Evaluation of Substation Reliability Indices and Outage Cost Based on Fuzzy Arithmetic

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***Abstract* — Substations and switching stations are critical segments in an electric power system. Their impacts on system operation and the consequential effects of their components failure are usually more dominant in overall power system reliability. There is always a degree of uncertainty in the practical input data used in reliability analysis. This paper presents a conceptual possibilistic approach using fuzzy set theory to manage the uncertainties in the reliability input data such as failure rate, repair time and operation of protective devices. Suitable fuzzy numbers are proposed for components reliability data. Load of feeders are also represented by fuzzy numbers and their impact on ENS (Energy Not Supplied) and OC (Outage Cost) are examined. The proposed approach is applied to a practical 230/63 KV substation and the results are presented.**

***Index Terms* — Substation Reliability, fuzzy Arithmetic, Outage Cost, Possibility Distribution.**

I. INTRODUCTION

Substation is that portion of an electric power system which forms a link between the bulk transmission system and the generation system. Substation is therefore an essential and critical segment of an electric power system. Its reliability can have significant impact on overall power system reliability. Substation comprises of more complex switching arrangements than other segments of a power system. This makes it more difficult to assess the reliability of substations. The reliability of substations depends primarily on the reliability of the basic station components and the arrangement of these components in the station [1].

There are two fundamental approaches to substations reliability evaluation, analytical and Monte Carlo simulation methods. These approaches require sufficient data for probabilistic behavior of each system component. Due to the high availability of system components, there are not sufficient statistical data to represent the failure and repair process of components. Also statistics- that are gathered by the utility into the form of a database- may not be accurate [2]. Therefore the calculated reliability indices suffer from a degree of uncertainty. Also it is possible that in a substation there are a number of types of one component, each of which has its own failure rate. Therefore component reliability data vary in certain ranges. Moreover, for different ages of the same type of equipment, the failure rates vary. Therefore the parameters used in calculating the reliability indices of substation such as failure rates, repair times, operation of protective devices, may include considerable uncertainty, which subsequently create uncertainty in the calculated indices. An adequate tool for incorporating these uncertainties is to use fuzzy arithmetic. Some application in reliability analysis of power systems using fuzzy arithmetic have been suggested in the past[2][3].Fuzzy sets were also successfully used in reliability evaluation of power distribution systems[4][5].Also in substations reliability evaluation, fuzzy sets were applied and failure rates and repair times of components are considered fuzzy numbers and possibility distributions of three basic indices of load points-failure rate, outage duration and annual outage duration-are determined [6]. In this paper, a method for building membership functions of components is presented. In order to conduct comprehensive studies of effects of all uncertainties on reliability indices, "the probability of operation of protective devices" and the "load of feeders" are modeled by fuzzy numbers. In addition to the possibility distributions of three basic indices, the possibility distribution of ENS and OC of load points and the substation are also calculated.

For defuzzification of fuzzy indices, center of area (COA) approach is selected. Finally, the applicability and flexibility of the proposed method are illustrated by calculating the three basic reliability indices, energy not supplied and outage costs of a practical substation (230/63 KV Hamedan substation).

II. Fuzzy Reliability

Conventional reliability theory is based on two fundamental assumptions: probability assumption and binary state assumption. By considering fuzzy sets in reliability theory, fuzzy reliability, may exhibit three forms [7]:

(1)-PROFUST reliability theory, based on the probability assumption and the fuzzy state assumption.

(2)-POSBIST reliability theory, based on the possibility assumption and the binary state assumption.

(3)-POSFUST reliability theory, based on the possibility assumption and the fuzzy state assumption.

In this paper, from these basis, the *PROFUST* is selected which, the states of components are well defined and fuzzy number (over interval of confidence) is used to account for imprecision and uncertainties in probability of finding a particular component at a given state.

III. Membership Functions of Input Reliability Data

A. Component Reliability Data

There are some sources for constructing membership functions of component reliability data such as: statistics events (that are gathered by the utility in the form of a database), expert opinions and so on. From each source, one membership functions can be determined. A convenient fuzzy number, for membership functions is triangular or trapezoidal fuzzy numbers; because these fuzzy numbers fit most of the cases and provide fast computation time [8]. Using the priority of sources, membership functions that are obtained from different sources must be combined. There are a number of approaches for combining membership functions which can be used [9].

B. Load of Feeders

In conventional reliability evaluation of substations (analytical method), average load of feeders are used for calculation of reliability indices such as ENS. In the long term planning, load values can be affected by many factors some of them being hardly forecasted [10]. In power system operation, due to variation of demand consumption in a period due to daily cycle or to weather condition, load of feeders of substations vary. Therefore the use of deterministic value, expected value, for load of feeders may include considerable uncertainty.

In this paper, to manage uncertainties in load of feeders in planning and operating phases, fuzzy set theory is used. A trapezoidal fuzzy number is convenient for modeling uncertainties of load points [10].

IV. Fuzzy Arithmetic to Reliability Evaluation of Substations

A. Analytical Method

Substations are complex systems; therefore to evaluate reliability of substations, it is necessary to find the failure modes of load points (feeders).The minimal cut set theory is a powerful computational tool for evaluating the failure modes of load points. A minimal cut set is a set of system components, when failed, cause failure of system but when any one the components works, cause system to operate. Consequently the components of the cut set are effectively connected in parallel. In addition, the system fails if any one of the cut sets fails. Consequently each cut set is effectively in series with the other cut sets.

In substations reliability evaluation, there are three types of basic indices for load points (feeders): average failure rate (λ), average outage time (*r*) and annual outage time (*U*). For i^{th} load point, these indices are given by

$$\lambda_i = \sum_{j\in m(i)} \lambda_j \tag{1}$$

$$U_i = \sum_{j\in m(i)} \lambda_j \cdot r_j \tag{2}$$

$$\mathrm{r_i} = \frac{U_i}{\lambda_i} \tag{3}$$

Where m(i): the set of failure modes of the i^{th} load point ; λ_j , r_j ,U_j : are the equivalent indices of the j^{th} failure mode and λ_i , r_i ,U_i: are the average failure rate(λ), average outage time(*r*) and annual outage time(*U*) of the i^{th} load point respectively. Also, in addition to these indices, "ENS"(energy not supplied) and "OC"(outage cost) are calculated for load points and substation

$$ENS_i = L_i . U_i \tag{4}$$

$$\mathrm{ENS} = \sum_{\mathrm{i}} \mathrm{ENS_i} \tag{5}$$

$$\mathrm{OC_i} = \sum_{\mathrm{j\in m(i)}} \lambda_{\mathrm{j}} . r_j . L_i . IC_i(r_j) \tag{6}$$

$$\mathrm{OC} = \sum_{\mathrm{i}} \mathrm{OC_i} \tag{7}$$

Where L_i: is the average load of i^{th} load point, ENS_i: is the energy not supplied of i^{th} load point, ENS: is total energy not supplied for the substation, $IC_i(r_j)$: is the interruption cost-in dollar per KW per minute- of i^{th} load point for outage duration of r_j, OC_i: is interruption cost of i^{th} load point and OC: is the outage cost of the substation.

B. Fuzzy Arithmetic

By considering fuzzy number for component reliability indices, such as failure rate, repair time and operation of protective devices, the equations adopted are similar to the one used in the "crisp" traditional approach except that fuzzy numbers are used instead. However as those equations include products and divisions, a special care must be taken in the order of performing them, so that no unnecessary uncertainty is added to the results. For example, fuzzy repair time of a parallel system must be given by $(\tilde{r}_i^{\,-1} + \tilde{r}_j^{\,-1})^{-1}$

instead of by

$\tilde{r}_i \cdot \tilde{r}_j \cdot (\tilde{r}_i + \tilde{r}_j)^{-1}$

For evaluating fuzzy reliability indices, the arithmetic operation on fuzzy number must be done. The arithmetic operation on interval of confidence can be used for arithmetic operation on fuzzy numbers (Appendix A).

After deducing equivalent fuzzy indices of failure modes, fuzzy reliability indices of load points and the substation can be determined by

$$[\lambda_{i,L}^{(\alpha)},\lambda_{i,U}^{(\alpha)}]=\sum_{j\in m(i)}[\lambda_{j,L}^{(\alpha)},\lambda_{j,U}^{(\alpha)}] \tag{8}$$

$$[U_{i,L}^{(\alpha)},U_{i,U}^{(\alpha)}]=\sum_{j\in m(i)}[U_{j,L}^{(\alpha)},U_{j,U}^{(\alpha)}] \tag{9}$$

$$[r_{i,L}^{(\alpha)},r_{i,U}^{(\alpha)}]=\sum_{j\in m(i)}[\lambda_{j,L}^{(\alpha)}\div r_{j,U}^{(\alpha)},\lambda_{j,U}^{(\alpha)}\div r_{j,L}^{(\alpha)}] \tag{10}$$

$$[ENS_{i,L}^{(\alpha)},ENS_{i,U}^{(\alpha)}]=[U_{i,L}^{(\alpha)}\cdot L_{i,L}^{(\alpha)},U_{i,U}^{(\alpha)}\cdot L_{i,U}^{(\alpha)}] \tag{11}$$

$$[ENS_{L}^{(\alpha)},ENS_{U}^{(\alpha)}]=\sum_{i}[ENS_{i,L}^{(\alpha)},ENS_{i,U}^{(\alpha)}] \tag{12}$$

$$\begin{aligned}[\mathrm{OC}_{i,L}^{(\alpha)},\mathrm{OC}_{i,U}^{(\alpha)}]=\sum_{\mathrm{j\in m(i)}}[&\lambda_{j,L}^{(\alpha)}\cdot r_{j,L}^{(\alpha)}\cdot L_{i,L}^{(\alpha)}\cdot IC_i(r_{j,L}^{(\alpha)}),\\ &\lambda_{j,U}^{(\alpha)}\cdot r_{j,U}^{(\alpha)}\cdot L_{i,U}^{(\alpha)}\cdot IC_i(r_{j,U}^{(\alpha)})]\end{aligned} \tag{13}$$

$$[\mathrm{OC}_{L}^{(\alpha)},\mathrm{OC}_{U}^{(\alpha)}]=\sum_{i}[\mathrm{OC}_{i,L}^{(\alpha)},\mathrm{OC}_{i,U}^{(\alpha)}] \tag{14}$$

Where ($\widetilde{\lambda}_j$, $\widetilde{r}_j$, $\widetilde{U}_j$) :fuzzy numbers of three basic indices of the jth failure mode;

$IC_i(r_{j,L}^{(\alpha)})$: the interruption cost-in dollar per KW per minute- of ith load point for outage duration of $r_{j,L}^{(\alpha)}$

$IC_i(r_{j,U}^{(\alpha)})$: the interruption cost-in dollar per KW per minute- of ith load point for outage duration of $r_{j,U}^{(\alpha)}$.

Subscript L (U) denotes lower (Upper) band of parameter and superscript (α) denotes to confidence level.

Therefore the algorithm for deducing fuzzy reliability indices of load points and the substation as follows:

1-Determine fuzzy number of component reliability indices.

2-Determine interval of confidence in any confidence level from 0 to 1(for example in steps 0.1).

3-Deduce the failure modes of a load point. In [11] an efficient algorithm, used in this paper, is presented for determining minimal cut sets of substations, including active failure modes.

4-Evaluate the fuzzy number of λ,*r*,*U*,OC for any failure mode of load point using arithmetic operation on interval of confidence for any confidence factor α, α ∈ [0 1].

5-Calculate the fuzzy number of λ,*r*,*U*,OC,ENS for a load point

6-Repeat the steps 3-5 for other load points.

7-Calculate the fuzzy number of OC, ENS for the substation

V. Case study

A utility substation, 230/63 KV Hamedan substation, is used for case study. The single line diagram of this substation is shown in Fig. 1. The components fuzzy reliability data are triangular or trapezoidal fuzzy numbers and are shown in Table I. These fuzzy numbers are obtained by combining information from different sources such as statistical events, expert opinion and so on [8]. In this table, data of maintenance components are related to uncoordinated state. For coordinated maintenance, which is used in many conditions, components fuzzy reliability data is shown in Table II. In Table III the average load of feeders, fuzzy load of feeders and consumption load of each type customer is shown [8]. Customer interruption costs per minute are shown in Table IV [12].

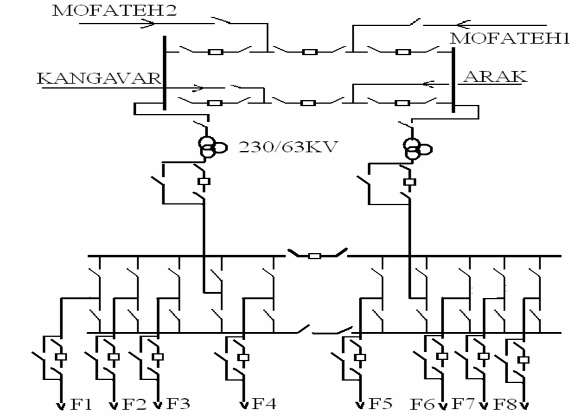

Fig. 1. 230/63KV Substation Configuration

The following failure modes are considered in the studies performed.

a) - First-order events:

1-A component failed.

2-A component actively failed.

b) - Second-order events:

1- Permanent failure of one component overlapping permanent failure of another component.

2- Active failure of one component overlapping Active failure of another component.

3- Permanent failure of one component overlapping Active failure of another component.

4- Permanent failure of one component overlapping scheduled maintenance outage of another component.

5- Active failure of one component overlapping scheduled maintenance outage of another component.

c) - Stuck breaker:

1- A breaker stuck overlapping a first order cut set.

2-A breaker stuck overlapping a second order cut set.

In calculation of fuzzy indices, the confidence level is changed in steps (Δ=0.1) from 0 to 1. Fig. 2 shows possibility distributions of failure rate, outage duration and annual outage duration of load point F1.These fuzzy numbers show the lower and upper band values of failure rate, outage duration and annual outage duration in any confidence level from 0 to 1. However, if it is necessary, these membership functions can be defuzzified by COA (center of area) approach. The defuzzified values of reliability indices of load point F1 are: λ= 0.4049[f/y]; *r* = 2.5446[hr/f]; *U* =0.9024[hr/yr].

Table I. Components Fuzzy Reliability Data[b]

Component	Transformer	Breaker 230	Breaker 63	Bus	Sectionalizer
λ (f/yr)	(0.2,0.25,0.3 ,0.35)	(0.01,0.011,0.012,0.013)	(0.01,0.012, 0.013,0.015)	(0.008,0.009,0.01)	(0.004,0.006,0.008)
λ^a(f/yr)	(0.2,0.25,0.3,0.35)	(0.006,0.0065,0.007,0.008)	(0.006,0.0072,0.0078,0.009)	(0.008,0.009,0.01)	(0.004,0.006,0.008)
λ_m (f/yr)	(1,1.2,1.4)	(1,1.2,1.4)	(1 ,1.2 ,1.4)	(1,1.5,2)	(1,1.5,2)
r (hr/f)	(36,48,72,84)	(5,5.5,6,7)	(5,5.5,6,7)	(2.5,3,3.5)	(1,1.5,2)
r_m (hr/f)	(9,10, 11,12)	(5,6,7)	(5,6,7)	(2.5,3,3.5)	(1,1.5,2)
Stuck prob. (P_c		(0,0.003,0.006)	(0,0.003,0.006)		
T_{sw} (hr)	(1.5,2,2.5)	(1,1.5,2)	(1,1.5,2)	(1,1.5,2)	

b; λ: total failure rate of component, λ^a: active failure rate of component, λ_m: the maintenance rate of component,r: average failure duration of component, r_m: the average maintenance of component, p_c : probability of stuck breaker, T_{sw}: the switching time of component

Table II. Components Fuzzy Reliability Data (Coordinated Maintenance)

	Branch1 (Breaker+ Bus+ Breaker)	Branch 2 (Transformer+Breaker)
λ_m(f/yr)	(1,1.5,2)	(1,1.5,2)
r_m (hr/f)	(6,7,8,9)	(12,13,14,15)

Table III. Average and Fuzzy load of Feeders and Load Demand of Each Type of Customer

Feeders	Average Load(MW)	Fuzzy Load of feeders	S. Ind.	Resdl.	Comml.	Off. bidg
F1	10	(4,9,11,16)	20%	30%	30%	20%
F2	12	(4,10,14,20)	65%	0	10%	25%
F3	15	(6,13,17,24)	20%	35%	30%	15%
F4	8	(3,7,9,13)	15%	40%	25%	20%
F5	13	(5,11,15,21)	15%	40%	25%	20%
F6	20	(8,17,23,32)	30%	30%	20%	20%
F7	15	(6,11,19,24)	15%	35%	35%	15%
F8	18	(7,15,21,29)	60%	5%	15%	20%

Table IV. Customer Damage per Minute ($/KW/min)

Customer Damage Per Minute($/KW/min)				
Duration(min)	Comml.	Off. Bidg	Resdl.	S. Ind.
1	0.381	4.778	0.001	1.625
20	0.148	0.494	0.005	0.193
60	0.143	0.351	0.008	0.151
240	0.130	0.287	0.020	0.105
480	0.137	0.248	0.033	0.116

Table V
Defuzzified Values of ENS

ENS [MWh/yr]	Load Point F1	Total Load Points	Load Point F1	Total Load Points
	With Expected Loads		With Fuzzy Loads	
	9.024	100	10.68	134.582

Table VI. Defuzzified Values of Outage Cost

OC[$/yr]	Load Point F1(*)	Total Load Points(*)	Load Point F1 (**)	Total Load Points (**)
	With Expected Values of Indices		With Fuzzy Indices	
	107.05×1000	1123.61×1000	119.925×1000	1605.147×1000

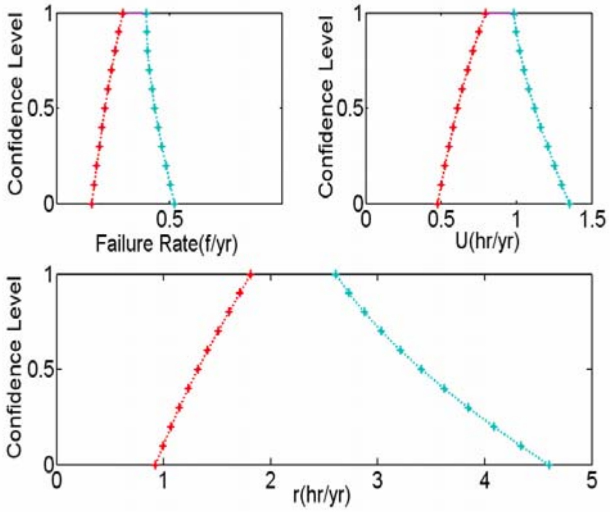

Fig. 2. Possibility Distributions of Indices of Load Point F1

In Fig. 3 possibility distribution of "ENS" of load point F1 is shown. Figs. 3.a and 3.b depict possibility distributions of ENS, when a singled value (the average value) for load or fuzzy load is considered respectively. Also, Figs. 4.a and 4.b show possibility distributions of ENS for total load points, when expected values or fuzzy numbers are considered for load points respectively. Table V shows defuzzified values of ENS. It can be seen that defuzzified value of ENS, when load is represented by fuzzy number 18.3% (34.58%) is greater than, when load is presented by expected value for load point F1(total load points). Therefore fuzzy load has considerable effect on ENS and for true evaluation, it is necessary that load of feeders are represented by fuzzy numbers.

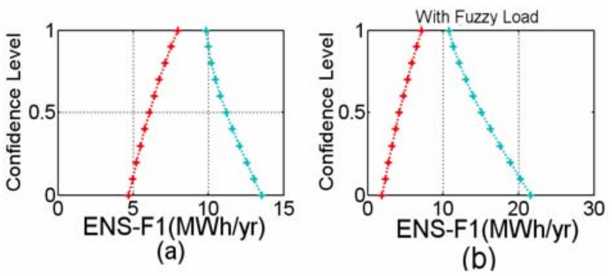

Fig. 3. Possibility Distribution of ENS of Load Point F1; a) With Expected Load; b) With Fuzzy Load

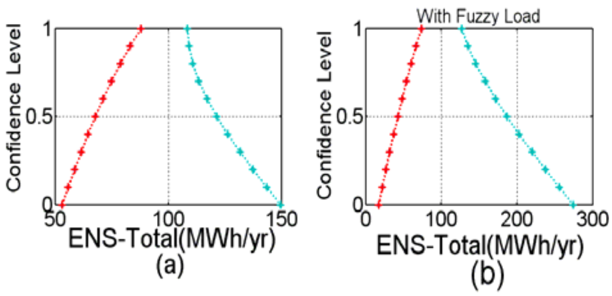

Fig. 4. ENS of Total Load Points; a) With Expected Load; b) With Fuzzy Load

Figs. 5.a and 5.b show possibility distributions of "OC"(outage cost) of load point F1 and total load points respectively by considering fuzzy load. These membership functions show the lower and upper band values of outage cost in any confidence level from 0 to 1.The defuzzified value of OC is calculated and shown in Table VI by (*). Also the reliability indices of OC is calculated by analytical method, when expected values are considered for components reliability data, and is shown in Table VI by (**). By comparing, it can be seen that for load point F1 (total load points) the defuzzifed value of OC 12% (30%) is greater than the value of OC, which calculated by analytical method. Therefore for true evaluation of interruption cost, it is necessary that all uncertainties of components reliability data and also load of feeders are considered and represented by fuzzy numbers.

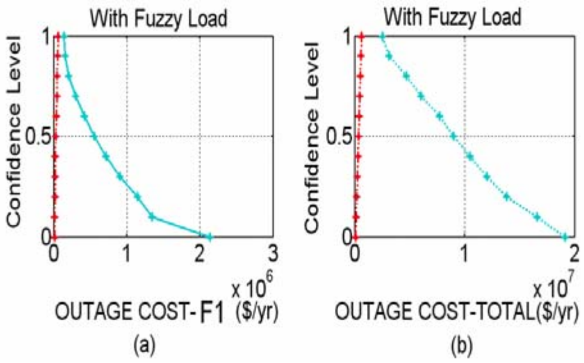

Fig. 5. Outage Cost; a) For Load Point F; b) For Total Load Points

VI. Conclusion

A conceptual possiblistic approach has been presented in this paper to model the uncertainties of components reliability data using fuzzy set theory. Trapezoidal (triangular) fuzzy numbers are used for components reliability data such as failure rate, repair time and operation of protective devices. Load of feeders are also represented by fuzzy numbers. Using Fuzzy Arithmetic, an algorithm is presented for calculation of fuzzy indices. In addition to three basic fuzzy reliability indices, ENS (energy not supplied) and OC (outage cost) of feeders and substation are also calculated. Possibility distributions of reliability indices would aid power system planner and operators in their decision making process. The Results presented indicate that fuzzy load representation has considerable effect on ENS. Therefore, for true evaluation of ENS, it is necessary to model load of feeders by fuzzy numbers.

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Appendix A) Fuzzy sets and fuzzy numbers

A fuzzy set is a set of ordered pairs with each containing an element and the degree of membership for that element. For fuzzy set $\tilde{A}$:

$$\tilde{A}=\{(x,\mu_{\tilde{A}}(x))|x\in U\} \quad (15)$$

Where $\mu_{\tilde{A}}(x)$ is called the membership functions of x in $\tilde{A}$,and indicate the degree that x belong to $\tilde{A}$ .An α-cut of a fuzzy set is a set containing all the elements of the universe U that has membership grade in $\tilde{A}$ equal or grater than α:

$$\tilde{A}^{(\alpha)}=\{x\in U|\mu_{\tilde{A}}(x)\geq\alpha,\alpha\in[0,1]\} \quad (16)$$

Therefore an α-cut($\tilde{A}^{(\alpha)}$) can be represented by $\tilde{A}^{(\alpha)}=[a_1^{(\alpha)}, a_2^{(\alpha)}]$ (Fig. 6.).

A fuzzy number is a *normal* and *convex* fuzzy set of the real line such that its membership function is piecewise continues.

A trapezoidal fuzzy number, usually represented by a quadruplet (a_1,a_2,a_3,a_4) and is sketched in Fig. 6. When $a_2=a_3$ then this fuzzy number is called triangular fuzzy number. A trapezoidal fuzzy number can be defined by the interval of confidence at level α as shown by

$$\tilde{A}^{(\alpha)}=[a_1^{(\alpha)}, a_2^{(\alpha)}]=[(a_2-a_1)\alpha+a_1,-(a_4-a_3)\alpha+a_4] \quad (17)$$

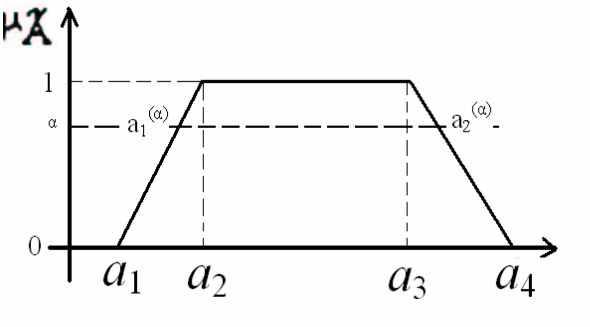

Fig. 6. A Trapezoidal Fuzzy Number

The arithmetic of fuzzy numbers depends on the arithmetic of the interval of confidence. Some main operations for fuzzy number $\tilde{A}$ and $\tilde{B}$, described by the interval of confidence, are [13]:

$$\tilde{A}^{(\alpha)}=[a_1^{(\alpha)}, a_2^{(\alpha)}]\ ,\ \tilde{B}^{(\alpha)}=[b_1^{(\alpha)}, b_2^{(\alpha)}] \quad (18)$$

Addition:

$$\tilde{A}^{(\alpha)}+\tilde{B}^{(\alpha)}=[a_1^{(\alpha)},a_2^{(\alpha)}]+[b_1^{(\alpha)},b_2^{(\alpha)}]=[a_1^{(\alpha)}+b_1^{(\alpha)},a_2^{(\alpha)}+b_2^{(\alpha)}] \quad (19)$$

Subtraction:

$$\tilde{A}^{(\alpha)}-\tilde{B}^{(\alpha)}=[a_1^{(\alpha)},a_2^{(\alpha)}]-[b_1^{(\alpha)},b_2^{(\alpha)}]=[a_1^{(\alpha)}-b_2^{(\alpha)},a_2^{(\alpha)}-b_1^{(\alpha)}] \quad (20)$$

Multiplication:

$$\tilde{A}^{(\alpha)}\times\tilde{B}^{(\alpha)}=[a_1^{(\alpha)},a_2^{(\alpha)}]\times[b_1^{(\alpha)},b_2^{(\alpha)}]=[a_1^{(\alpha)}\times b_1^{(\alpha)},a_2^{(\alpha)}\times b_2^{(\alpha)}] \quad (21)$$

Division:

$$\tilde{A}^{(\alpha)}\div\tilde{B}^{(\alpha)}=[a_1^{(\alpha)},a_2^{(\alpha)}]\div[b_1^{(\alpha)},b_2^{(\alpha)}]=[a_1^{(\alpha)}\div b_2^{(\alpha)},a_2^{(\alpha)}\div b_1^{(\alpha)}] \quad (22)$$