# Successive Blind Recursive Constant Modulus Detectors for DS/CDMA Signals with BPSK Modulation

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***Abstract* — A successive detection technique using a multi-stage finite impulse response filter, controlled by the constant modulus algorithm (CMA), and a signal canceller (SC) to enable the blind detection of DS-CDMA signals is presented. This CMA/SC multi-stage system shows robustness of detection of different users in dense environments, it is easily implemented by simple gradient algorithms, and it requires lower computational effort than other reported algorithms. In addition, it does not require a knowledge of the spreading codes of the different users. A drawback of this multi-stage CMA/SC system is the relatively slow convergence of the CMA algorithm, which may not be suitable for some applications. This paper describes in detail a recursive constant-modulus filter/signal canceller (RCMA/SC) for the blind detection of DS/CDMA signals over an additive white Gaussian noise (AWGN) channel. The RCMA is derived by analogy to the recursive least squares algorithm (RLS) as a fast version of the CMA. The convergence properties of the algorithm are analyzed and compared with the conventional CMA. Simulation examples, for both the CMA and the RCMA, are given, to demonstrate the robustness of the proposed algorithm and its superiority from the point of view of fast convergence.**

## I. Introduction

We are interested in the following problem: given multiple digitally-modulated signals being heard simultaneously by a receiver, how does the receiver reliably demodulate a particular user (or users)? This problem is currently receiving intensive interest because of the rapid growth of DS-CDMA systems, where the detection of a specified user, while canceling the multiple access interference (MAI) caused by other users, is fundamental. Adaptive interference suppression is analogous to adaptive equalization of a time invariant channel by virtue of the analogy between MAI and intersymbol interference (ISI). Application of these methods to DS-CDMA is a relatively recent concept proposed by a number of different authors at approximately the same time, [1-3]. These authors propose adaptive receivers based on the linear minimum mean squared error (MMSE) criterion. These receivers require only the code sequence of the desired user and a coarse knowledge of the timing of the desired user. They can be implemented adaptively using standard algorithms such as least mean squares (LMS) or recursive least squares (RLS) [4-6].

The preceding arguments motivate the subject of this paper, namely, blind adaptive multi-user detection , which at its minimum means that the receiver does not require a training sequence for the desired user (in addition to not requiring knowledge of the interference parameters) [7,8]. Blind reception is of interest in its own right in broadcast or multicast settings, since it enables receivers to asynchronously tune into a transmission of interest at any time, without any requirement for a training sequence, thus offering better spectrum efficiency. The most representative methods for blind multi-user detection include the minimum output energy (MOE) [10] and subspace approach [11]. However these methods assume a knowledge of the spreading waveform, which is not available in some applications like electronic intelligence (ELINT). The constant modulus (CM) receiver can perform almost as well as the non-blind/trained receiver except that the CM Algorithm (CMA) captures an arbitrary signal from the combined received CDMA signals, which may be of no interest [12]. We describe a successive blind detection scheme for all the active users in a DS/CDMA system [13]. Our principle of system operation is based on utilizing identical successive stages; each stage consisting of a CM filter followed by a signal canceller, allowing for the detection of different signals. In each stage, the CMA detects an arbitrary signal, which is subtracted from the input of the stage by a signal canceller and the remainder is applied to the next stage to permit detection of another signal. It has been shown that in a synchronous system, the CMA exhibits a lock convergence behavior when the filter is at steady state; that is it locks onto one signal and nulls all other interfering signals. The converged filter is equivalent to the well-known decorrelator and is orthogonal to the MAI space. The signal canceller utilizes this orthogonality to remove the detected signal and permits another one to be detected. The performance of a CM-based receiver is limited by the received power of the desired user. Another drawback of the multi-stage CMA/SC system is the relative slow convergence of the CM algorithm, which may make it not suitable for some applications. This paper describes an investigation into the performance of a multi-stage recursive constant-modulus filter/signal canceller (RCMA/SC) for the blind detection of DS/CDMA signals over an additive white Gaussian noise (AWGN) channel. The RCMA is derived, as a fast version of the CMA, by analogy to the RLS. The RCMA shows superiority over the original CMA from the point of view of fast convergence, at the expense of some increase in complexity.

The paper is organized as follows; section 2 is a description of the multi-stage system, its principle of operation and the signal model; the performance of the multi-stage system, based on the RCMA, is analyzed in section 3; simulation examples which demonstrate the robustness and efficiency of the proposed system are presented in section 4; and finally conclusions are given in section 5.

## II. System and Signals Descriptions

The proposed multi-stage RCMA/SC system, for the separation of co-channel DS/CDMA signals, is shown in Fig.1. The system consists of M-identical stages; each stage consists of an adaptive finite impulse response (FIR) filter followed by an adaptive signal canceller. The adaptive FIR filter is controlled by the recursive constant modulus algorithm (RCMA), which captures one of the DS/CDMA signals at the input of the stage. The signal canceller subtracts the detected signal from the combined DS/CDMA received signals at the input of the stage. This permits the next stage to capture another signal from the multiplexed CDMA input signals.

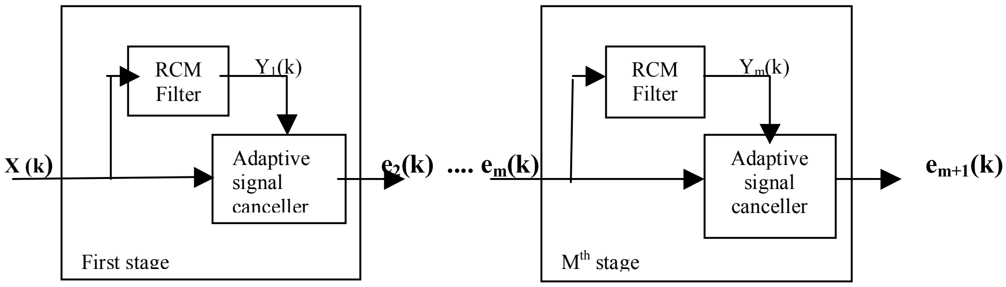


Fig. 1. Multi-stage RCMA/SC for separation of co-channel CDMA signals.

Assume that, there are $K$ active users in the observed DS/CDMA system. The signal received by the first stage is represented as

$$x(t) = \sum_{k=1}^{K} r_k(t) + n(t) \tag{1}$$

Where, $n(t)$ is the additive white Gaussian noise (AWGN) with zero-mean and variance $\sigma^2$. The $k^{th}$ user signal $r_k(t)$ in a DS/CDMA system is given by

$$r_k(t) = \sum_{j=-\infty}^{\infty} A_k b_k(j) s_k(t - jT_b - \tau_k)\cos(\omega_c t + \theta_k) \tag{2}$$

Where, $A_k$, $b_k$, $\tau_k$ and $\theta_k$ are the amplitude, data symbols, delay and phase of the $k^{th}$ user signal respectively. Each user data symbols $b_k$ is assumed to be binary phase shift keying (BPSK) signal. $\omega_c$ is the angular carrier frequency and $T_b$ is the symbol duration. The spreading code of the $k^{th}$ user is given by

$$s_k(t) = \sum_{n=0}^{N-1} u_k(n) p(t - nT_c) \tag{3}$$

Where, $u_k(n) = \pm 1$ is the $n^{th}$ element of the spreading sequence for the $k^{th}$ user and the chip waveform $p(t)$ is a rectangular waveform of duration $T_c$. The chip duration $T_c$ is assumed to be, $T_c = T_b/N$; where $N$ is the processing gain of the spreading code.

The equivalent discrete synchronous model of the received signal at the $j^{th}$ data symbol is given by

$$\underline{x}(j) = b_1(j) A_1 U_1 + \sum_{l=2}^{K} b_l(j) A_l U_l + \underline{n}(j) \tag{4}$$

Where Ul is the spreading sequence of the $l^{th}$ user represented by the vector

$$U_l = [u_l(1)\ u_l(2)\ .........u_l(N)]^T \tag{5}$$

The first term in equation (4) represents the data symbol of the captured user by the first stage $b_1$ modulated by its spreading signal vector $U_1$. While the second term represents the multiple access interference, (MAI), from other users and the third one is the AWGN vector. This equation can be written in matrix form as

$$X(j) = U\,A(n) \quad (6)$$

Where $U$ is the (N x K) code matrix of the spreading codes of all users and $A\ (n)$ is the (Kx1) vector of symbols of the active users at the n$^{th}$ time instant with their corresponding amplitudes. $U$, $A\ (n)$ are defined as

$$U = [U_1\ U_2\ ...........U_k\ ] \quad (7)$$

$$A(n) = [A_1 b_1(n)\ \ A_2 b_2(n)\ \ ........A_K b_K(n)] \quad (8)$$

The output of the CM filter of the m$^{th}$ stage, at the n$^{th}$ time instant, is given by

$$y_m(n) = \underline{w}_m^T(n)\ \ \underline{e}_m(n) \quad (9)$$

- $\underline{w}_m^T(n)$ is the weight vector of the CM filter of the m$^{th}$ stage, represented as

$$\underline{w}_m^T(n) = [w_{m,1}(n)\ \ w_{m,2}(n)\ ......w_{m,N}(n)]^T \quad (10)$$

- $\underline{e}_m(n)$ is the input signal vector of the m$^{th}$ stage at the n$^{th}$ time instant.

Note that for the first stage $\underline{e}_1\ (n)$ is $\underline{x}\ (n)$ and the output of the signal canceller of the m$^{th}$ stage, $\underline{e}_{m+1}(n)$, represents the input of the next stage, and it can be written as

$$\underline{e}_{m+1}(n) = \underline{e}_m(n) - y_m(n)\,\underline{z}_m(n)\ =\ T_m(n)\,\underline{e}_m(n)$$
(11)

Where we have substituted $y_m\ (n)$ from equation (9) and define the signal transfer matrix

$$T_m(n)\ =\ I - \underline{z}_m(n)\ \ \underline{w}_m^*\ \ (n) \quad (12)$$

The signal canceller weight vector $\underline{z}_m\ (n)$ is given by

$$\underline{z}_m(n)\ = [z_{m,1}(n)\ z_{m,2}(n)\ .........z_{m,N}(n)]^T \quad (13)$$

Using this notation, it is straightforward to write the output of the m$^{th}$ stage in terms of the system input $\underline{e}_1\ (n) = \underline{x}\ (n)$ as follows

$$y_m(n) = \underline{w}_m^*(n) T_{m-1}(n).....T_1(n)\underline{e}_1(n) = \underline{w}_m^*(n)\{\prod_{j=1}^{m-1} T_j(n)\}\underline{e}_1(n)$$

(14)

Associate with each stage an effective code matrix (Ũ$^m$), which is analogous to $U$ for the first stage, and an input correlation matrix ($R_m$). Clearly $\tilde{U}^1 = U$ and for notational convenience and with out loss of generality we will assume that the signal $s_m\ (n)$ is the captured signal by the m$^{th}$ stage (for m =1,2,……..K). It has been shown that the construction of the effective code matrices $\tilde{U}^m$ (m>1) is similar to $\tilde{U}^1$, except that, the earlier (m-1) columns are replaced by zeros, since their corresponding signals are cancelled [13].

## III. Analysis of System Behavior

The Recursive constant modulus algorithm (RCMA) in the m$^{th}$ stage minimizes the weighted mean square error between the actual amplitude of the output of the controlled FIR filter and the desired amplitude which is normalized to be unity. Thus, the objective function of the RCMA is defined as

$$\Phi = \sum_{k=0}^{N} \alpha^{N-k}\ \varepsilon^2(k) \quad (15)$$

Where $N$ is the observation length and $\alpha$ is a positive weighting (forgetting) factor, $0<\alpha<1$. The error signal, $\varepsilon\ (k)$ is defined as

$$\varepsilon\ (k) = 1 - |y(k)|^2 \quad (16)$$

For simplicity we consider the behavior of the first stage. The behavior of other stages are similar, with the minor change of the code matrix $\tilde{U}^1$ by the corresponding code matrix $\tilde{U}^m$ and the input vector $\underline{e}_1\ (k) = \underline{x}\ (k)$ by its corresponding input vector $\underline{e}_m\ (n)$. The performance function $\Phi$ penalizes the weighted square difference between the desired envelope and the actual one at the output of the FIR filter. It is required to determine the weights vector $w$, which minimizes the function, $\Phi$ as defined in equation (15), thus we set the gradient of the cumulative error squares $\Phi$ with respect to the weights vector to zero which implies that

$$\nabla_w \Phi = 2\sum_{k=0}^{N} \alpha^{N-k}\ \varepsilon(k)\frac{\partial\varepsilon(k)}{\partial w} = 0 \quad (17)$$

The error signal could be rewritten as

$$\varepsilon(k) = 1 - |y(k)|^2 = 1 - w^H x^*\ x^T\ w \quad (18)$$

Where * denotes the complex conjugate operation and $H$ denotes the complex conjugate matrix transpose operation. The derivatives of the error signal with respect to the weights vector is given by [17-18]

$$\frac{\partial\varepsilon(k)}{\partial w} = -x^*(k)x^T(k)\,w(k) = y(k)x^*(k) \quad (19)$$

$$-2\sum_{k=0}^{N}\alpha^{N-k}(1-|y(k)|^{2})\,y(k)\,x^{*}(k)=0 \tag{20}$$

It is clear that equation (20) is a non linear equation in the filter weights. Even if they were linear there would be a solution only for the limited case that the length of the observation equaled the number of FIR weights. Instead of solving these equations we choose the FIR weights, $w$, to obtain a least-squares fit between the desired output envelope and the actual one. Define the matrix $R\ (N)$ and the column vector $D\ (N)$ such that,

$$D(N)=R(N)\,w(N) \tag{21}$$

Where $R\ (N)$ and $D\ (N)$ are given by the forms,

$$R(N)=\sum_{k=0}^{N}\alpha^{N-k}\,S(k)\,S^{H}(k) \tag{22}$$

$$D(N)=\sum_{k=0}^{N}\alpha^{N-k}\,S(k) \tag{23}$$

Where the vector $S\ (k)$ is defined as:

$$S(k)=y(k)\,x^{*}(k) \tag{24}$$

A generalized solution of the system in (21) exists if the matrix $R\ (N)$ is non-singular and is given by

$$w(N)=R^{-1}(N)\,D(N) \tag{25}$$

However, the system of equations in (21) are non-linear equations in the filter weights vector $w$ since $y\ (k)$ itself is a function of $w$. In order to solve this conflict we utilize a gradient search to find $w$, which minimizes the performance function $\Phi$ in an iterative manner as follows; first suppose that we have an initial value $w(N-1)$ for the FIR filter weights vector $w$, which satisfies the system in (21) at the $(N-1)^{th}$ iteration thus

$$w(N-1)\overset{\Delta}{=}R^{-1}(N-1)\,D(N-1) \tag{26}$$

Second we note the recursive nature of both $D\ (N)$ and $R\ (N)$ as:

$$D(N)=\alpha\,D(N-1)+S^{*}(N) \tag{27}$$

$$R(N)=\alpha\,R(N-1)+S^{*}(N)\,S^{T}(N) \tag{28}$$

Define the matrix $P=R^{-1}$ and take the matrix inversion of both sides of (28) we can verify that [19]

$$P(N)=\frac{1}{\alpha}\Big[P(N-1)-\frac{P(N-1)S^{*}(N)S^{T}(N)P(N-1)}{\alpha+S^{T}(N)P(N-1)S^{*}(N)}\Big] \tag{29}$$

Before we continue we further simplified $P\ (N)$ as follows; define the column vector $K\ (N)$ as:

$$K(N)=\frac{1}{\alpha+\Delta(N)}P(N-1)S^{*}(N) \tag{30}$$

Where the scalar $\Delta(N)$ is given by

$$\Delta(N)=S^{T}(N)\,P(N-1)\,S^{*}(N) \tag{31}$$

So we can rewrite $P\ (N)$ as

$$P(N)=\frac{1}{\alpha}[P(N-1)-K(N)S^{T}(N)P(N-1)] \tag{32}$$

Substituting $P$ and $D$ from (32) and (27) and simple manipulation results in an updated equation for the FIR filter weights vector at the $m^{th}$ stage:

$$w_m(n)=w_m(n-1)+K_m(n)(1-|y_m(n)|^{2}) \tag{33}$$

By inspection of equation (33) we can verify that the vector of the FIR filter weights changes with time by an amount equal to the error signal multiplied by the gain vector $K(n)$. Since $K\ (n)$ is an $N$ dimensional vector each element of the weights vector, in effect, is controlled by one of the elements of $K(n)$. Consequently rapid convergence is expected for the RCMA. This is in contrast to the steepest decent algorithm used for the original CMA where we have a fixed step size [19]. The corresponding update of the signal canceller weights is given by [34]

$$\underline{z}_m(n+1)=\underline{z}_m(n)+\mu_{sc}\,y_m^{*}(n)\,\underline{e}_{m+1}(n) \tag{34}$$

Where $\mu_{sc}>0$ is the step size of the LMS controlling the signal canceller. Because there is only one input $y_m\ (n)$ and $N$ error signals, (contained in vector $\underline{e}_{m+1}(n)$), the recursion in (34) actually corresponds to $N$ independent LMS updates.

The steady state behavior of the $m^{th}$ stage and its optimum weight vectors $\underline{w}_m$, $\underline{z}_m$ are obtained by the orthogonality principle [14]. For the signal canceller weights, the orthogonality condition is applied to the gradient estimate in (34),

$$E\{e_{m+1}(k)\,y_m(k)\}=\underline{0} \tag{35}$$

This orthogonal property yields

$$\underline{z}_m=R_m\,\underline{w}_m\,/\,\sigma_{Y_m}^{2} \tag{36}$$

Where $\sigma_{Y_m}^{2}$ is the mean power of the $m^{th}$ captured signal. To obtain the steady state weights of the CM filter of the $m^{th}$ stage we replace the error signal $\varepsilon_m$ by $s_m(k)-y_m(k)$. Using this approximation, we obtain the optimum weights of the CM filter as

$$\underline{w}_m=\sigma_{s_m}^{2}\,R_m^{-1}\,U_m \tag{37}$$

Where $U_m$ is the m[th] column of the matrix $\tilde{U}^m$ and $\sigma^2_{s_m}$ is the mean power of the m[th] signal. Substituting this result into (36) yields

$$z_m = (\sigma^2_{s_m} \;/\; \sigma^2_{Y_m})\, U_m = (\frac{1}{G^2_m})\, U_m \qquad (38)$$

Where $G_m$ is the gain of the m[th] stage. This result can be explained as follows; the m[th] stage captures the m[th] user signal, the signs of the impulse response of the optimum CM filter weight vector is typical the spreading code of that user, i.e. sign($\underline{w}_1$) = $U_I$ [16]. The output power of this stage is amplified by the square magnitude of the CM filter, $G^2_m = |\underline{w}_m|^2$. Evidently, implementing the signal canceller as (38) will remove the components of the m[th] captured user from the input to the next stage.

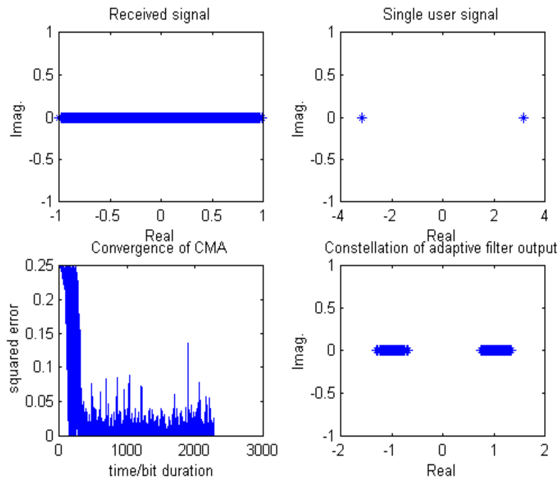


Fig.2. Behavior of the first CMA/SC stage (Equal power signals).

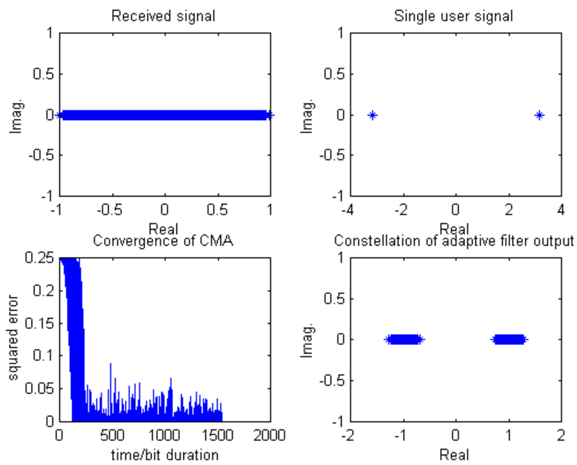


Fig.3 Behavior of second CMA/SC stage (Equal power signals).

## IV. Simulation Results

It is assumed that 2 active users contribute to the combined, received signal at the input of the first stage. Each user transmits BPSK data signals modulated by a 15 chips Gold codes. Each user signal is 10 dB above the background AWGN. Furthermore the data signals are perfectly synchronized and the received signal is sampled at the code chip rate. As a comparison with the previous results of the CMA/SC system [13], the behavior of the multi-stages CMA/SC is shown in Fig.2 & 3, While the behavior of the RCMA/SC are shown in Fig. 4 & 5. In each of the mentioned figures the corrupted received signal constellation, and the corresponding single user BPSK signal constellation are shown respectively. One can see the severe effect of MAI on the signal constellation. Comparison between the convergence of the CMA and RCMA shows the superiority of the RCMA, which reduces the convergence time by approximately an order of magnitude, compared to the CMA. The CM filter reduces the effect of the MAI introduced by the other active user, which is clear from the spectrum of figures 2, 3, 4, and 5. Comparison of the convergence curves of the two stages for each algorithm indicates that, the smaller the number of active users, the faster the convergence of the algorithms. An interesting point to be analyzed in future work is; which user is detected first? The simulations indicate that, when there is power difference between the users, the signal with the higher power is detected by the earlier stages, as might be expected.

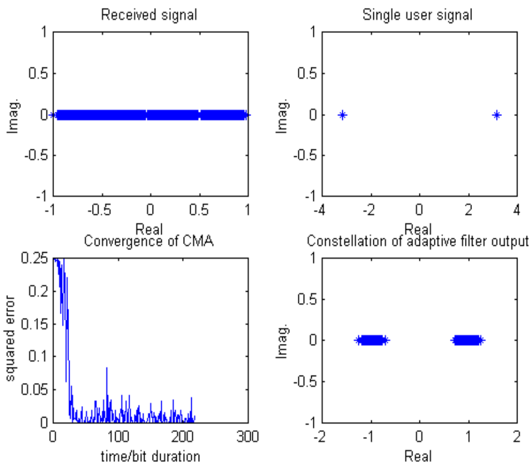


Fig.4 Behavior of RCMA/SC first stage.

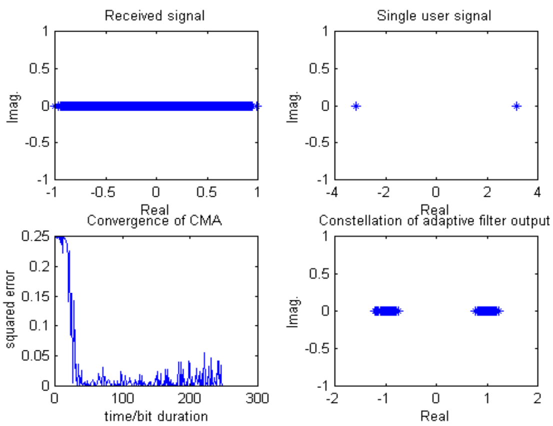


Fig.5 Behavior of RCMA/SC second stage.

## V. Conclusions

We have presented a proposal for successive multi-user detection using a multi-stage CM filter and signal canceller. It has been shown that in a synchronous system, the CM filter exhibits a lock convergence behavior when the filter, at steady state, can lock onto one user and null all other interfering users. The converged filter, at lock convergence, is equivalent to the well-known decorrelator and it is orthogonal to the MAI space. The signal canceller utilizes this orthogonality to remove the detected user signal and permits another one to be detected. Both the CMA/SC and RCMA/SC multi-stage systems show efficient performance in the successive detection of DS-CDMA signals. Further both of them do not require a knowledge of the spreading codes of the different users or the timing of the waveforms. However, the RCMA offers a better steady-state performance and has a higher convergence rate, relative to the CMA.

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