Reduction of Peak-to-Average Power Ratio of OFDM Symbols using Phasing Schemes combined with Companding

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***Abstract* — In this paper various phasing schemes that reduce the Peak-to-Average Power Ratio (PAPR) within OFDM systems are discussed. Simulation results on the effect of these phasing schemes in reducing PAPR are also presented. Companding OFDM symbols prior to transmission using the µ-Law has also been shown in previous papers to further reduce the PAPR. This paper now presents the results of combining both companding with various phasing schemes. The Bit-Error Rate (BER) performance of various PAPR reduction techniques in an Additive White Gaussian Noise (AWGN) channel is also presented. It is shown that combining companding and the application of phasing schemes can significantly improve further PAPR reduction as well as improve the BER performance.**

***Index Terms* — Companding, OFDM, Peak-to-Average Power Ratio, Phasing Schemes**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is known to be a robust high data rate transmission technique which combats the effects of multipath and fading channels in wireless communication applications. In addition to popular applications like digital audio broadcasting (DAB) and terrestrial digital video broadcasting (DVB-T), OFDM is also implemented within wireless local area network standards such as the IEEE 802.11a/g and HIPERLAN/2. However, one major problem within OFDM systems is the existence of large Peak-to-Average Power Ratio (PAPR) which arises as a consequence of the coherent addition of multiple subcarrier amplitudes and phases from the system. A large PAPR limits the range of linear operation of power amplifiers in transmitters, thus reducing the efficiency of the system. It also increases the complexity of analogue-to-digital and digital-to-analog converters in wireless digital systems.

A number of PAPR reduction techniques have been published in the literature with many new techniques also being proposed. These techniques can be broadly classified into signal predistortion techniques [1], coding techniques [2], signal scrambling techniques [3,4], and phasing techniques [5-8] etc.

Signal predistortion techniques such as clipping, peak windowing and peak cancellation aim to reduce the peak amplitudes of the transmitted signals by nonlinearly distorting the OFDM signal at or around the peak values. Coding techniques use a forward-error correcting code set that excludes the OFDM symbols with a high PAPR. Symbol scrambling techniques can be considered as a special case of coding techniques, in which the input sequence is scrambled by a certain number of scrambling sequences. The sequence which produces the smallest PAPR is the one used for transmission.

In phasing schemes, the complex symbols that modulate the orthogonal subcarriers are given a phase shift in such a way that the resultant OFDM symbols have a reduced PAPR. With the Crest Factor (CF) defined as a measure of the PAPR of the RF envelope, Shapiro and Rudin [5] have shown that for a multi-tone signal with N tones, the CF could be reduced to 3 dB by setting the phase of the subcarriers to 0 or π in a particular sequence. However, in relation to the CF discussed in this present paper, the CF will be calculated from the baseband signal, as simulating an RF waveform requires an extremely high sample rate and excessive computational overhead. For a complex baseband signal, the CF is the ratio of the peak envelope power to the mean envelope power, taken over a given time period, usually the OFDM symbol duration. (1) below gives an expression for the CF for a complex baseband signal. This is an equivalent measure for PAPR, hence, in this paper we will use the term PAPR.

$$CF_{dB} = 10\log_{10}\left(\frac{\max\limits_{0<t<T}\left(x^2(t)\right)}{\underset{0<t<T}{mean}\left(x^2(t)\right)}\right) \qquad (1)$$

Newmann [6], and Narahashi and Nojima [7,8], have proposed changing the phase of the complex signal in a quadratic fashion, where the phase shift is increased with the index of the subcarrier. The phasing scheme proposed by Narahashi and Nojima provides a slightly better PAPR reduction than that proposed by Newmann.

Companding, in which the OFDM signal peaks are compressed at the transmitter and expanded at the receiver, is classified as a predistortion technique. Companding has in fact been shown to outperform clipping as an effective tool for reducing the PAPR of OFDM signals. Huang et. al. have shown in [9] that a nonlinear-quasi-symmetrical companding transform can outperform a clipping-filtering scheme by 4.6dB in relation to Signal-to-Noise Ratio (SNR) for a BER of 10^{-4} in an Additive White Gaussian Noise (AWGN) channel. The techniques proposed by these authors enlarge the amplitudes of small signals while the peaks remain unchanged after companding. The average power is thus increased and the PAPR is reduced. Companding therefore has potential to reduce the PAPR.

Vallavaraj et. al. have shown in [10], that a further improvement in BER performance can be achieved by choosing an optimized or better companding profile. Their work has shown that a companding profile with a Peak Ratio (PR) greater than 1, and typically 2, will result in significant improvement in BER performance and reduction in PAPR.

The purpose of this paper is now to investigate the possibility of improving BER performance and also PAPR reduction through combining companding of the OFDM signal with known phasing schemes. There appears to be no published literature which undertakes such an investigation so the results presented in this paper are therefore new to the field of OFDM studies.

The paper is structured as follows. Section II describes briefly various phasing schemes that reduce PAPR. Section III defines PR and presents various companding profiles. It also discusses briefly how a PR > 1 can significantly reduce PAPR and improve BER performance. Section IV details the Matlab/Simulink model used for simulation and evaluation of BER and PAPR and also presents and discusses the simulation results. Section V is a conclusions section and summarizes the main results from the paper.

II. PHASING SCHEMES FOR PAPR REDUCTION

Several phasing schemes have been developed for producing lower PAPR in multicarrier transmissions. These are now briefly described.

(a) Shapiro-Rudin Phasing Scheme

Shapiro and Rudin [5] have shown that by adding a specified phase sequence of 0s and πs to the phase of the subcarriers of a multi-tone signal with N tones, a CF of 3 dB may be realised. The sequence starts with a string m = {11}, and repeatedly concatenates to m a copy of m with its second half negated. Table 1 gives an example of Shapiro-Rudin sequences for different values of N. The added phase of the subcarriers is set as shown in (2),

$$\theta_k = \begin{cases} 0, & s_k = 1 \\ \pi, & s_k = -1 \end{cases} \tag{2}$$

where θ_k is the added phase, and s_k is the cosine of the phase. The Shapiro-Rudin phasing scheme uses discrete phase angles and are semi-random.

Table 1 Shapiro-Rudin Sequences as a function of N

N	Shapiro-Rudin Phase Sequence
2	S = {1 1}
4	S = {1 1 1 -1}
8	S = {1 1 1 -1 1 1 -1 1}
16	S = {1 1 1 -1 1 1 -1 1 1 1 1 -1 -1 -1 1 -1}

(b) Newmann Phasing Scheme

Newmann phases [6] are varied in a quadratic fashion as shown in (3).

$$\theta_k = \frac{\pi(k-1)^2}{N}. \tag{3}$$

Here N is the number of subcarriers and k is the index of the particular subcarrier. It is reported in [6] that the worst case CF is 3.6 dB for $N = 3$, whilst for $N > 10$, a CF less than 2.8 dB could be achieved. The CF approaches an asymptote of 2.6 dB as N approaches infinity.

(c) Narahasi-Nojima Phasing Scheme

Narahasi and Nojima have proposed in [7] a new phasing scheme that is similar to Newmann's phasing scheme, and is given by

$$\theta_k = \frac{\pi(k-1)(k-2)}{N-1}. \tag{4}$$

The CF is very similar to that obtained through Newmann's phasing scheme, albeit a slightly better PAPR reduction is achieved over Newmann's scheme when N < 6.

Narahasi and Nojima have also proposed in [8] an algorithm that starts with a random phase sequence, and uses steepest descend minimization on the variance of the instantaneous envelope. This process is repeated for many random phase sequences and the best resulting sequence is kept. This technique has resulted in much lower CF than previous methods.

(d) Phase Shift to Alternate QPSK symbols

It is well known that the Fourier Transform of a symbol of constant amplitude, say a step input, is a signal concentrated at just one point in frequency. The spectrum of this signal would have a maximum PAPR. For the spectrum to have a reduced PAPR the signal in the frequency domain should have its power distributed in the spectrum, which is possible when the signal in the time domain has varying amplitudes. By purposely introducing a phase shift to alternate Quadrature Phase Shift Keying (QPSK) signals, the alternate signals are ensured to have different amplitudes. The phase angle is chosen in such a way that the alternate QPSK signals will have a constellation different from that of a normal QPSK.

(e) Random Phase Shift Addition to all QPSK symbols

This is a slight variation of the technique explained earlier. Each QPSK signal is given a random phase shift that results in each QPSK signal having different amplitudes.

III. COMPANDING

In companding the OFDM signal is compressed at the transmitter and expanded at the receiver. Compression is performed according to the well known μ-Law viz.

$$y = V \frac{\log\left(1 + \mu \frac{|x|}{V}\right)}{\log(1+\mu)} \operatorname{sgn}(x) \qquad (5)$$

where V is the peak amplitude of the signal, and x is the instantaneous amplitude of the input signal. Decompression is simply the inverse of (5).

Compression improves the quantization resolution of small amplitude signals at the cost of lowering the resolution of large signals. This also introduces quantization noise, however, the effect of the quantization noise due to reduction in resolution of the peaks is relatively small as the peaks occur less frequently. The compression algorithm as described by (5) amplifies the signals of lower amplitude with the peaks remaining unchanged.

It has been shown in [10] that all amplitudes including the peaks of the input signal can be amplified by changing the companding profile. A Peak Ratio (PR) is defined as the ratio of peak amplitude V of the signal specified for the μ-Law compressor to the peak amplitude of the actual signal to be compressed i.e.

$$PR = \frac{Peak\ \ Amplitude\ \ of\ compressor}{Peak\ \ of\ \ actual\ \ signal} = \frac{V}{peak\ (x)} \qquad (6)$$

Choosing PR = 1 results in a transformation gain of unity for the peak amplitude of the signal, which means the lower amplitude signals are amplified and the peaks remain unchanged. Choosing PR = 2 results in a gain greater than unity for the peaks and a much higher gain for the lower amplitude signals. Fig. 1 shows the compressor profiles for different values of PR.

It is shown in [10] that choosing a PR = 2 results in a reduction of PAPR by about 6.88 dB.

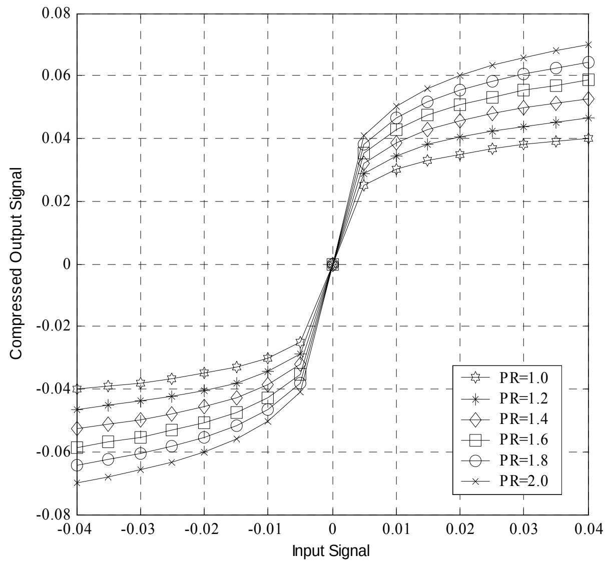

Fig.1. Compressor profiles for PR ranging from 1 to 2

IV. SIMULATION AND RESULTS

An OFDM scheme comprising 16 subchannels was developed for simulation using Simulink. The maximum theoretical PAPR for this system should be 10log (16) = 12.04 dB. The simulation model produces this maximum theoretical value when all the input bits are set to either 0 or 1, thus validating the model. Fig. 2 shows the system block diagram of the developed simulation model.

The random Bernoulli input data bits are mapped onto complex symbols by the QPSK block. The complex QPSK symbols are passed through an inverse Fast Fourier Transform (IFFT) block, which produces OFDM subcarriers modulated by the QPSK symbols. The μ−Law compressor block developed allows the user to specify the value of PR thus allowing the user to choose a specific compression profile.

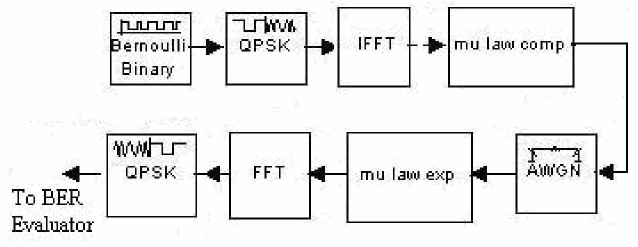

Fig.2. Simulink model developed for OFDM

A compressor profile PR = 2 is used in this simulation. The transmission channel includes an AWGN block to allow BER performance to be evaluated as a function of SNR. The receiver comprises the inverse processes of the transmitter and includes a μ-Law expansion block, an FFT block, a QPSK demodulator, and a BER evaluation.

(A) PAPR Evaluation of the Various Techniques

The results presented here were obtained by transmitting 30000 frames of OFDM symbols; each frame consisting of 16 subcarriers resulting in almost a million bits. The data transmission was set with the probability of a "0" or a "1" equal to 0.5 this providing a random sequence of transmitted 0s and 1s. The PAPR for each phasing scheme was determined and the results are presented in Table 2. PAPR is calculated by measuring the peak and the average power for each frame. The maximum frame PAPR over 30000 frames was then determined.

Table 2 PAPR evaluation for each phasing scheme

Phasing Scheme	PAPR without companding (dB)	PAPR with companding (dB)
No phasing scheme	10.5	5.01
Shift alternate QPSK	10.61	4.925
Random Phase shift	11.15	5.128
Shapiro - Rudin	2.323	0.7967
Narahashi - Nojima	1.335	0.3506
Newmann	2.753	1.034

It can be seen that companding once again appears to be an effective and superior technique in reducing PAPR. The reduction in PAPR when only companding is considered with no phasing scheme is about 5.5 dB. As presented in [5-7] the phasing schemes like Shapiro-Rudin, Narahashi-Nojima and Newmann show very good PAPR reduction capabilities. However, the proposition of introducing either random phase shift to all QPSK symbols or to alternate QPSK symbols has not shown any reduction in PAPR.

A critical and important result for OFDM systems is that further significant reduction in PAPR can be achieved by employing a companding profile with Shapiro-Rudin, Narahashi-Nojima, and Newmann phasing schemes. This is a principal result from the paper. It also appears that companding together with Narahashi-Nojima phasing could reduce the PAPR level down to 0.3506 dB for an OFDM system with 16 subcarriers. It is expected that PAPR could even be less than this value for N > 16, however, this has not yet been confirmed through simulation.

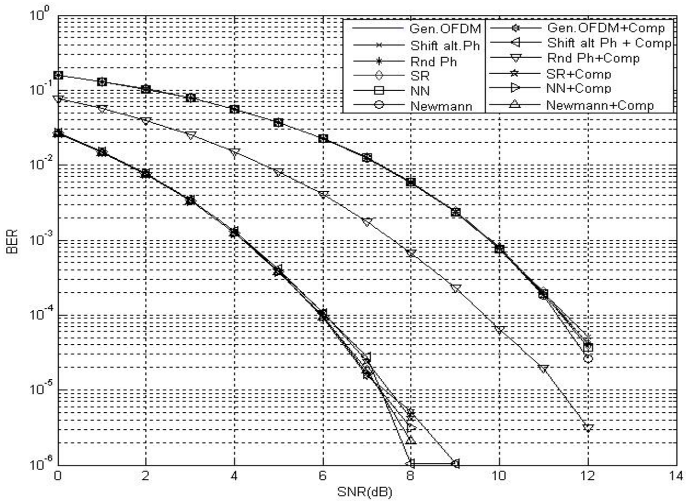

Fig. 3 BER performance of companded and non companded OFDM signals for various phasing schemes in an AWGN channel.

(B) BER Performance of Various Phasing Schemes

Fig. 3 shows the BER performance of the OFDM system with 16 subcarriers for various phasing schemes with and without companding as a function of SNR (equal to the signal energy over the noise density i.e. Eb/No). The upper "curves" in Fig. 3 are all the different forms of OFDM transmissions with no companding. The middle curve is the OFDM transmission with random phasing and companding, whilst the lower "curves" are the OFDM transmissions implementing companding and all other phasing schemes. It can be seen that all phasing schemes have almost the same BER performance in an AWGN channel when no companding is employed. However, a very significant improvement in SNR of about 4 dB for a BER of 10^{-6} could be achieved when companding is combined with most of the various phasing schemes. Interestingly, it is observed that OFDM with companding and random phases has only an SNR improvement of about 2 dB, making it not an optimum technique as far as combined PAPR reduction and BER performance are concerned. Thus a key result appears to be that implementation of a companding profile along with a suitable phasing scheme improves the BER rate performance of OFDM systems.

V. CONCLUSIONS

A technique of reducing the PAPR of OFDM signals by combining companding with various phasing schemes has been presented in this paper. The PAPR and BER performance of μ-Law companding with a PR=2 combined with various phasing schemes was contrasted and compared with the performance of various phasing schemes when no companding is implemented.

Simulation study has shown that the companded OFDM transmissions could overcome peak power problems, and when suitable phasing schemes are applied there could be a further reduction in PAPR to as low as 0.3506 dB. The results also show an improvement in BER performance when companding is used in conjunction with suitable phasing techniques.

In general, these results have demonstrated that reduction in PAPR and improvement in BER rates can be significant when companding is combined with an appropriate phasing scheme. This is a significant result which has implications for further improving the quality of OFDM transmissions over non-linear transmitter amplifier systems and noisy channels.

Future research will concentrate on investigating and quantifying further the influence of PAPR and BER as a function of different modulation mapping schemes, OFDM subcarrier levels, companding PR levels and phasing schemes.

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