of the proposed method. The number of sampling times needed for convergence of the proposed method is shown in Table III, for different values of AOA. It is observed that the number of sampling times needed for convergence of the proposed method increases when the AOA of the desired signal becomes larger. This increase is from 15 for AOA=0 degree to 41 for AOA=60 degrees. In other words, the proposed method needs more time to converge for larger AOAs.

TABLE III

NUMBER OF SAMPLING TIMES NEEDED FOR CONVERGENCE OF THE PROPOSED METHOD

AOA (degree)	Number of sampling times
0	15
10	32
20	39
30	40
40	40
50	41
60	41

Now we evaluate the performance of the proposed method in alleviating the probability of bit error (P_{be}). We consider a propagation environment consisting of 21 active users (1 desired user and 20 undesired ones). The processing gain (*G*) is 31. First, we verify the proposed method capability in reducing P_{be} in a non-fading channel. Fig. 3 shows the probability of bit error (P_{be}) vs. Number of antennas in the array, for 3 different SNRs of 1.5, 4, and 7dB. It is observed from Fig. 3 that for SNR=7 dB, we can achieve the P_{be} less than 10^{-6} by using 6 antennas in the antenna array. For SNR=4 dB, $P_{be} = 10^{-6}$ can be achieved by using 10 antennas in the array. But, for SNR=1.5 dB, using 10 antennas in the array leads to the P_{be} near 10^{-4}. In the case of not using beamforming (using single antenna), P_{be} s for SNRs of 1.5, 4, and 7 dB are 0.27, 0.06, and 0.016 respectively.

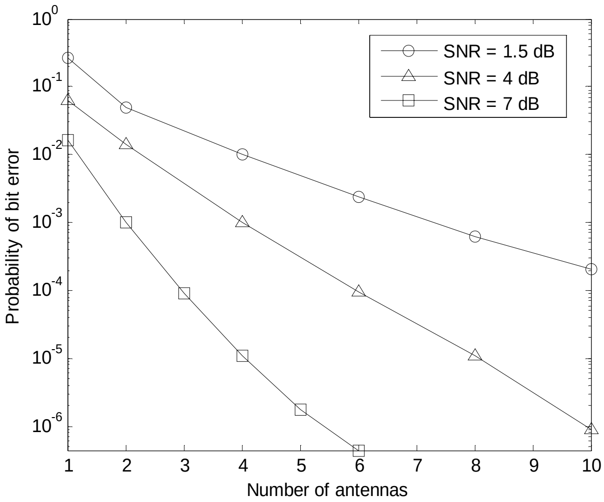

Fig. 2. Probability of bit error vs. the number of antennas in the array, for the non-fading channel. 20 interference signals, and *G*=31.

The probability of bit error (P_{be}) vs. the number of antennas in the array, for a fading channel, has been illustrated in Fig. 4. In this case, the environment characteristics is the same as previous example (i.e. 20 interference signals, *G*=31). Moreover, the channel has a maximum Doppler frequency of 100 Hz. We assume that the channel does not change during each symbol time, but it can change from one symbol time to another. As the previous section, P_{be} has been shown for different SNRs of 1.5, 4, and 7 dB. It is observed from Fig. 4 that although propose method reduces P_{be} in a fading channel, this reduction is less than the case of non-fading channel. In the case of SNR=7 dB, for the fading channel, the P_{be} less than 10^{-2} can be achieved by using 10 antennas in the array. For SNR=7 dB, in the fading channel characterized before, the P_{be} without beamforming is 0.08. In the specified fading channel the P_{be} s for SNRs of 1.5 and 4dB decrease from 0.18 and 0.12, for the case of no beamforming (using single antenna), to 0.038 and 0.019 for the case of using 10 antennas in the array, respectively

V. Conclusions

In this paper, we proposed a novel adaptive beamforming algorithm for DS-CDMA communication systems. This method works based on Xu algorithm for finding the eigenvectors of a matrix. The proposed method needs only 2*N*+1 complex multiplications and 2*N*-1 complex additions for a single run, where *N* is the number of antennas in the array. Therefore, our proposed method has the lowest computational load among other presented beam forming methods.

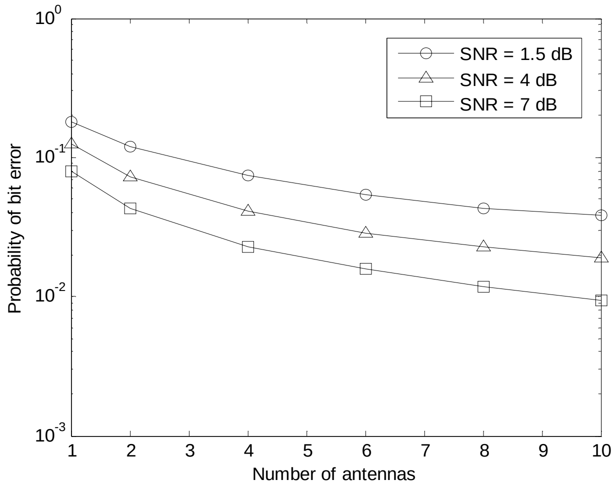

Fig. 3. Probability of bit error vs. the number of antennas in the array, for the fading environment with maximum Doppler frequency of 100 Hz, 20 interference signals, and *G*=31.

It was shown that the proposed method has satisfactory performance in estimating the AOA of the desired signal. The difference between the exact AOA and estimated AOA is 0.1 degrees, in the worst case. Moreover the proposed method produces high quality beam patterns for AOA below 30 degrees.

Furthermore, we observed that the proposed method can reduce the probability of bit error in both fading and non-fading channel. But this reduction is much less in a non-fading channel

Acknowledgement

We hereby thank Iran Telecommunication Research Center (ITRC) for its partial support of the research reported in this paper.

References

[1] A. Naguib, A. Paulraj, and T. Kailath, “Capacity improvement with base-station antenna arrays in cellular CDMA,” *IEEE Trans. Veh. Tech.*, vol. 43, pp. 691-698., Aug. 1994.

[2] J. H. Winters, “On the capacity of radio communication systems in a Rayleigh fading environment,” *IEEE J. Select. Areas Commun.*, vol. 5, pp. 871-878, June 1987.

[3] S. Anderson, M. Millnert, M. Viberg, and B. Wahlberg, ”An adaptive array for mobile communication systems,” *IEEE Trans. Veh. Tech.*, vol. 40, pp. 230-236, Feb. 1991.

[4] S. Kim, W. Yang, and G. Zu, “On the performance of adaptive smart antenna systems in multipath environments,” *IEEE Proc. Conf. Military Commun.*, Oct.-Nov. 1999, vol. 2, pp. 934-938.

[5] D. H. Johnson and S. DeGraaf, “Improving the resolution of bearing in passive sonar arrays b eigenvalue analysis,” *IEEE Trans. Acoust. Speech, Signal Processing*, vol. ASSP-30, pp. 638-647, Aug. 1982.

[6] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust. Speech, Signal Processing*, vol. 37, pp. 984-995, July 1989.

[7] T. Dendini and G. Y. Delisle, "A nonlinear algorithm for output power maximization of an indoor adaptive phased array," *IEEE Trans. Electromagn. Compat*., vol. 37, pp. 201-209, May 1995.

[8] R. Schmidt and R. Franks, "Multiple DF signal processing: an experimental system," *IEEE Trans. Antennas Propagat.*, vol. 34, pp 281-290, March 1986.

[9] A. Paulraj, R. Roy, and T. Kailath, "Estimation of signal parameters by rotational invariance techniques (ESPRIT)," *Proc. ICASSP'89*, Glasgow Scotland, 1985, vol. IV, pp. 2807-2810.

[10] M. Viberg and B. Ottersten, "Sensor array processing based on subspace fitting," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 39, pp. 1110-1121, May 1991.

[11] W. Y. Shiu and S. D. Blostein, "Adaptive digital beamforming in cellular CDMA systems using noniterative signal subspace tracking," *IEEE Int. Conf. Towards the Knowledge Millennium, ICC 97*, Montreal, Canada, June 1997, vol. 2, pp. 652-656.

[12] S. Choi and D. Yun ,"Design of an adaptive antenna array for tracking the source of maximum power and its application to CDMA mobile communications," *IEEE Trans. Antennas Propagat*., vol. 45, pp. 1393-1404, Sep. 1997.

[13] S. Choi and D. Shim, "A novel adaptive beamforming algorithm for a smart antenna in a CDMA mobil communication environment," *IEEE Trans. Veh. Technol*., vol. 49, pp. 1793-1806, Sep. 2000.

[14] F. Tavassoli, H. Oraizi, and B. Abolhassani, "A simple adaptive beamforming algorithmfor CDMA wireless communication systems," Submitted to ICC'06.

[15] E. Oja, and J. Karhunen, "On stochastic approximation of the eigenvectors and eigenvalues of the expectation of a random matrix," *J. Mathematical Analysis and Applications*, vol. 106, pp. 69-84, Feb. 1985.

[16] L. Xu, "Least MSE reconstruction for self-organization: (II) Further theoretical and experimental studies on one layer nets," *Proc. Int. Conf. Neural Networks*, Singapore, pp. 2368-2373, 1991.

[17] R. L. Peterson, R. E. Ziemer, and D. E. Borth, *Introduction to spread-spectrum communications*. New Jersey: Prentice-Hall, 1995.

[18] A. F. Naguib, "Adaptive antennas for CDMA wireless networks," Ph.D. dissertation, Dept. Elec. Eng., Stanford University, Aug. 1996