Optimum design of high frequency transformer for compact and light weight switch mode power supplies (SMPS)

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Abstract — in this paper a new approach for optimization of high frequency transformer design is presented. The presented design method is based on a restatement of the traditional transformer design equations to include non-sinusoidal switching waveforms and high frequency skin and proximity effects. In this optimization procedure both electric and thermal effects in the transformer is considered. Wave form of voltage and current, and maximum acceptable temperature rise, are used as input data. The aim of this procedure is the selection of the smallest core that can deliver desired power, and determination of optimum flux density and current density to reach a transformer with high power density and admissible temperature rise. Since the transformer is the major contributor to the volume and weight of the Power Supply, the results of this transformer analysis can be used for entire power supply optimization as well. Finally the validity of presented method is analyzed.

Index terms — **losses, optimization method, proximity, skin, switch mode power supplies, thermal model, transformer**

I. INTRODUCTION

Magnetic component technology has received considerable attention in recent years. It is widely recognized that the ability to manufacture small and efficient magnetic components is the key to achieving high power density that is important factor for power supply in application such as satellite, airplane, computer or any compact and mobile systems. It is a well-known fact that reduction in the size of magnetic components have been achieved by operating at higher frequencies, mainly in switching circuits. However the high-frequency transformer is the major contributor in the size of any SMPS since it determines about 25% of the overall volume and more than 30% of the overall weight [1]. Fundamental issues in the design of any high-power, high-frequency transformer are to minimize the power loss, volume and weight. This paper explores the optimum design of a high-power and high-frequency transformer, which contains:

1) Selection of the smallest standard core size relevant to the throughput power, frequency, and transformer operating temperature.

2) Calculation of the optimum flux and current density providing minimum transformer loss.

In order to accomplish such a complex optimization procedure, investigations were carried out in the following areas:

1) Core loss determination
2) Copper loss determination
3) Thermal modeling
4) Optimization

II) LOSS COMPONENT

The total loss of a transformer consists of two basic parts: copper loss and core loss. In two following subsection both of these components are discussed in detail.

A) Core loss

The total core loss is a result of three loss mechanisms: hysteresis, eddy current and stray losses. Each of them computed as bellow [2]:

$$P_{hys} = \alpha . f . B_{\max}^{n} \tag{1}$$

$$P_{eddy} = \frac{(\pi B_{\max} fh)^2}{6\rho} \tag{2}$$

$$P_{stray} = \left[1.628(\frac{2l}{h})-1\right]P_{eddy} \tag{3}$$

Where α, n, ρ, h and l are constants depend on material property and dimensions of the core, f is frequency and B_{max} is flux density. As can be seen, all above equation are function of f and B_{max} thus we can approximate the sum of them by:

$$P_{coreloss} = c.V_{core}.f^{x} B_{\max}^{y} \tag{4}$$

Where V_{core} is the volume of the core and c, x and y result from the manufacturer data sheet by curve fitting method [2,3].

B) Copper loss

By definition, the copper loss in a two winding transformer carrying high frequency current with an effective value I_{rms1} and I_{rms2} in primary and secondary side is:

$$P_{cu} = R_1 I_{rms1}^2 + R_2 I_{rms2}^2 = F_1 R_{dc1} I_{rms1}^2 + F_2 R_{dc2} I_{rms2}^2 \tag{5}$$

Where:

$$F_i = R_{effi} / R_{dci} \quad (6)$$

Is called an ac-resistance coefficient for i^{th} harmonic and model skin and proximity effect in the conductor [3]. This coefficient is calculated by improved formulation based on Dowell work [4]. This formulation is as follows:

$$F_i = \frac{1}{2}\sum_{n=1}^{\infty} K_n \left(\frac{I_{ni}}{I_{rms}}\right)^2 \quad (7)$$

Where I_{ni}, because of the nonsinuosoidal nature of the current wave form of these transformers, are n^{th} harmonic component of the current of i^{th} winding. K_n is calculated as follows[3]:

$$K_n = \sqrt{n}\Delta\left[\frac{\sinh 2\sqrt{n}\Delta + \sin 2\sqrt{n}\Delta}{\cosh 2\sqrt{n}\Delta - \cos 2\sqrt{n}\Delta} + \frac{2(m^2-1)}{3}\times\frac{\sinh\sqrt{n}\Delta - \sin\sqrt{n}\Delta}{\cosh\sqrt{n}\Delta + \cos\sqrt{n}\Delta}\right] \quad (8)$$

Where m is the number of layer and Δ is normalized diameter and shown as:

$$\Delta = \frac{h}{\delta}\sqrt{K_{layer}} \quad (9)$$

Where $\delta = \sqrt{2/\sigma\mu_0\omega}$ is skin depth at fundamental frequency, h is thickness of the layer, μ_0 is permeability of air, σ is the conductivity of the conductor and K_{layer} is the porosity factor and is equal to the ratio of the total net height of the copper to the height of the core window. Following picture illustrate these parameters.

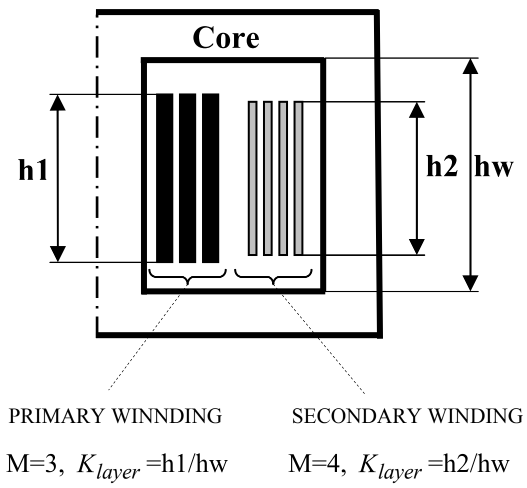

Fig. 1.schematic diagram of transformer core window

III) TRANSFORMER TEMPERATURE RISE

Transformer temperature estimation is needed in design process in order to verify that material temperature specifications are not exceeded and also to take temperature dependence of loss mechanisms into account.

Linear thermal resistances are frequently used in electronic design. However, these should be used only for conductive heat transfer. But in transformer thermal modeling there are another mechanisms in heat transfer that are convection and radiation. These two phenomena are nonlinear and depend on the instantaneous temperature differences between the object and ambient. In this work heat transfer mechanisms is considered and the transformer temperature rise is calculated as follows [6].

Dissipated power = Pcnvection + Pradiation

A) Convection

Convective heat transfer capacity $P_{convection}$ (W) of a cube with dimensions in millimeter is:

$$P_{conv} = P_{conv-up} + P_{conv-bottom} + P_{conv-side} \quad (10)$$

Those are dissipated power from up, bottom and side surface of the body. Each of them is:

$$P_{conv-up} = 2.34 \times A_{up} \times \Delta T^{1.25} \quad (11)$$

$$P_{conv-bottom} = 1.26 \times A_{bottom} \times \Delta T^{1.25} \quad (12)$$

$$P_{conv-side} = 1.91 \times A_{side} \times \Delta T^{1.25} \quad (13)$$

Where A_{up} is the area of the upper surface of the body and so on. ΔT is temperature rise and is:

$$\Delta T = T_{surface} - T_{ambient} \quad (14)$$

Substituting (11), (12) and (13) into (10) yield:

$$P_{conv-up} = (2.34 A_{up} + 1.26 A_{bottom} + 1.91 A_{side})\Delta T^{1.25} \quad (15)$$

B) Radiation

Radiative heat transfer capacity $P_{radiation}$ (w) of an object with emissive factor of 0.85 and dimensions in millimeter is:

$$P_{rad} = 4.845 \times 10^{-8} \times A_{total}\left[(T_{surface})^4 - (T_{ambient})^4\right] \quad (16)$$

IV) THERMAL SYSTEM

Total dissipated power is a function of three parameters as:

$$P_{total} = f(T_{surface}, T_{ambient}, \text{object dimention}) \quad (17)$$

We can utilize of this function in two forms as shown below:

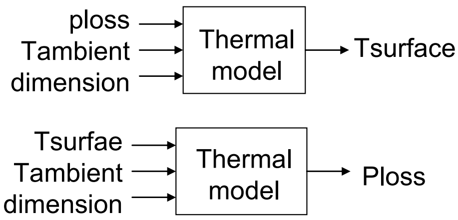

Fig. 2. Thermal system

V) OPTIMIZATION

The aim of this paper is to reach minimum size and maximum efficiency for transformer, thus the object function that has to minimize is as:

$$f = volum \times P_{loss} \quad (18)$$

Since McLyman's work [7] it is obvious that the product of core area and winding area is a stand for transformer volume. Thus we use area product, A_p instead of volume in our object function. A_p is calculated as below:

$$A_P = A_{core} \times A_{cu} \quad (19)$$

$$A_{core} = \frac{E_1}{K_f N_1 B_{\max} f} \quad (20)$$

Where E_1 is the effective value of primary voltage, N_1 is number of turn of primary and K_f is the form factor of the voltage waveform that is 4.44 for sinusoidal and 4 for square waveform [7,8].

$$A_{cu} = \frac{N_1 I_1}{K_{cu} j} + \frac{N_2 I_2}{K_{cu} j} \quad (21)$$

Where K_{cu} is copper fill factor and j is the current density that assume to be equal at primary and secondary winding. Substitution of (20) and (21) into (19) yields:

$$A_P = \frac{E_1 I_1 + E_2 I_2}{K B_{\max} J.f} \quad (22)$$

And $K = K_f K_{cu}$

Substitution (4), (5) and (22) into (18) for given frequency results in :

$$f = f(j, B_{\max}) = \frac{E_1 I_1 + E_2 I_2}{K B_{\max} j.f} (F_1 R_{dc1} I_1^2 + F_2 R_{dc2} I_2^2 + c.V_{core} f^x B_{\max}^y) \quad (23)$$

In (23) R_{dc1} and R_{dc2} are directly depend on j and indirectly to $B_{\max}$. To replace them with their equivalents the following substitution is done:

$$R_{dc1} = \rho \frac{l_1}{a_1} = \rho \frac{l_{av1} N_1}{a_1} \quad (24)$$

Where l_{av1} is average length of one turn of the primary winding and N_1 and a_1 are number turn and copper cross section of it respectively and so on for secondary. Now there is:

$$f = f(j, B_{\max}) = \frac{E_1 I_1 + E_2 I_2}{K.f} \left(\frac{F_1 \rho l_{av1} N_1 I_1^2}{a_1 B_{\max} j} + \frac{F_2 \rho l_{av2} N_2 I_2^2}{a_2 B_{\max} j} + \frac{c.V_{core} f^x B_{\max}^{y-1}}{j} \right) \quad (25)$$

Where:

$a_1.j = I_1$, $a_2.j = I_2$ and $N_i = \dfrac{E_i}{K_v.f.A_{core}.B_{\max}}$

Substituting these into (25) yields:

$$f = f(j, B_{\max}) = \frac{E_1 I_1 + E_2 I_2}{K.f} \left(\frac{F_1 \rho l_{av1} E_1 I_1}{K_v A_{core} B_{\max}^2} + \frac{F_2 \rho l_{av2} E_2 I_2}{K_v A_{core} B_{\max}^2} + \frac{c.V_{core} f^x B_{\max}^{y-1}}{j} \right) \quad (26)$$

In (26) all the parameters except j and $B_{\max}$ are constant thus it can be written:

$$f = f(j, B_{\max}) = K_1 \left(\frac{K_2}{B_{\max}^2} + K_3 \frac{B_{\max}^{y-1}}{j} \right) \quad (27)$$

Where:

$$K_1 = \frac{E_1 I_1 + E_2 I_2}{K.f} \quad (28)$$

$$K_2 = \left[F_1 \rho l_{av1} E_1 I_1 + F_2 \rho l_{av2} E_2 I_2 \right] / (K_v A_{core}) \quad (29)$$

$$K_3 = c V_{core} f^x \quad (30)$$

Eq. (27) is a two variable function that can be optimized by partial derivation but since j is in the denominator of one term, if it become larger, the better result obtained. But $B_{\max}$ is in denominator of one term and nominator of another, thus it has an optimum value and is our derivative variable:

$$\frac{\partial f}{\partial B_{\max}} = K_1\left[-\frac{2K_2}{B_{\max}^3} + K_3(y-1)\frac{B_{\max}^{y-2}}{j}\right] = 0 \quad (31)$$

$$\Rightarrow B_{\max\cdot opt} = \left[\frac{2K_2}{(y-1)K_3}\,j\right]^{\frac{1}{y+1}} \quad (32)$$

Substitution of (32) into equation that is the sum of all loss component yield

$$P_{loss.total} = \left[\left(\frac{(y-1)K_3}{2}\right)^{\frac{1}{y+1}} + K_3^{\frac{1}{y+1}}\left(\frac{2}{y-1}\right)\right]^{\frac{y}{y+1}} (K_2 j)^{\frac{y}{y+1}} \quad (33)$$

If:

$$K_4 = \left[\frac{y-1}{2}\right]^{\frac{1}{y+1}} + \left[\frac{2}{y-1}\right]^{\frac{y}{y+1}} \quad (34)$$

Then:

$$P_{loss.total} = K_4 K_3^{\frac{1}{y+1}} (K_2 j)^{\frac{y}{y+1}} \quad (35)$$

$$\Rightarrow j = \frac{1}{K_2}\left[\frac{P_{loss}}{K_4} K_3^{\frac{-1}{y+1}}\right]^{\frac{y+1}{y}} \quad (36)$$

From thermal model we can calculate the maximum value of $P_{loss.total}$ that reach transformer temperature rise to the safety margin. Substitution this $P_{loss.total}$ in (36) yield maximum admissible j and by (32) the optimum $B_{\max}$ can be calculated. Substituting this j and $B_{\max}$ in (22) give minimum A_p or minimum core size. The Fig.3 flowchart, shows the optimization procedure .

VI) OPTIMIZATION METHOD VALIDATION

The proposed optimization procedure is utilized for design a transformer that must operate in the circuit that shows in Fig 4. the voltage and current wave form of this transformer is illustrated in Fig.5. Because the rms value of voltage and current are 182.58 volt and 6.83 ampere respectively, apparent power of the designed transformer is 1247.048 VA at 20 kHz. It is assumed that transformer temperature not exceeds of 100^oC. This means that $T_{surface} = 100^oC$. For this transformer, EE ferrite core made of manifer198A material is used. These cores are available in standard range such as E25, E32 , E42, E55, E65, and other standard size. The optimization routine propose E55, B_{max} = 3357 gauss and j = 3.23 a/mm^2 . Power loss and temperature rise for E42, E55 and E65 are calculated. The result is showed in Table I. as can be seen from this table, to achieve desired throughput power T_{max} will be higher than 100^oC for E42 core that is smaller than E55. In case E65, surface temperature is 89^oC that is smaller than 100^oC. This means that E65 core don't use in optimum operating point and its full capacity is not used.

TABLE I. optimum design for EE core

Core type	$B_{max\ opt}$ (g)	J_{opt} (a/mm^2)	T_{max} °c
E42	3826	3.81	115.1
E55	3357	3.23	97.8
E65	2994	2.98	89

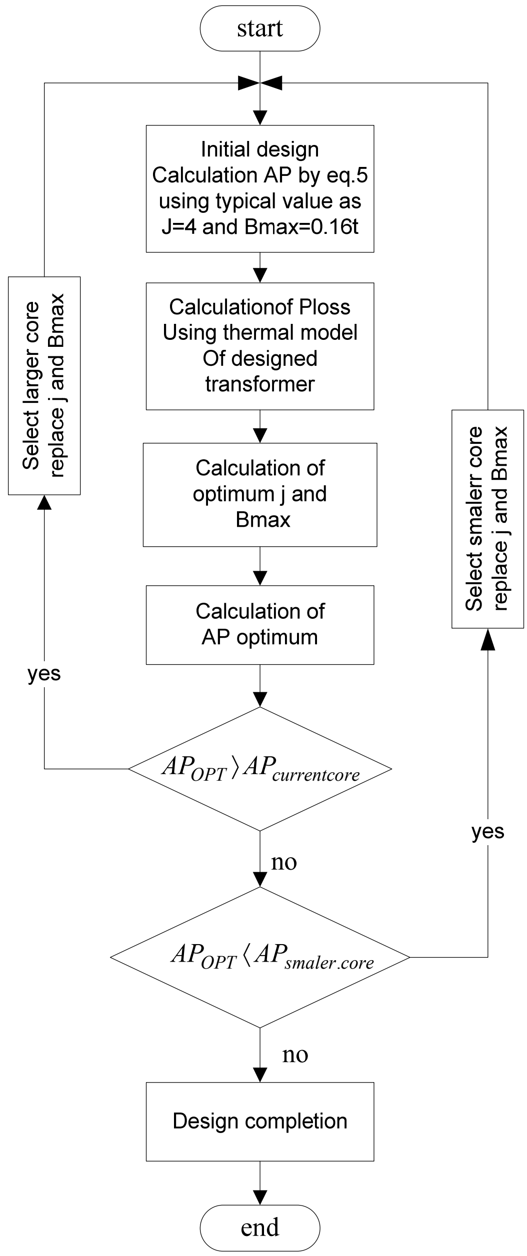

Fig. 3.Optimization procedure

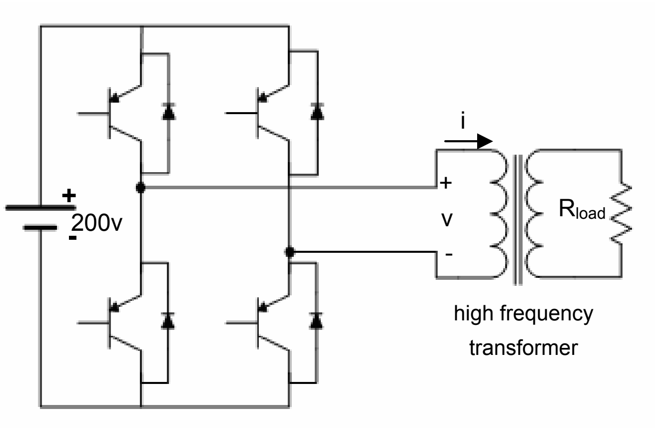

Fig. 4 Simulated circuit for high frequency voltage generation

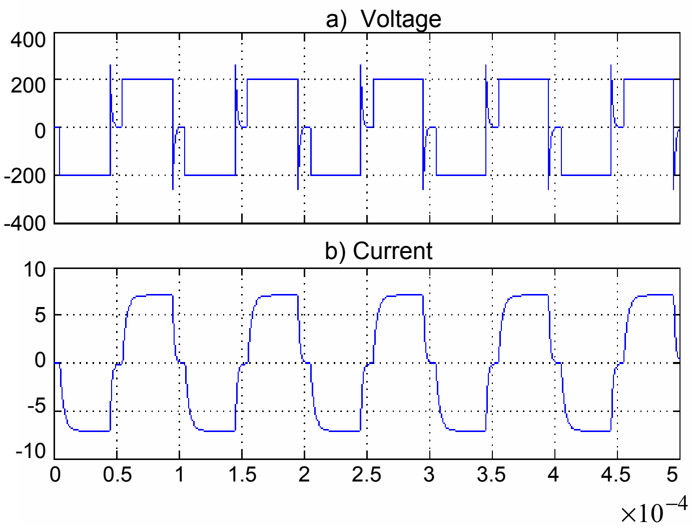

Fig. 5. a)Voltage (volt), b)current (ampere),vs. time (sec)

In another test an investigative analysis is done by plotting power loss versus B_{max} and j for E55 (which according to Table I and because of obtained temperature, is the optimum core size) around optimum operating point. For this, because of close relation between B_{max} and j, at first B_{max} is set to given value by change in primary turn, then since dimensions of core window are specified, j and P_{loss} can be calculated. Table II shows the result of this investigation.

TABLE II. operation of E55 around optimum point

B_{max} (G)	j(A/mm^2)	P_{loss} (w)	η %	$T_{surface}$ °c
3967	2.6358	196.34	86.42	152.6
3795	2.7556	148.27	88.11	134.0
3637	2.8754	140.29	88.75	126.9
3491	2.9952	122.7	90.16	111.3
3357	3.230	107.49	91.38	97.8
3233	3.2848	122.2	90.21	110.89
3117	3.3546	147.89	88.14	1333.3
3010	3.4744	158.12	87.86	142.7

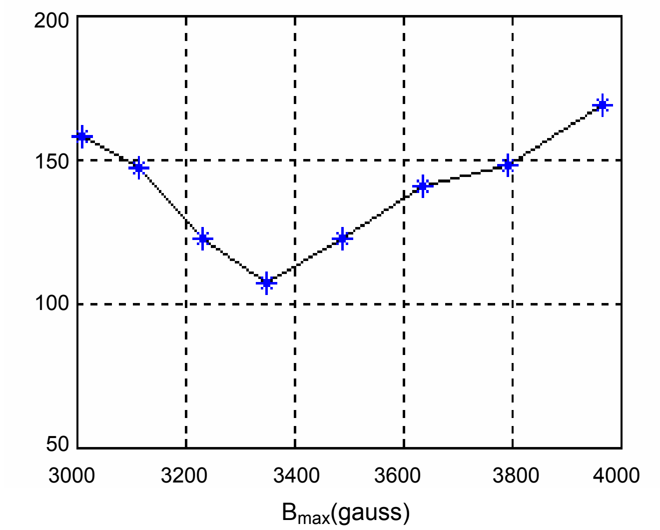

Fig.6 P_{loss} (watt) vs. B_{max}(gauss)

Fig. 6 shows the result of this analysis in a graph. As can be seen from Table I, Table II and Fig. 6 the optimization algorithm accurately determine the smallest core that can operate at given condition, and the design parameter (B_{max} and j) for selected core. However it must be considered that correspondence between analytical result and experimental result is depend on the accuracy of equation that use for calculation of transformer specification such as core and copper loss and thermal model. Since these equation can predict the transformer specification with error less than 10 %[6], the whole procedure is reliable up to 90%. The authors is engaged in implementation a set up to test prototype optimized transformers and evaluation of validity of proposed optimization procedure in practice.

VII) CONCLUSION

A new methodology for designing transformers has been described. The design procedure so optimized to determine the smallest possible core and to minimize the core and winding losses by selection optimum current density and flux densitie based on the electrical and thermal analysis of the transformer. The design procedure flowchart is suitable for use in a computer application in conjunction with a database of core size and materials. Analytical investigation showed that optimization procedure is accurate but because of the approximation in thermal and loss equations, accordance between analytical result and practical result could not be completely expected. In terms of that, the main advantage of the design procedure developed is that it allows us to select the core and calculate winding parameters, i.e., it provides (with acceptable accuracy) the most necessary design information. This information can be further used for more precise (finite element or finite difference) analysis, if necessary.

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