

# Performance of Short Transmission Lines Models

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**Abstract** — Performance of a short transmission line models is investigated in this paper. The performance accuracy is assessed by considering three dynamic models for the line. Computer simulations show that the results of the dynamic models agree with the results of the standard line model during steady state operations but they disagree with it during transient circumstances. In controlling the level of transients, the paper results recommend that in energizing a line, it is preferable to switch-on the poles of circuit breaker in a sequential manner rather than in a simultaneous manner. Also, simulation results show that transient voltages in a loaded line case are more severe than the unloaded line case.

**Index Terms** — Transmission Line Modeling, Fault Studies.

## I. INTRODUCTION

It is quite known that transmission lines represent the largest portion of any power system network. Consequently, it makes sense that proper modeling of such lines in different power analysis studies is one of the mandatory issues that should be given clear-cut attention. Such modeling will be the key tool that will be used in the prediction of the line performance during transient studies as well as during steady state studies. Examining the literature, several texts reveal that transmission lines can be divided into three categories. One of the texts [1] specifies that the line length is used to categorize any transmission line:

- Short transmission line has a length not exceeding 80 km
- Medium transmission line length is bounded by 80 km and 250 km limits
- Long transmission line has a length beyond 250 km

Each category is characterized by a certain equivalent circuitry. The short transmission line model ignores all shunting (leakage) effects and hence it represents the line by a simple R-L series equivalent circuit which will be referred in this paper by the term standard model. The medium transmission line model has a  $\pi$  (pi) geometric configuration consisting of 1- a series of resistance and inductance and 2- shunt admittance's reflecting the line capacitances. The long

transmission line model is a set of cascaded (lumped)  $\pi$  configurations.

Checking the accuracy of each model is justified by comparing the results of the investigated model with practical measurements or with the results obtained when considering the line to be made of a sufficient infinite number of cascaded sections.

In this paper, the originality of work [2] is extended to investigate the performance of a short transmission line following certain regular power systems maneuvers. Three dynamic models for the transmission line are developed. Computer simulations will show that steady state results of the three models agree with each other but transient results can be incomparable.

## II. ESTIMATION OF TRANSMISSION LINE PARAMETERS

A first step preceding the development of line models persists in the estimation of transmission line parameters. Figure 1 represents the position of conductor wires in a tower of a short transmission line. The line is composed of three conductor phases and two parallel shield wires. The phases are of a bundle type and having each two conducting wires. The shield wires are grounded at different points along the path of the transmission line. The length of the line is assumed to be 80 km. The line is also assumed to be a transposed one. The dimensions and necessary data required to estimate the line parameters are provided in the following table (Table 1).

TABLE I  
DIMENSIONS AND CONDUCTORS DATA OF THE  
INVESTIGATED LINE

H1=	H2=	D1=	D2=
16.764 m	3.048 m	6.096 m	3.048 m
<b>Phase Conductors:</b> Resistance = 0.100559 $\Omega$ /km, bundle spacing = 20 cm, GMR= 1.02 cm, Radius=1.5 cm			
<b>Shield Wires:</b> Resistance = 2.48 $\Omega$ /km, GMR= 3.048 *10 <sup>-2</sup> cm, Radius=0.25 cm			
Earth Resistivity $\delta$ = 100 $\Omega$ -m Frequency f= 1000 Hz			

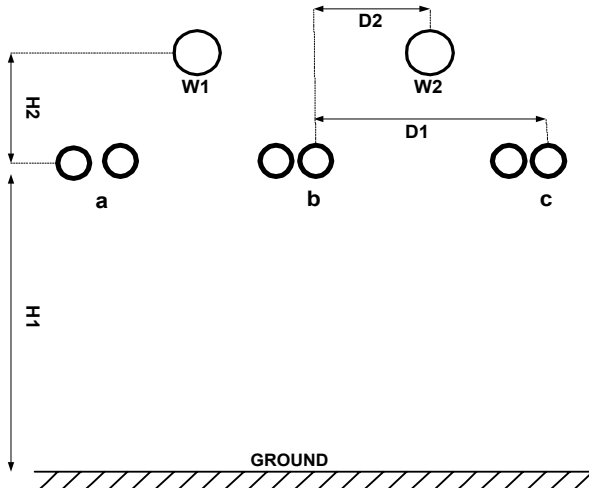


Figure 1: Position of Conductors and Shield Wires in Short Transmission Line

The estimation of the line parameters are evaluated according to the steps enumerated in chapter 3 of reference [3]. That is, the elements of 5X5 resistance matrix, in  $\Omega$  per meter, are estimated as follows:

$R_{ii} = R_i + R_e$  for self resistances and  $R_{ij} = R_e$  for mutual resistances.  $R_e$  is the earth resistance and it can be expressed as:  $R_e = 9.869 \cdot 10^{-7} f$ . The elements of the 5X5-inductance matrix, in Henry per meter, are calculated as:

$L_{ii} = 2 \cdot 10^{-7} \ln(D_e / GMR_i)$  for self inductances and  $L_{ij} = 2 \cdot 10^{-7} \ln(D_e / D_{ij})$  for mutual inductances.  $D_e$  represents the equivalent depth of the conductor carrying the returning current. It can be expressed as:  $D_e = 658.368 \cdot \delta / \sqrt{f}$   $D_{ij}$  is the distance between conductor  $i$  and conductor  $j$ . The effect of bundling is included in the value of the  $GMR_i$ . For the line under investigation, the  $GMR_i$  of the phase conductors will be changed to:

$D_{ii} = \sqrt{GMR_i \cdot \text{bundle spacing}}$  as suggested in section 5 of reference [1].

The elements of the 5X5 permeance matrix, in meter per Farad, are calculated as:

$P_{ii} = 1.79836 \cdot 10^{-10} \ln(2H_i / GMR_i)$  for self-permeances and  $P_{ij} = 1.79836 \cdot 10^{-10} \ln(D_{ij}' / D_{ij})$  for mutual permeances.  $H_i$  is the height of conductor  $i$  above the ground.  $D_{ij}'$  is the distance between conductor  $i$  and the image of conductor  $j$  whereas  $D_{ij}$  is the distance between conductor  $i$  and conductor  $j$ . Similarly, the effect of bundling is taken into consideration by changing the value of the  $GMR_i$  of the phase conductors by  $D_{ii} = \sqrt{GMR_i \cdot \text{bundle spacing}}$ .

The capacitance matrix is calculated by inversion of the permeance matrix.

Based on the previous guidelines and on the provided data, the different parameters matrices are tabulated and are shown in the following table (table II):

TABLE II  
LINE PARAMETERS

Line Resistance Matrix [R] ( $\Omega/\text{km}$ )	1.0875	0.9869	0.9869	0.9869	0.9869
Line Inductance Matrix [L] (H/km)	0.0017	0.0007	0.0006	0.0008	0.0006
Line Permeance Matrix [C] ( $\mu\text{F}/\text{km}$ )	1.189	0.309	0.193	0.385	0.245

Since the shield wires are grounded, the previous parameters matrices can be reduced to 3X3 matrices. The results of such reductions are shown in table III.

TABLE III  
REDUCED LINE PARAMETERS

Line Resistance Matrix [R] ( $\Omega/\text{km}$ )	0.6501	0.5495	0.5495
Line Inductance Matrix [L] (H/km)	0.0014	0.0004	0.0003
Line Capacitance Matrix [C] ( $\mu\text{F}/\text{km}$ )	0.00948	-0.00174	-0.00069

In obtaining the reduced capacitance matrix the reader is informed that the original 5X5 permeance matrix has to be reduced first to 3X3 matrix then the inversion of the result will be the one shown in last right cell of table III.

### III. TRANSMISSION LINE MODELS

Figure 2 represents the one line diagram of the line under investigation. It consists of a three phase source feeding a three phase load.

In order to predict the expected voltages at any location of the line and in particular at the source ( $V_s$ ) and load ( $V_l$ ) sides, it is required to develop a dynamic model of the line. Three possible models are developed:

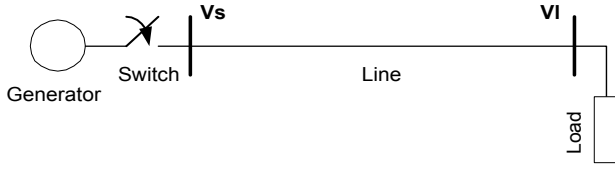


Figure 2: Investigated System

#### A- Standard Line Model:

In developing this model for the short line, all shunt elements are neglected. The state equations representing the phase currents can be easily derived to obey the following system:

$$\left[ \frac{di_\phi}{dt} \right] = [L_s + L + L_l]^{-1} \{ [e_{s_\phi}] - [R_s + R + R_l] [i_\phi] \} \quad (1)$$

where:

$e_{s_\phi}$  represent the source voltages,

$R$  and  $L$  are the total line resistance and inductance matrices respectively,

$R_s$  and  $L_s$  are the source resistance and inductance matrices respectively,

and  $R_l$  and  $L_l$  are the load resistance and inductance matrices respectively.

The subscript  $\phi$  is referred to the phase number which can be either phase a, or phase b or phase c.

Having predicted the values of the state currents and their corresponding derivatives, the sending and load voltages values are tabulated as:

$$[V_{s_\phi}] = [e_{s_\phi}] - [R_s] [i_\phi] - [L_s] \left[ \frac{di_\phi}{dt} \right] \quad (2)$$

$$[V_{l_\phi}] = [R_l] [i_\phi] + [L_l] \left[ \frac{di_\phi}{dt} \right] \quad (3)$$

#### B- $\pi$ Line Model:

The line is divided into "n" sections. The sections are lumped together. Figure 3 represents one of these sections. Such section is attributed the letter "p".

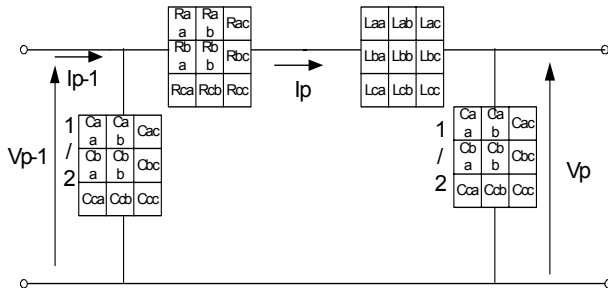


Figure 3: Pi Model of section "p"

The system of state equations that can be used to model such "p" section can be easily derived. It is in the form:

$$\left[ \frac{di_{p,\phi}}{dt} \right] = -[L]^{-1} [R] [i_{p,\phi}] + [L]^{-1} \{ [V_{(p-1),\phi}] - [V_{p,\phi}] \} \quad (4)$$

$$\left[ \frac{dv_{p,\phi}}{dt} \right] = [C]^{-1} \{ [i_{p,\phi}] - [i_{(p+1),\phi}] \} \quad (5)$$

$R$ ,  $L$ , and  $C$  should be equal to the total section resistance, inductance, and capacitance matrices respectively.

At the beginning of the line (i.e. generator side) and in which the phase currents will be assigned a subscript '0', the following system is obtained:

$$\left[ \frac{di_{0,\phi}}{dt} \right] = -[L_s]^{-1} [R_s] [i_{0,\phi}] + [L_s]^{-1} \{ [e_{s,\phi}] - [V_{0,\phi}] \} \quad (6)$$

$$\left[ \frac{dv_{0,\phi}}{dt} \right] = 2[C]^{-1} \{ [i_{0,\phi}] - [i_{1,\phi}] \} \quad (7)$$

Similarly at the load side in which the subscript 'n+1' is attributed to the load phase currents, the following system is derived:

$$\left[ \frac{di_{(n+1),\phi}}{dt} \right] = -[L_l]^{-1} [R_l] [i_{(n+1),\phi}] + [L_l]^{-1} \{ [V_{n,\phi}] \} \quad (8)$$

$$\left[ \frac{dv_{n,\phi}}{dt} \right] = 2[C]^{-1} \{ [i_{n,\phi}] - [i_{(n+1),\phi}] \} \quad (9)$$

#### C- T Line Model:

The line can be similarly divided into "n" sections. The sections are lumped together. Figure 4 represents the equivalent circuit of one of the line sections.

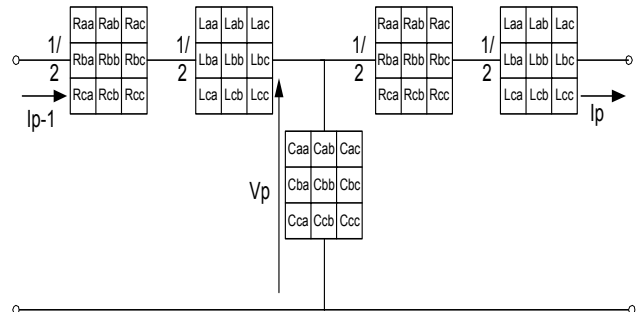


Figure 4: T Model of section "p"

The system of state equations that can be used to represent such equivalent circuit is of the form:

$$\left[ \frac{di_{p,\phi}}{dt} \right] = -[L]^{-1} [R] i_{p,\phi} + [L]^{-1} \{ [V_{p,\phi}] - [V_{(p+1),\phi}] \} \quad (10)$$

$$\left[ \frac{dv_{p,\phi}}{dt} \right] = [C]^{-1} \{ [i_{(p-1),\phi}] - [i_{p,\phi}] \} \quad (11)$$

At the generator side in which the current is attributed the subscript "0", the following system can be found:

$$\left[ \frac{di_{0,\phi}}{dt} \right] = - \left[ L_s + \frac{1}{2} L \right]^{-1} [R_s] i_{0,\phi} + \left[ L_s + \frac{1}{2} L \right]^{-1} \{ [e_{s,\phi}] - [V_{1,\phi}] \} \quad (12)$$

$$\left[ \frac{dv_{1,\phi}}{dt} \right] = [C]^{-1} \{ [i_{0,\phi}] - [i_{1,\phi}] \} \quad (13)$$

At the load side, the following two systems can be obtained:

$$\left[ \frac{dv_{n,\phi}}{dt} \right] = [C]^{-1} \{ [i_{(n-1),\phi}] - [i_{n,\phi}] \} \quad (14)$$

$$\left[ \frac{di_{n,\phi}}{dt} \right] = - \left[ L_l + \frac{1}{2} L \right]^{-1} [R_l] i_{n,\phi} + \left[ L_l + \frac{1}{2} L \right]^{-1} [V_{n,\phi}] \quad (15)$$

#### IV. SIMULATIONS

In order to test the performance of the developed models, a series of simulations are done. The simulations consist of energizing the short line. As it was mentioned earlier the line is 80 km long and it is made of 16 cascaded sections (i.e. each section is therefore 5 km long). The first test is done by assuming that the line is initially loaded and it is desired to put it into service through a simultaneous closing of all three poles of the circuit breaker. The circuit breaker is located at the generator side. The results of the receiving voltage (i.e. load voltages) of the previous three dynamic models are depicted in figure 5. The left column of the figure represent phase **a**, phase **b**, and phase **c** receiving voltages respectively. As it is observed, the responses of the  $\pi$  and **T** line models are almost superimposed on each other under transient moments as well as later under steady state conditions. The response of the first model (standard model) shows discrepancy with the responses of the two other line models during transients but agree with them during steady states. To get a clear comparison between the three models, the first few moments of the transient portion of the three phase voltages are zoomed out and are shown in the right column of the figure (figure 5).

In targeting the control of transient levels encountered during the energizing process, the same previous test is repeated but this time the circuit

breaker three poles are assumed to close in a sequential manner. That is 120 degree apart from each other. The obtained results are graphed in figure 6. Once again all three models responses agree with each other in steady state cycles but in the transient portion only the  $\pi$  and the **T** models agree and differ clearly from the first model response. But, what it is interesting from such test, is that the transients encountered during the initial moments are much lowered down when compared to figure 5 responses.

In the third test, the line is initially unloaded and the circuit breaker three poles are closed simultaneously, figure 7 represents the results of the phase voltages at the receiving end. It is seen that the transients in phases **b** and **c** are much better than their corresponding ones in the case of loaded line as has been depicted previously in figure 5. To decrease the observed transients even further, a fourth test is held on the unloaded line by pretending that the circuit breaker poles are closed in a sequential manner. Figure 8 represents the results of the simulation. Such results strengthen the argument: " it is preferable to switch-on the line under no-load and in a sequential manner".

To investigate the effect of the number of sections representing the short line, the  $\pi$  line model has been tested and that is by assuming that the line is made of 16 sections one time and of two sections the other time. Figure 9 depicts the results. It is clearly shown that the results of the 16 sections agree with the ones of the two sections under steady state as well under the first transient moments.

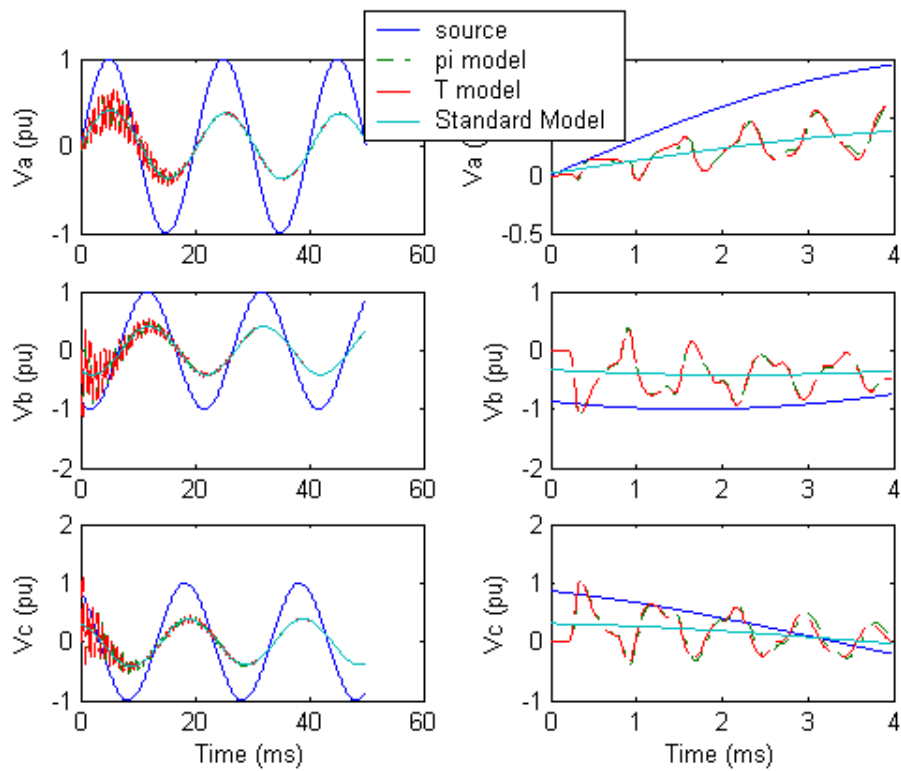
#### V. CONCLUSION

The accuracy of certain short transmission line models has been investigated in this paper. This has been done through the development of three dynamic models of the line. Computer simulations of the three models indicate all three models end up with superimposed results during steady states, but differ in transient moments. Levels of transient over voltages can be monitored efficiently by switching-on line phases sequentially.

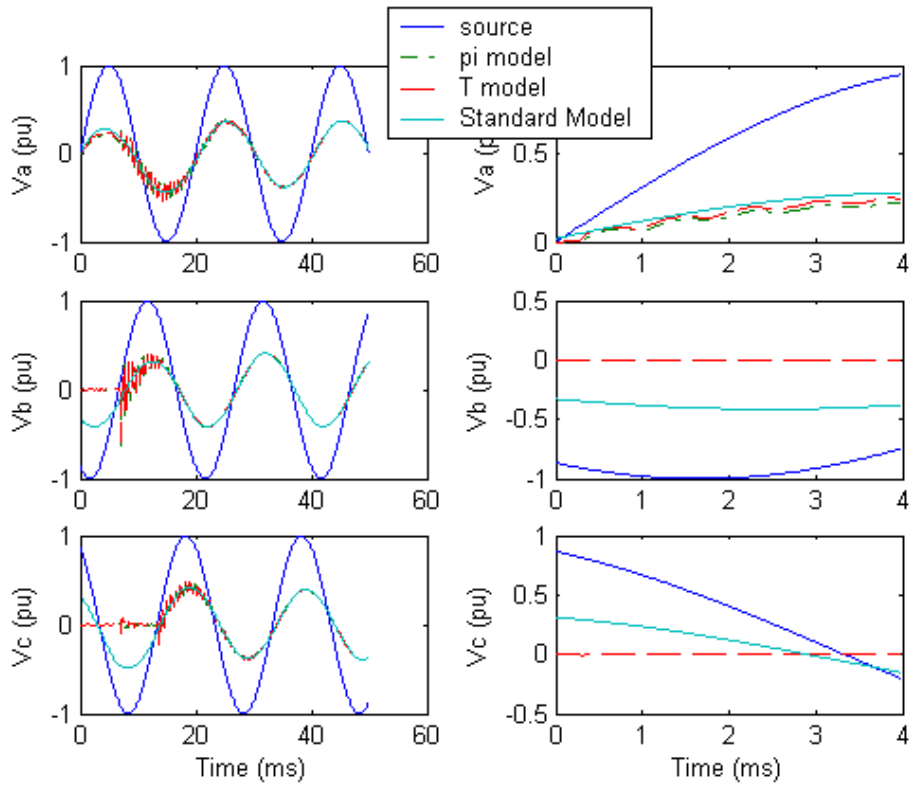
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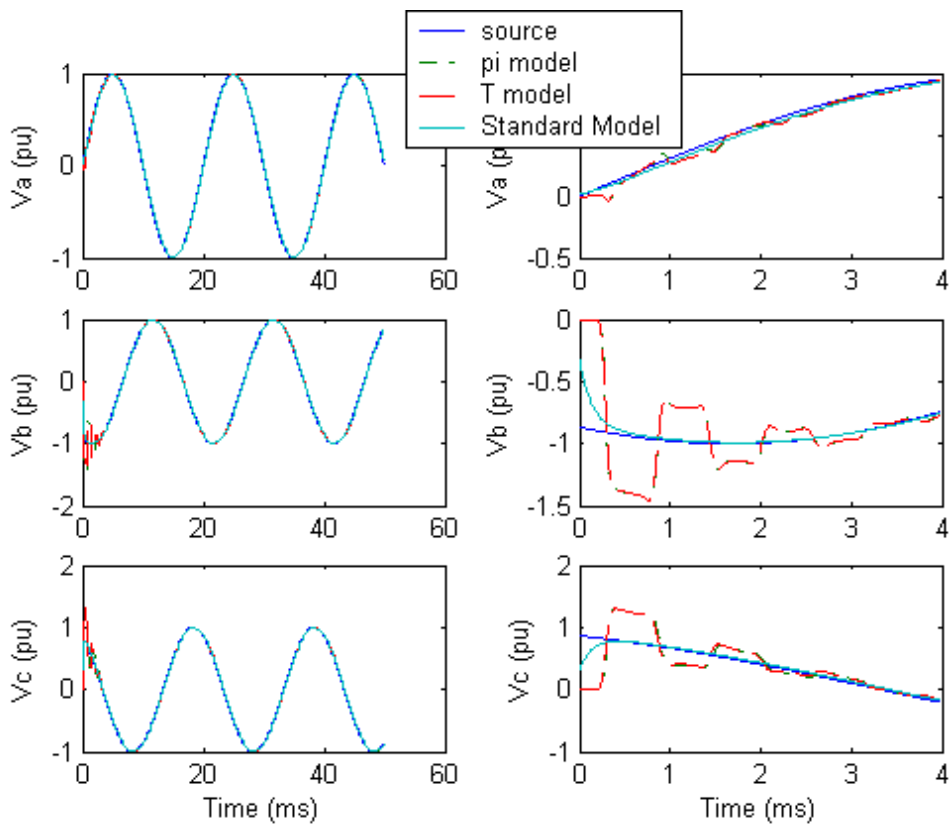
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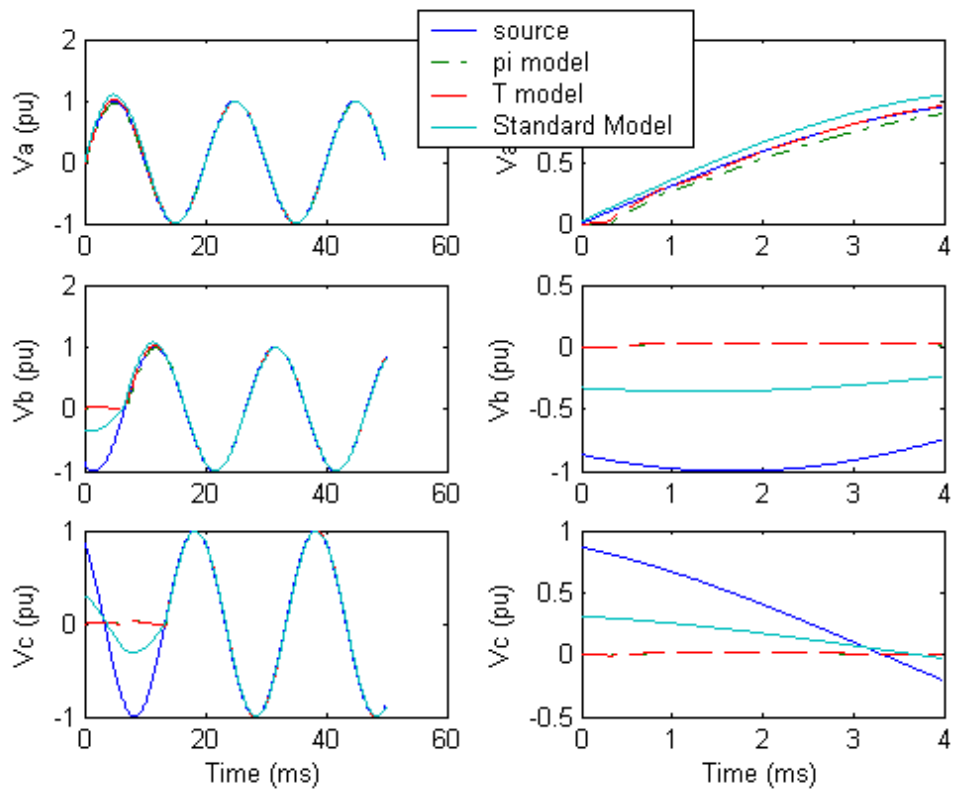
**Figure 5:** Receiving phase a, phase b, and phase c voltages for pre-loaded short line and circuit breaker poles closed simultaneously.



**Figure 6:** Receiving phase a, phase b, and phase c voltages for pre-loaded short line and circuit breaker poles closed sequentially.



**Figure 7:** Receiving phase a, phase b, and phase c voltages for unloaded short line and circuit breaker poles closed simultaneously.



**Figure 8:** Receiving phase a, phase b, and phase c voltages for unloaded short line and circuit breaker poles closed sequentially