# Performance Analysis of Layered Space-Time Codes in Wireless Communications Channels

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***Abstract* — The objective of this paper is to simulate numerically the wireless channel and the V-BLAST architecture to compare the performance of these systems using linear nulling and symbol cancellation along linear nulling for a zero forcing (ZF) and a minimum mean-squared error (MMSE) receiver. We will also compare the performance of V-BLAST and the Successive Cancellation Receiver (SUC) against the QR decomposition as an approximation of V-BLAST, trying to develop a less calculation intensive algorithm. The simulation will compute the frame error rate in both cases for different values of SNR in a flat fading channel. The transmitted symbols will be modulated using a QPSK constellation with 4 transmitters and 6 receivers. The results will be compared to those measured in the laboratory of the optimum V-BLAST.**

***Index Terms* — Layered Space-time Codes, V-Blast, QR Decomposition, Sorted QR Decomposition.**

## I. INTRODUCTION

Applying multiple antennas at both the transmitter and receiver side can greatly improve the capacity and throughput of a wireless communication link in flat-fading [1], as well as frequency-selective fading channels [2], especially when the environment provides rich scattering. The Vertical Bell Laboratories Layered Space-Time (V-BLAST) architecture was introduced as a simplified version of the diagonal BLAST (D-BLAST), which was first introduced by [1]. Layered space-time codes have been designed to exploit the capacity advantage of multiple antenna systems in Rayleigh fading environments. In this paper, we are comparing different codes based on the Layered Space-Time techniques: Successive Cancellation (SUC), Vertical BLAST (V-BLAST), QR Decomposition and Sorted QR decomposition. These kinds of Space-Time codes have been introduced to use space as a second dimension of coding. Layered Space-Time codes are special kinds of Space-Time codes with the advantage of a feasible decoding complexity [1]. The mathematical model for all schemes presented here consists of a single data stream that is demultiplexed into $n_T$ substreams and each substream is then modulated into symbols and fed to its respective transmitter. The $n_T$ transmitters operate co-channel with synchronized symbol timing. Each transmitter is itself an ordinary QPSK transmitter. We assume that transmissions are organized into bursts of $L$ symbols. The power launched by each transmitter is proportional to $1/n_T$ so that the total radiated power is constant and independent of $n_T$. The $n_R$ receivers are, individually, conventional QPSK receivers. These receivers also operate co-channel, each receiving signals radiated from all $n_T$ transmit antennas. For simplicity in the sequel, flat fading is assumed and the matrix channel transfer function $\mathbf{H}$, is zero-mean, white Gaussian distributed.

The organization of the paper is as follows. In section 2, the system description and notations are introduced. In section 3, the capacity of a multi-input multi-output system is given. In order to simplify the derivation, the linear ZF and MMSE with the detection of BLAST systems using the QR decomposition of the channel matrix are investigated in sections 4 and 5, respectively. The simulation results are introduced in section 6, while we conclude in section 7.

## II. SYSTEM DESCRIPTION

In what follows, we take a discrete-time baseband view of the detection process for a single transmitted vector symbol, assuming symbol-synchronous receiver sampling and ideal timing. Let $\mathbf{a} = [a_1 \cdots a_{n_T}]^T$ denote the vector of transmit symbols, then the corresponding received $n_R$-vector is:

$$\mathbf{r} = \mathbf{H} \cdot \mathbf{a} + \mathbf{n} \qquad (1)$$

where $\mathbf{n}$ is a noise vector with components drawn from i.i.d. wide-sense stationary processes with variance $\mathrm{N}_0$. We consider a flat fading multiple–input multiple–output (MIMO) channel as shown in Figure 1, which describes a system with $n_T$ transmit antennas and $n_R$ receive antennas. The tap gain from transmit antenna $j$ to receive antenna $i$ at time $k$ is denoted by $h_{i,j}^k$. Later, we will drop the time index $k$ if the taps are assumed to be constant over the time period considered. The antennas are separated far enough to ensure independently fading channels from each transmit to each receive antenna. Therefore, the channel taps are modeled as independent complex Gaussian random variables of equal variance and satisfy $E\{|h_{i,j}|^2\} = 1$. The symbol transmitted from antenna $j$ is denoted $a_j$. The mean energy per symbol $a_j$ is given by

$$E_j = E\{a_j \cdot \overline{a}_j\} \quad (2)$$

whereas

$$E = \sum_{j=1}^{n_T} E_j \quad (3)$$

is the total energy per use of the MIMO channel. A channel use is defined as the simultaneous transmission of a symbol $a_j$ from all transmit antennas $j = 1, \cdots, n_T$. The observed value at receive antenna $i$ is given by

$$r_i = \sum_{j=1}^{n_T} h_{i,j} \cdot a_j + n_i \,. \quad (4)$$

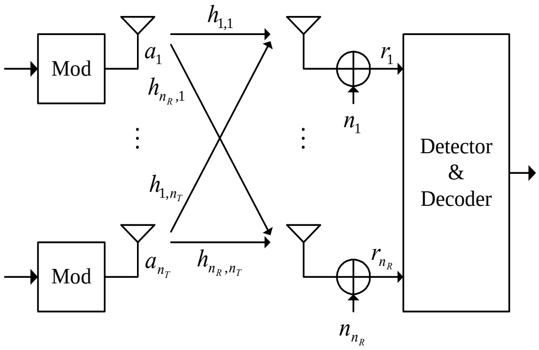


Figure 1: Flat fading MIMO channel model.

The additive noise $n_i$ at each receive antenna $i$ is assumed to be white and Gaussian with spectral power density $N_0$. Now, the expected Signal-to-Noise Ratio (SNR) per receiving antenna, i.e. the SNR for each component of $\mathbf{r}$, can be found and is equal to:

$$\rho = \frac{E_s}{N_0} \quad (5)$$

where $E_s$ stands for the signal power per receive antenna and $\mathrm{N}_0$ denotes the noise power per receive antenna.

## IV. LINEAR DETECTION

The V-BLAST detection algorithm [3] bases on the linear zero-forcing solution, but detects the signals one after another and not in parallel. In order to achieve the best performance, it is optimal to choose always the layer with the largest signal-to-noise-ratio (SNR), or equivalently with the smallest estimation error. The adaptation to the MMSE criterion was presented in [11], where the optimal sequence maximizes the signal-to-interference-and- noise ratio (SINR) in each detection step. The main drawback of the V-BLAST detection algorithms lies in the computational complexity, as it requires multiple calculations of the pseudo-inverse of the channel matrix [3]. By introducing an extended system model, we show the similarity of both criteria. This analogy will play a key role for the introduction of the MMSE based QR detection algorithm.

### *A. Zero-Forcing Detector (ZF)*

In a linear detector, the receive signal vector $\mathbf{a}$ is multiplied with a filter matrix $\mathbf{G}$, followed by a parallel decision on all layers. Zero-forcing means that the mutual interference between the layers shall be perfectly suppressed. This is accomplished by the Moore-Penrose pseudo-inverse (denoted by $(\cdot)^+$) of the channel matrix [5]

$$\mathbf{G}_{\text{ZF}} = \mathbf{H}^+ = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \,, \quad (6)$$

where we assumed that $\mathbf{H}$ has full column rank. The decision step consists of mapping each element of the filter output vector

$$\tilde{\mathbf{a}}_{\text{ZF}} = \mathbf{G}_{\text{ZF}} \mathbf{r} = \mathbf{H}^+ \mathbf{r} = \mathbf{a} + (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{n} \quad (7)$$

onto an element of the symbol alphabet by a minimum distance quantization. The estimation errors of the different layers correspond to the main diagonal elements of the error covariance matrix

$$\mathbf{\Phi}_{\text{ZF}} = \mathbf{E}\left\{(\tilde{\mathbf{a}}_{\text{ZF}} - \mathbf{a})(\tilde{\mathbf{a}}_{\text{ZF}} - \mathbf{a})^H\right\} = \sigma_n^2 (\mathbf{H}^H \mathbf{H})^{-1} \quad (8)$$

which equals the covariance matrix of the noise after the receive filter. It is obvious that small eigenvalues of $\mathbf{H}^H \mathbf{H}$ will lead to large errors due to noise amplification. This effect is especially observed in systems with equal number of transmit and receive antennas. In fact, using a result from random matrix theory [6], it can be shown that in the large system limit for $n_T = n_R \to \infty$ the noise amplification tends to infinity almost surely. In order to improve the performance, the noise term can be included in the design of the filter matrix $\mathbf{G}$. This is done by the MMSE detection scheme, where the filter represents a trade-off between noise amplification and interference suppression.

### *B. MMSE Detector*

The MMSE detector minimizes the mean squared error (MSE) between the actually transmitted symbols and the output of the linear detector and leads to the filter matrix [5]

$$\mathbf{G}_{\text{MMSE}} = (\mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{n_T})^{-1} \mathbf{H}^H \quad (9)$$

The resulting filter output is given by

$$\tilde{\mathbf{a}}_{\text{MMSE}} = \mathbf{G}_{\mathbf{MMSE}} \mathbf{r} = (\mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{n_T})^{-1} \mathbf{H}^H \mathbf{r} \,. \quad (10)$$

The estimation errors of the different layers correspond to the main diagonal elements of the error covariance matrix

$$\begin{aligned} \mathbf{\Phi}_{\text{MMSE}} &= (\mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{n_T})^{-1} \mathbf{H}^H \quad (11) \\ &= \sigma_n^2 (\mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{n_T})^{-1} \mathbf{H}^H \mathbf{r} \,. \end{aligned}$$

With the definition of a $(n_T + n_R) \times n_T$ extended channel matrix $\underline{\mathbf{H}}$ and a $(n_T + n_R) \times 1$ extended receive vector $\underline{\mathbf{r}}$ through

$$\underline{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_{n_T} \end{bmatrix} \text{ and } \underline{\mathbf{r}} = \begin{bmatrix} \mathbf{r} \\ \mathbf{0}_{n_T,1} \end{bmatrix}, \qquad (12)$$

the output of the MMSE filter given by (10) can be rewritten as

$$\tilde{\mathbf{a}}_{\mathrm{MMSE}} = (\underline{\mathbf{H}}^H \underline{\mathbf{H}})^{-1} \underline{\mathbf{H}}^H \underline{\mathbf{r}} = \underline{\mathbf{H}}^{+} \underline{\mathbf{r}} \,. \qquad (13)$$

Furthermore, the error covariance matrix (11) becomes

$$\mathbf{\Phi}_{\mathrm{MMSE}} = \sigma_n^2 (\underline{\mathbf{H}}^H \underline{\mathbf{H}})^{-1} = \sigma_n^2 \underline{\mathbf{H}}^{+} \underline{\mathbf{H}}^{+H} \,. \qquad (14)$$

Comparing (13) and (14) to the corresponding expression for linear zero-forcing detector in (7) and (8), the only difference is that the channel matrix $\mathbf{H}$ has been replaced by $\underline{\mathbf{H}}$.

## V. DESCRIPTION OF THE CODES

### *A. Successive Cancellation (SUC)*

The key idea in the successive canceling technique is layer peeling where the symbol streams are successively decoded and stripped away layer by layer (Layered Space-Time).

For detecting signal $i$, we define: $\mathbf{G}_1 = \mathbf{H}^{+}$ for a ZF receiver or $\mathbf{G}_1 = \left[ \mathbf{H}^H \cdot \mathbf{H} + \frac{n_T}{\rho} \cdot \mathbf{I}_{n_T} \right]^{-1} \mathbf{H}^H$ for an MMSE receiver, where $\mathbf{H}^{+}$ represents the pseudo-inverse of matrix $\mathbf{H}$. Letting $i$ go from 1 to $n_T$, we extract the symbol from each $i$th stream:

$$z = \mathbf{g}_i \cdot \mathbf{r}_i \qquad (15)$$

where $\mathbf{g}_\mathrm{i}$ is the $i$th row of the ZF/MMSE receiver $\mathbf{G}_1$. $n_T$ is the number of transmit antennas and $\rho$ is the signal to noise ratio $\mathrm{E_b/N_0}$. The obtained $z$ is then sliced to obtain the received symbol $\mathbf{a}_i$. Assume that the decision on $\mathbf{a}_i$ is correct, remodulate to get $\mathbf{a}_i$ and subtract its contribution from the received signal $\mathbf{r}_i$. The reduced signal model is:

$$\mathbf{r}_{i+1} = \mathbf{r}_i - \mathbf{h}_i \cdot \mathbf{a}_i = \mathbf{H}_{i+1} \cdot \mathbf{a}_{i+1} + \mathbf{n} \qquad (16)$$

where $\mathbf{r}_{i+1}$ is the $n_R \times 1$ received vector with the contribution of $\mathbf{a}_i$ removed, $\mathbf{h}_i$ is the $i$th column of $\mathbf{H}$, and $\mathbf{H}_{i+1}$ is a reduced channel matrix of dimension $n_R \times (n_T - (i+1))$ with:

$$\mathbf{H}_{i+1} = \left\lfloor \mathbf{h}_{i+1} \cdots \mathbf{h}_{n_T} \right\rfloor \qquad (17)$$

and $\mathbf{a}_{i+1}$ is a reduced signal vector of dimension $(n_T - (i+1)) \times 1$ given by:

$$\mathbf{a}_{i+1} = \left\lfloor a_{i+1} \cdots a_{n_T} \right\rfloor^T \qquad (18)$$

Recalculate $\mathbf{G}_{i+1}$, using the expressions given for $\mathbf{G}_1$, now using $\mathbf{H}_{i+1}$, for either ZF or MMSE receivers.

### *B. V-BLAST (Ordered Successive Cancellation)*

The V-BLAST technique, also called Ordered Successive Cancellation (OSUC), is a slightly better approach than SUC. The principle behind OSUC is that at the beginning of each stage, the stream with the highest SNR is selected for peeling. This improves the quality of the decision and has been shown to be optimal for the SUC approach. The SUC algorithm requires only a small change, wherein, the SNR of the remaining streams are calculated at each stage and the stream with the highest SNR is selected for decoding.

For detecting signal $i$, we define $\mathbf{G}_1$ as before, for either the ZF or the MMSE receiver. Before iterating we must first define the vector $\mathbf{k}$, which will record the proper ordering of decoding.

$$k_1 = \arg \min_j \left\| \mathbf{g}_j \right\|^2 \qquad (19)$$

where the notation $\mathbf{g}_\mathrm{i}$ is used to denote the $j$th column of matrix $\mathbf{G}_1$. Letting $i$ go from 1 to $n_T$, we extract the symbol from each $k_i$th stream:

$$z = \mathbf{g}_{k_i} \cdot \mathbf{r}_i \qquad (20)$$

where $\mathbf{g}_{\mathrm{k_i}}$ is the $k_i$th row of the ZF/MMSE receiver $\mathbf{G}_1$, $n_T$ is the number of transmit antennas and $\rho$ is the signal to noise ratio $\mathrm{E_b/N_0}$. The obtained $z$ is then sliced to obtain the received symbol $\hat{\mathrm{a}}_{k_i}$. Assume that the decision on $\hat{\mathrm{a}}_{k_i}$ is correct, remodulate to get $\mathrm{a}_{k_i}$ and subtract its contribution from the received signal $\mathbf{r}_\mathrm{i}$. The reduced signal model is:

$$\mathbf{r}_{i+1} = \mathbf{r}_i - \mathbf{h}_{k_i} \cdot \hat{a}_{k_i} = \mathbf{H}_{i+1} \cdot \mathbf{a}_{i+1} + \mathbf{n} \qquad (21)$$

where $\mathbf{r}_{\mathrm{i+1}}$ is the $n_R \times 1$ received vector with the contribution of $\mathrm{a}_{k_i}$ removed, $\mathrm{h}_{k_i}$ is the $k_i$th column of $\mathbf{H}$, and $\mathbf{H}_{\mathrm{i+1}}$ is a reduced channel matrix of dimension $n_R \times (n_T - (i+1))$ with:

$$\mathbf{H}_{i+1} = \left\lfloor \mathbf{h}_j \right\rfloor_{j \notin \{k_1 \cdots k_i\}} \qquad (22)$$

and $\mathbf{a}_{\mathrm{i+1}}$ is a reduced signal vector of dimension $(n_T - (i+1)) \times 1$ given by:

$$\mathbf{a}_{i+1} = \left\lfloor \mathbf{a}_j \right\rfloor^T_{j \notin \{k_1 \cdots k_i\}} \qquad (23)$$

Recalculate $\mathbf{G}_{\mathrm{i+1}}$, using the expressions given for $\mathbf{G}_1$, now using $\mathbf{H}_{\mathrm{i+1}}$, for either ZF or MMSE receivers. Calculate the new value $k_{\mathrm{i+1}}$, for the next iteration:

$$k_{i+1} = \arg \min_{j \notin \{k_1 \cdots k_i\}} \left\| \mathbf{g}_j \right\|^2 \qquad (24)$$

where $\mathbf{g}_\mathrm{j}$ denotes now the $j$th column of $\mathbf{G}_{\mathrm{i+1}}$, calculated in the previous step.

This algorithm uses the sequence $k_i$ to determine the optimal ordering, compute the ZF or MMSE vector $\mathbf{g}_\mathrm{i}$, the decision statistic and the estimated component of $\mathbf{a}$. Then cancellation of the detected component is done, and the new $\mathbf{G}_{\mathrm{i+1}}$ is calculated for the next iteration. This new matrix is calculated using a reduced version of $\mathbf{H}$, where columns $\mathbf{k}_1 \ldots \mathbf{k}_i$ are eliminated.

### C. Zero-Forcing BLAST with QR Decomposition

It was shown (e.g. [8], [9]) that the BLAST algorithm can be restated in terms of the QR decomposition of the channel matrix **H**, i.e.

$$\mathbf{H} = \mathbf{QR}, \tag{25}$$

where the $n_R \times n_T$ matrix $\mathbf{Q}$ has orthogonal columns with unit norm and the $n_T \times n_T$ matrix $\mathbf{R}$ is upper triangular. By multiplying the received signal **x** with the Hermitian transpose of $\mathbf{Q}$, the sufficient statistic

$$\tilde{\mathbf{a}} = \mathbf{Q}^H \mathbf{r} = \mathbf{Ra} + \boldsymbol{\eta} \tag{26}$$

for the transmit vector **a** is obtained. Note that the statistical properties of the noise term $\boldsymbol{\eta} = \mathbf{Q}^H \mathbf{n}$ remain unchanged. Due to the upper triangular structure of $\mathbf{R}$, the $k$th element of $\tilde{a}$ is

$$\tilde{a}_k = r_{k,k} \cdot a_k + \sum_{i=k+1}^{n_T} r_{k,i} \cdot a_i + \eta_k \tag{27}$$

Thus, $\tilde{a}_{n_T}$ is free of interference and can be used to estimate $a_{n_T}$ after appropriate scaling with $1/r_{n_T,n_T}$. Proceeding with $\tilde{a}_{n_T-1},\ldots,\tilde{a}_1$ and assuming correct previous decisions, the interference can be perfectly cancelled in each step. Then it follows from (27) that the SNR of layer $k$ is determined by the diagonal element $|r_{k,k}|^2$.

As already mentioned, the detection sequence is crucial due to the risk of error propagation. It can be modified by permuting elements of **a** and the corresponding columns of **H** prior to the QR decomposition, leading to different matrices **Q** and **R** [8]. In order to find the optimum sequence, $|r_{k,k}|$, which represents the length of the component of the column vector $\mathbf{h}_k$ that is perpendicular to the space spanned by $\mathbf{h}_1,\ldots,\mathbf{h}_{k-1}$, needs to be maximized for $k = n_T,\ldots,1$. This may be accomplished in a straight forward way by performing $O(n_T^2/2)$ different QR decompositions of permutations of $\mathbf{H}$ [3]. A far more efficient approach is based on the easily verified relation

$$\mathbf{G}_{\mathrm{ZF}} = \mathbf{H}^+ = \mathbf{R}^{-1}\mathbf{Q}^H \tag{28}$$

and the fact that the row norms of $\mathbf{G}_{\mathrm{ZF}}$ equal those of $\mathbf{R}^{-1}$. Keeping in mind that the signal $a_{n_T}$ is detected first and recalling the optimal ordering criterion from Section 3-B, the last row of $\mathbf{R}^{-1}$ must have minimum norm. If necessary, rows of $\mathbf{R}^{-1}$ as well as the corresponding columns of **R** have to be exchanged at the expense of destroying the upper triangular structure. However, by right multiplying the permuted version of $\mathbf{R}^{-1}$ with a proper unitary $n_T \times n_T$ Householder matrix $\boldsymbol{\Theta}$, a block triangular matrix is achieved. Finally, $\mathbf{Q}$ has to be updated to $\mathbf{Q\Theta}$ while the permuted $\mathbf{R}$ is left multiplied with $\boldsymbol{\Theta}^H$. These steps are then iterated for the upper left $(n_T-1)\times(n_T-1)$ sub matrices of the such modified matrices $\mathbf{R}^{-1}$, $\mathbf{R}$ and the first $n_T-1$ columns of the new matrix $\mathbf{Q}$, resulting in the QR decomposition of the optimally ordered channel matrix $\mathbf{H}$.

The computational effort is made up of an initial **QR** decomposition, the inversion of $\mathbf{R}$, and the subsequent ordering, which is dominated by the multiplications of $\mathbf{R}^{-1}$, $\mathbf{R}$, and $\mathbf{Q}$ with the Householder matrix $\boldsymbol{\Theta}$ in each step. Although this is much better than computing the pseudo-inverse over and over again as in the original ZF-BLAST, a suboptimal algorithm proposed by the authors [8] requiring only a single *sorted* **QR** decomposition is reviewed in the next section.

### D. Zero-Forcing Sorted QR Decomposition

In order to obtain the optimal detection order, first $|r_{n_T,n_T}|$ has to be maximized over all possible permutations of the columns of the channel matrix $\mathbf{H}$, followed by $|r_{n_T,n_T}|$, and so on. Unfortunately, using standard algorithms for the **QR** decomposition, the diagonal elements of $\mathbf{R}$ are calculated just in the opposite order, starting with $r_{1,1}$. This makes finding the optimal order of detection such a difficult task.

The sorted **QR** decomposition (SQRD) algorithm is basically an extension to the modified Gram-Schmidt procedure [10] by reordering the columns of the channel matrix prior to each orthogonalization step. The fundamental idea is that $|r_{k,k}|$ is *minimized* in the order it is computed (from 1 to $n_T$) instead of being *maximized* in the order of detection (from $n_T$ to 1). This is motivated by the fact that the layers detected last affect only few other layers through error propagation and may therefore have rather small SNR, which increases the probability of large SNR for the first layers. Now, $r_{1,1}$ is simply the norm of the column vector $\mathbf{h}_1$, so the first optimization in the SQRD algorithm consists merely of permuting the column of $\mathbf{H}$ with minimum norm to this position. During the following orthogonalization of the vectors $\mathbf{h}_2,\ldots,\mathbf{h}_{n_T}$ with respect to the normalized vector $\mathbf{h}_1$, the first row of $\mathbf{R}$ is obtained. Next, $r_{2,2}$ is determined in a similar fashion from the remaining $n_T-1$ orthogonalized vectors, et cetera. Thereby, the channel matrix **H** is successively transformed into the matrix **Q** associated with the desired ordering, while the corresponding $\mathbf{R}$ is calculated row by row. Note that the column norms have to be calculated only once in the beginning and can be easily updated afterwards. Hence, the computational overhead due to sorting is negligible.

*E. MMSE BLAST with QR Decomposition Detection*

In order to extend the QR based detection with respect to the MMSE criterion, we can apply the similarity of ZF and MMSE detection noted in Section V.*A*. We introduce the QR decomposition of the extended channel matrix (12)

$$\underline{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_{\mathbf{n_T}} \end{bmatrix} = \underline{\mathbf{Q}}\underline{\mathbf{R}} = \begin{bmatrix} \mathbf{Q_1} \\ \mathbf{Q_2} \end{bmatrix} \underline{\mathbf{R}} = \begin{bmatrix} \mathbf{Q_1}\underline{\mathbf{R}} \\ \mathbf{Q_2}\underline{\mathbf{R}} \end{bmatrix}, \quad (29)$$

where the $(n_T + n_R) \times n_T$ matrix $\underline{\mathbf{Q}}$ with orthonormal columns was partitioned into the $n_R \times n_T$ matrix $\mathbf{Q}_1$ and the $n_T \times n_T$ matrix $\mathbf{Q}_2$. Obviously,

$$\underline{\mathbf{Q}}^H \underline{\mathbf{H}} = \mathbf{Q_1}^H \mathbf{H} + \sigma_n \mathbf{Q}_2^H = \underline{\mathbf{R}} \quad (30)$$

holds and from the relation $\sigma_n \mathbf{I}_{n_T} = \mathbf{Q_2}\underline{\mathbf{R}}$ it follows that

$$\underline{\mathbf{R}}^{-1} = \frac{\mathbf{1}}{\sigma_n}\mathbf{Q_2} \quad (31)$$

i.e. the inverse $\underline{\mathbf{R}}^{-1}$ is a byproduct of the $\mathbf{QR}$ decomposition and $\mathbf{Q_2}$ is an upper triangular matrix. Using (30) and (31), the filtered receive vector becomes

$$\tilde{\mathbf{a}} = \underline{\mathbf{Q}}^H \underline{\mathbf{r}} = \mathbf{Q}_1^H \mathbf{r} = \underline{\mathbf{R}}\mathbf{a} - \sigma_n \mathbf{Q}_2^H \mathbf{a} + \mathbf{Q}_1^H \mathbf{n}. \quad (32)$$

The second term on the right hand side of (32) including the lower triangular matrix $\mathbf{Q}_2^H$ constitutes the remaining interference that can not be removed by the successive interference cancellation procedure. This points out the trade-off between noise amplification and interference suppression.

The optimum detection sequence now maximizes the signal to-interference-and-noise ratio (SINR) for each layer, leading to minimal estimation error for the corresponding detection step. The estimation errors of the different layers in the first detection step correspond to the diagonal elements of the error covariance matrix (28)

$$\mathbf{\Phi} = \sigma_n^2 (\underline{\mathbf{H}}^H \underline{\mathbf{H}})^{-1} = \sigma_n^2 \underline{\mathbf{R}}^{-1} \underline{\mathbf{R}}^{-H}. \quad (33)$$

The estimation error after perfect interference cancellation is given by $\sigma_n^2 / \left|\underline{r}_{k,k}\right|^2$. Thus, it is again optimal to choose the permutation that maximizes $\left|\underline{r}_{k,k}\right|$ in each detection step. The algorithm in the next section determines an optimized detection sequence within a single *sorted* QR decomposition and thereby significantly reduces the computational complexity in comparison to standard MMSE-BLAST algorithms.

*F.MMSE BLAST Sorted QR Decomposition Detection*

In order to obtain the optimal detection order, first $\left|\underline{r}_{n_T,n_T}\right|$ has to be maximized over all possible permutations of the columns of the extended channel matrix $\underline{\mathbf{H}}$, followed by $|\underline{r}_{n_T-1,n_T-1}|$, and so on. Unfortunately, using standard algorithms for the QR decomposition, the diagonal elements of $\underline{\mathbf{R}}$ are calculated just in the opposite order, starting with $\underline{r}_{1,1}$. This makes finding the optimal order of detection a difficult task.

The fundamental idea is that $|\underline{r}_{k,k}|$ is *minimized* in the order it is computed $(1,\cdots,n_T)$ instead of being *maximized* in the order of detection $(n_T,\cdots,1)$. This is motivated by the fact that the layers detected last affect only few other layers through error propagation and may therefore have rather small SINR, which increases the probability of large SINR in the first layers. Now, $\underline{r}_{1,1}$ is simply the norm of the column vector $\underline{\mathbf{h}}_1$, so the first optimization in the SQRD algorithm consists merely of permuting the column of $\underline{\mathbf{H}}$ with minimum norm to this position. During the following orthogonalization of the vectors $\underline{\mathbf{h}}_2,\cdots,\underline{\mathbf{h}}_{n_T}$ with respect to the normalized vector $\underline{\mathbf{h}}_1$, the first row of $\underline{\mathbf{R}}$ is obtained. Next, $\underline{r}_{2,2}$ is determined in a similar fashion from the remaining $n_T - 1$ orthogonalized vectors, et cetera. Thereby, the extended channel matrix $\underline{\mathbf{H}}$ is successively transformed into the matrix $\underline{\mathbf{Q}}$ associated with the desired ordering, while the corresponding $\underline{\mathbf{R}}$ is calculated row by row. Note that the column norms have to be calculated only once in the beginning and can be easily updated afterwards. Hence, the computational overhead due to sorting is negligible.

## VI. SIMULATION RESULTS

The error performances of the proposed four coding schemes are compared for QPSK modulation. The block error rates of the simulation for a system with $n_T = 4$ and $n_R = 6$ antennas are shown in the next figure. The block error rates are calculated defining a block as a set of $\mathbf{L} = 120$ symbols. Therefore, a block will be in error in any of the L symbols in it is in error. The strong impact of ordering the QR decomposition is obvious and only a small difference with the V-BLAST is noticed at higher SNR. The same thing can be said if we compare V-BLAST and Successive Cancellation. We notice an important increase of performance between V-BLAST and SUC, due to the optimal ordering that is being done in V-BLAST that is not done in SUC. As can be seen from Figures 2 and 3, there is a coding gain of about 3.5 dB $(@\text{FER} = 0.01)$ between the sorted and unsorted methods (between V-BLAST and Successive Cancellation and between QR Decomposition and Sorted QR Decomposition).

As said before, the simulation is to compute the block error rate in both cases for different values of SNR in a white Gaussian channel. In these calculations, SNR is defined as:

$$\mathrm{SNR} = 10 \cdot \log\left(\frac{E_s}{N_o}\right) \tag{34}$$

where $E_s$ is the symbol energy, normalized to 1 for this constellation, and $N_0$ is the noise variance. The transmitted symbols are encoded using the QPSK constellation with mean symbol energy of 1, 4 transmitters and 6 receivers.

## VII. CONCLUSIONS

We have described different wireless architectures, capable of realizing different spectral efficiencies over a rich scattering wireless channel. The general V-BLAST, SUC, QR and Sorted QR decompositions were described in detail and the results in the comparison on error performances of the schemes were reported. Results show that sorting can bring an important improvement over performance for these codes. We also showed that the Sorted QR algorithm requires less computational effort, and brings little loss in performance compared to the most optimum of the four codes; The V-BLAST scheme.

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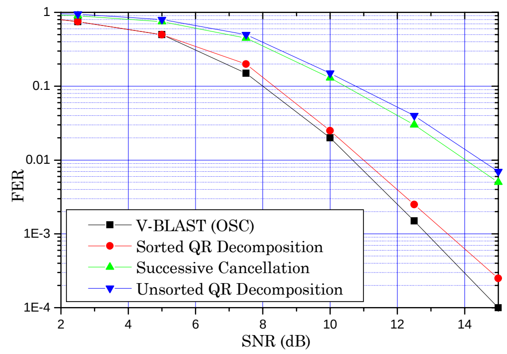


Figure 2: FER versus SNR for different types of LST codes for ZF detection.

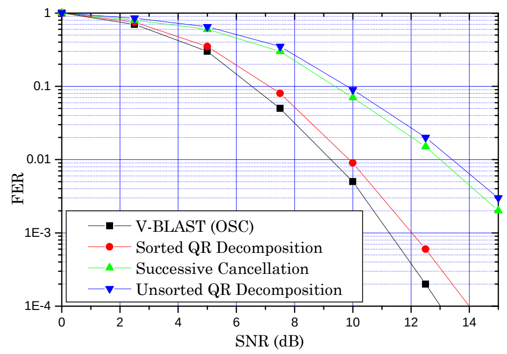


Figure 3: FER versus SNR for different types of LST codes for MMSE detection.