Perturbation Method Based Evaluation of Power System Voltage Security

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***Abstract* — Security in stressed power systems is seriously affected by voltage collapse phenomena. This paper proposes a new algorithm for determining the voltage security margin. The algorithm is based on the perturbation method and has a very high computational efficiency. Therefore it can be used for on-line and real-time voltage evaluation. The proposed approach has been validated using IEEE14, IEEE30 and IEEE57 Bus systems**
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Index Terms* — *Voltage security, Voltage stability, Perturbation parameter

I. Introduction

Even if the power systems designers' efforts are focused on basis of powerful engineering assumptions, it is a fact that system performance is subjected to be disturbed or reach to overall down status, according to unforeseen events.
Nowadays, power system stabilizers are widely used adjacent to rotor angle stability boundaries [1], referred to as stressed power systems, according to reliable performance of power system stabilizers and growing trend of electric loads utilization. Stressed power system's security is seriously affected by voltage instability phenomena that are one of the key parameters determining secure loading margins of power systems. Power-phase stability, and voltage stability concepts are considered as subset of power systems' stability space and could be recognized by simple examples described at the following [2].
Rotor angle stability: Assume a synchronized generator that is connected to an infinity bus via a line with reactance of X_L. Stability in such a system is referred to power-phase stability. This concept is discussed in [3] and [4].
Voltage stability: In the case that system under study is a synchronous generator supplying a static load via a line with X_L, concept of stability for such a system is voltage stability. For more illustration assume the load to be gradually increased, then working point of the system will be changed instant by instant, finally will reach to such a point that afterward any partially change in load volume, will make a huge change in system characteristics, this point is named boundary of voltage stability that is referred to as saddle-node bifurcation, in which the Jacobian matrix of equations set for load flow becomes singular with an eigenvalue of zero [5]. So far, many of voltage instabilities is occurred at even large power systems: voltage collapse in western France at 1987 due to outage of 9000MW of generators [6], voltage collapse in Tokyo at 1987 due to unusual overload rate with 400 MW per minute at noon of a hot day [7], voltage instability occurrence of northern California network at 1983 for 2 minutes due to outage of HVDC lines, not proceeded to collapse because of conducting the network to a new stable working point after a whole volume of load detached [8].
Therefore, it is necessarily advised to evaluate voltage stability of the network in an on-line real-time basis, to be able to prepare assured reliability to the system. Voltage stability phenomena can be studied in two methods of dynamic and static. In dynamic type of study of voltage stability, non-linear differential equations are analyzed, and it is conveyed the solvability of algebraic flow equations in static form of studies. According to importance of voltage stability, it is necessary to add a tool into the energy management system of the power systems for evaluating the voltage security assessment (VSA). Such a tool is called VSA environment [9] that consists of five steps as shown in Figure 1 and are notified further.
Step1 "Evaluation of voltage stability at current working point": Using sensitivity analysis on eigenvalues of Jacobian Matrix, in case one of these values equals with zero, it means that the system resides in boundary of voltage stability.
Step2 "Selection of Contingency": Due to working point circumstances and information received from VSA database, critical Contingencies are selected.
Step3 "Ranking of Contingencies": Contingencies are sorted in correspondence with their severity.

Step4 “Evaluation of Contingencies”: Using the sorted list obtained from step3, contingencies are evaluated in an exact manner.
Step5 “Applying Corrective/ Preventive strategy”: Corrective solutions like optimum flow of reactive power after contingency, or preventive strategies like urgent cut-off of the load before occurring any damage, are applied to the power system.

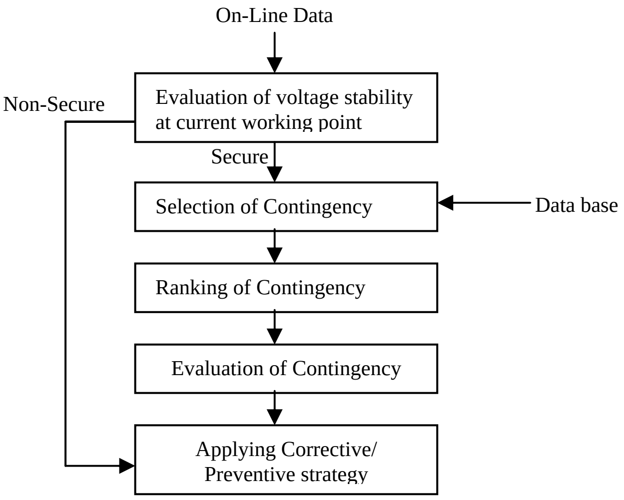

Fig.1. VSA Environment

To determine voltage stability boundary or Saddle-Node-Bifurcation (SNB) point, research methods are mostly categorized within two items: 1. direct methods [10,11], 2. Indirect methods [12,13]. Appropriate terms are added - in direct methods - to power flow equations at saddle-node bifurcation point, correspondent with power system conditions, then this point is achieved via solving the equation set. Utilizing individual root achieved from power flow - via indirect methods – the rest of the roots are continuously derived one by one until the power system reaches to the boundary of stability.
One of the generally used indirect methods is known as continuation power flow (CPF) [13], in which power system’s splitting point is recognized through selecting a given parameter and applying gradually variation on it. In this paper, by using perturbation theorem, a new method is developed for evaluation of power system voltage security. Considering flexibility of developed method, it can be easily applied so that to determine the system working point distance with boundary of voltage stability in a very high convergence speed. The new method is validated via IEEE-14, 30 and 57 BUS power systems. Proposed method is compared with CPF method as well.

II. EQUATION EXPRESSION OF POWER SYSTEM

Power systems general load flow equations can be stated by the equation below:

$$F(\delta, V) = 0 \qquad (1)$$

Expression (1) includes $2n_1+n_2$ variables, and the same number of equations in which n_1 is number of PV Buss and n_2 number of PQ buss, and δ and V are voltage phase and magnitude of system buss. To obtain generalized form of load flow equations, parameter λ is included into equations:

$$F(\delta, V, \lambda) = 0 \quad ; \quad \lambda_0 \le \lambda \le \lambda_{critical} \qquad (2)$$

Expression (2) includes $2n_1+ n_2$ equations and $2n_1+ n_2 +1$ variables, $\lambda=0$ represents the base case initial value, and $\lambda=\lambda_{critical}$ indicates critical load or SNB point conditions. Considering λ in rectangular form of load flow equations we obtain generalized load flow equations for the i-th bus through combining the following expressions:

$$P_{Gi} - P_{Di} - P_{Ti} = 0 \qquad (3)$$

$$Q_{Gi} - Q_{Di} - Q_{Ti} = 0 \qquad (4)$$

$$P_{Ti} = \sum_k G_{ik}(e_i e_k + f_i f_k) + \sum_k B_{ik}(e_k f_i - e_i f_k)$$

$$Q_{Ti} = \sum_k G_{ik}(e_k f_i - e_i f_k) - \sum_k B_{ik}(e_i e_k + f_i f_k)$$

$$P_{Di} = P_{Di0} + \lambda(K_{Di} S_{\Delta base} COS\theta_i)$$

$$Q_{Di} = Q_{Di0} + \lambda(K_{Di} S_{\Delta base} SIN\theta_i)$$

$$P_{Gi} = P_{Gi0}(1 + \lambda K_{Gi})$$

where :
K_{Gi}: factor indicating variations in generated power at i-th bus
P_{Gi0}: real power generated at i-th bus in base case
Q_{Di0}: imaginary power dissipated at i-th bus in base case
P_{Di0}: real power dissipated at i-th bus in base case
K_{Di}: factor indicating variations in load at i-th bus
θ_i: angle of power factor indicating variations in load at i-th bus
V_i: voltage at i-th bus $S_{\Delta base}$: base apparent power due to λ
y_{ik}: (i,k)-th element of y_{bus} matrix
$G_{ik} = real(y_{ik})$
$B_{ik} = imag(y_{ik})$
$e_i = real(V_i)$
$f_i = imag(V_i)$

III. CONTINUATION POWER FLOW

For evaluating static voltage stability of the power systems violated, the load flow equations are solved. In voltage stability boundary, Jacobian matrix for the system has singularity, so numerical solution process of load flow equations diverges adjacent to this point. An alternate to resolve this problem, is to apply CPF method instead of the conventional load flow. Equations used in CPF are the same with the generalized form of load flow equations. This is to be discussed at the following section.

A. Continuation Load Flow Algorithm

In CPF method, locally parameterized continuation (LPC) technique [14] is utilized to solve generalized load flow equations in order to evaluate stability boundaries. This is done through generation of continuation solutions in form of (V_1,δ_1,λ_1), (V_2,δ_2,λ_2), from base case solution of conventional load flow equations say (V_0,δ_0,λ_0), where $(\lambda_0<\lambda_1<\lambda_2)$ that is illustrated in Fig.2. Continuation Power Flow(CPF) consists of two stages: A. predicting stage, B. parameterization and correction stage.

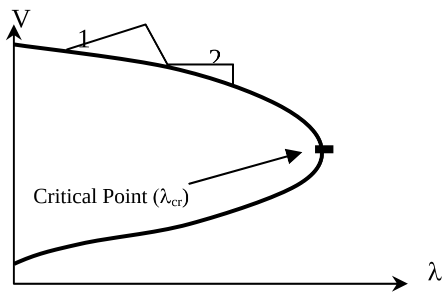

Fig.2 . Continuation Power Flow scheme (1: Predicting, 2: Correction)

❖ *Predicting Stage*

In this stage, an approximate solution is achieved from generalized load flow equations starting from the base case in direction of tangent vector to the V-λ curve. Hence the first job in predicting stage is to calculate tangential vector. By differentiation from both sides of (2) we obtain:

$$d\,F = F_\delta\, d\delta + F_V\, dV + F_\lambda\, d\lambda \qquad (5)$$

and after factorizing yields :

$$[F_\delta, F_V, F_\lambda]\ t \qquad (6)$$

$$t = [d_\delta, d_V, d_\lambda]^T \qquad (7)$$

where $[F_\delta,F_V,F_\lambda]$ is a $(2n_1+ n_2)$ x $(2n_1+ n_2+1)$ matrix and t is a $(2n_1+ n_2+1)$ x 1 vector, and T denotes transposition operator. Determining one of the variables from tangential vector t (e.g. k-th element of t) yields:

$$[F_\delta, F_V, F_\lambda]\ t = 0 \quad , \quad t_k = \pm 1 \qquad (8)$$

then, proceeded by explicitly solving the equation (8), solutions of predicting stage shall be achieved as :

$$\delta^* = \delta + \sigma\ d\delta \qquad (9\text{-a})$$

$$V^* = V + \sigma\ dV \qquad (9\text{-b})$$

$$\lambda^* = \lambda + \sigma\ d\lambda \qquad (9\text{-c})$$

As notation * is used in our literature to present solutions of predicting stage and σ is the step size, selected in a manner in which predicting stage solutions be located within convergence radius of the correction stage [15].

❖ *Parameterization and Correction Stage*

As the coordinates of the point achieved in predicting stage do not touch the V-λ curve, in correction stage, this solution is corrected and (V_1,δ_1,λ_1) point which is accurately located on V-λ curve achieved. Number of variables is also one more than number of equations and appropriate vector can be given as :

$$X = [\delta, V, \lambda]^T \qquad (10)$$

and X is a $(2n_1+ n_2+1)$ x 1 vector. Ascertaining one variable from vector X (e.g. $X_k = \mu$), and solving equations (11-a) and (11-b) using Newton-Raphson method, solutions of corrector stage are determined.

$$F(x)=0 \qquad (11\text{-a})$$

$$X_k - \mu = 0 \qquad (11\text{-b})$$

In LPC technique at each of corrector stages, only one of variables of X vector may be certain, that is denoted as continuation parameter, and component from t with maximum possible value shall be taken correspondent with that so called continuation parameter. Therefore, continuation parameter shall be the same variable in predicting stage and is determined as (12):

$$\mu = \{X_k : |\, t_k\,| = MAX,\ 2 \le k \le 2n_1 + n_2 + 1\} \qquad (12)$$

Continuation parameter μ in starting step is taken the same λ, but at next steps may also be chosen as voltage magnitude or voltage phase angle of buss. By approaching to voltage stability boundary (SNB), we shall have Δλ=0, and λ will reach to its maximum value. After passing from critical point, variation rate of λ goes negative and λ decreases. So Δλ can be taken as a criterion to discriminate boundary of voltage stability.

IV. Proposed Method Declaration

In evaluation of voltage security phenomena, it is necessary to solve the generalized load flow equations by increasing λ from λ_0 (load parameter value in base case) to λ_{cr} (load parameter critical value), inspiring $\lambda_{cr} - \lambda_0$ value to be considered as a criterion denoting marginal voltage security of power system. Generalized load flow equation set consists of $2n_1+ n_2$ equations and $2n_1+ n_2+1$ variable, therefore, number of equation is less than the number of variables by 1.

In this paper, by using perturbation method, a new method is developed in which by considering one of the variables as perturbation parameter, possibility to achieve the rest of solutions of generalized load flow equation is obtained. Perturbation theorem is declared hereby for further reference [16].

Consider the equation (13):

$$f(x,\varepsilon) = 0 \tag{13}$$

This expression is called a perturbed algebraic equation, with perturbation parameter ε and ε<1. let perturbation parameter to be zero, then equation (13) can be rewritten :

$$f(x)=0 \tag{14}$$

If solution of equation (14) equals to x_0, then solution of (13) in terms of ε shall be :

$$x = x_0 + \sum_{k=1}^{\infty} x_q \varepsilon^q \tag{15}$$

To calculate x_q , substitute x from (15) into (13) and set the coefficients of all powers of perturbation parameter ε to zero. Because the parameter ε<1 is too small, x can be written:

$$x \approx x_0 + \sum_{k=1}^{p} x_q \varepsilon^q \tag{16}$$

in which all terms with powers greater than p are ignored, say error orders to ε^{p+1}.

A. *Proposed Method's Formulation*

❖ *Theorem 1.*

Consider rectangular form of generalized load flow equations (3) and (4). If $\varepsilon_\lambda=\Delta\lambda$ be the perturbation parameter, then real part and imaginary part of buss voltage is derived from expressions (17) and (18):

$$e_i = e_{i,0} + e_{i,1}\varepsilon_\lambda{}^2 + \cdots + e_{i,p1}\varepsilon_\lambda{}^{p1} \tag{17}$$

$$f_i = f_{i,0} + f_{i,1}\varepsilon_\lambda{}^2 + \cdots + f_{i,p1}\varepsilon_\lambda{}^{p1} \quad i = 2, 3, \cdots, n \tag{18}$$

Where $e_{i,0}$ and $f_{i,0}$ are real and imaginary part of i-th bus voltage in base case, and $e_{i,1}$ to $e_{i,p1}$ and $f_{i,1}$ to $f_{i,p1}$ are derived from (19) to (24) and calculations afterward [18].

$$\begin{bmatrix} E_q \\ F_q \end{bmatrix} = J_0^{-1} \begin{bmatrix} \alpha_q \\ \beta_q \end{bmatrix} \quad q = 1, 2, 3, ..., p_1 \tag{19}$$

$$J_0 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{20}$$

$$E_q = \left[e_{2,q}, \cdots, e_{n,q}\right]^T \tag{21}$$

$$F_q = \left[f_{2,q}, \cdots, f_{n,q}\right]^T \tag{22}$$

$$\alpha_q = \left[\alpha_{2,q}, \cdots, \alpha_{n,q}\right]^T \tag{23}$$

$$\beta_q = \left[\beta_{2,q}, \cdots, \beta_{n,q}\right]^T \tag{24}$$

$$A_{i,i} = \sum_{j\neq 1}(G_{ij}e_{j,0} - B_{ij}f_{j,0}) + (G_{ii}e_{i,0} + B_{ii}f_{i,0}) + G_{i1}e_1$$

$$A_{i,j} = G_{ij}e_{i,0} + B_{ij}f_{i,0}$$

$$B_{i,i} = \sum_{j\neq 1}(G_{ij}f_{j,0} + B_{ij}e_{j,0}) + (G_{ii}f_{i,0} - B_{ii}e_{i,0}) + B_{i1}e_1$$

$$B_{i,j} = G_{ij}f_{i,0} - B_{ij}e_{i,0}$$

$$C_{i,i} = -\sum_{j\neq 1}(G_{ij}f_{j,0} + B_{ij}e_{j,0}) + (G_{ii}f_{i,0} + B_{ii}e_{i,0}) - B_{i1}e_1$$

$$C_{i.j} = G_{ij}f_{i,0} - B_{ij}e_{i,0}$$

$$D_{i,i} = \sum_{j\neq 1}(G_{ij}e_{j,0} - B_{ij}f_{j,0}) - (G_{ii}e_{i,0} + B_{ii}f_{i,0}) + G_{i1}e_1$$

$$D_{i,j} = -G_{ij}e_{i,0} - B_{ij}f_{i,0}$$

$$\alpha_{i,1} = K_{Gi} - K_{Di} \qquad \beta_{i,1} = -K_{Di}\ \mathrm{TAN}\ \theta_i$$

$$\alpha_{i,q} = -\sum_{j\neq 1}\{G_{ij}\sum_{q=1}^{p-1}(e_{i,q}e_{j,p-q} + f_{i,q}f_{j,p-q}) + B_{ij}\sum_{q=1}^{p-1}(e_{j,q}f_{i,p-q} - e_{i,q}f_{j,p-q})\}$$

$$\beta_{i,q} = -\sum_{j\neq 1}\{G_{ij}\sum_{q=1}^{p-1}(e_{j,q}f_{i,p-q} - e_{i,q}f_{j,p-q}) - B_{ij}\sum_{q=1}^{p-1}(e_{i,q}e_{j,p-q} + f_{i,q}f_{j,p-q})\}$$

❖ *Theorem 2.*

If we consider perturbation parameter in rectangular form of generalized load flow equations (3) and (4) as variation of real part of critical bus voltage, defined as $e_{indx} = e^* - \varepsilon_{eindx}$, then real and imaginary part of buss voltages, and load parameter λ can be derived from expressions (25) to (27), where $\varepsilon_e = \varepsilon_{eindx}$.

$$e_i = e_{i,0} + e_{i,1}\varepsilon_e + e_{i,2}\varepsilon_e{}^2 + \cdots + e_{i,p2}\varepsilon_e{}^{p2} \quad i \neq indx,\ i = 2, 3, ..., n \tag{25}$$

$$f_i = f_{i,0} + f_{i,1}\varepsilon_e + f_{i,2}\varepsilon_e{}^2 + \cdots + f_{i,p2}\varepsilon_e{}^{p2} \quad i = 2, 3, ..., n \tag{26}$$

$$\lambda = \lambda_0 + \lambda_1\varepsilon_e + \lambda_2\varepsilon_e{}^2 + \cdots + \lambda_{p2}\varepsilon_e{}^{p2} \tag{27}$$

Where λ_0 is the base load, $e_{i,0}$ and $f_{i,0}$ are real and imaginary part of i-th bus voltage in base case, and $e_{i,1}$ to $e_{i,p2}$ and $f_{i,1}$ to $f_{i,p2}$ and λ_1 to λ_{p2} are derived from (28) and (29).

$$\begin{bmatrix} E'_q \\ F_q \\ \lambda_q \end{bmatrix} = (J'_0)^{-1} \begin{bmatrix} \alpha'_q \\ \beta'_q \\ \gamma_q \end{bmatrix} \quad q = 1, 2, 3, ..., p_2 \tag{28}$$

$$J'_0 = \begin{bmatrix} A' & B' & M \\ C' & D' & N \\ H & J & L \end{bmatrix} \tag{29}$$

E'_k is defined by eliminating e_{indx} variable in E_k. By similar manner, discarding row and column regarding to indx in A, yields A'. As well as omitting the row presenting indx in B and the column presenting indx in C, shall make B' and C' respectively. Also consider that D'=D.

$$M_i = K_{Di} - K_{Gi} \quad ; \quad i \neq indx$$

$$N_i = K_{Di}\ \mathrm{TAN}\ \theta_i$$

$$H_i = G_{indx,i} e_{indx} + B_{indx,i} f_{indx,0}; \quad i \neq indx$$

$$J_i = G_{indx,i} f_{indx} - B_{indx,i} e_{indx,0}; \quad i \neq indx$$

$$I_{indx} = \sum_{j\neq 1}(G_{indx,j} f_{j,0} + B_{indx,j} e_{j,0}) + (G_{indx,indx} f_{indx,0} - B_{indx,indx} e_{indx})$$

$$L = K_{D,indx} - K_{G,indx}$$

For q =1, values of α'_q, β'_q, γ_q are expressed by:

$$\alpha'_{1,i} = G_{i,indx} e_{i,0} + B_{i,indx} f_{i,0} \quad ; \quad i \neq indx$$

$$\beta'_{1,i} = G_{i,indx} f_{i,0} - B_{i,indx} e_{i,0} \quad ; \quad i \neq indx$$

$$\beta'_{1,indx} = -\sum_{j\neq 1}(G_{indx,j} f_{j,0} + B_{indx,j} e_{j,0}) + (G_{indx,indx} f_{indx,0} - B_{indx,indx} e_{indx}) - B_{indx,1} e_1$$

$$\gamma_{1,indx} = \sum_{j\neq 1}(G_{indx,j} e_{j,0} - B_{indx,j} f_{j,0}) + (G_{indx,indx} e_{indx} + B_{indx,indx} f_{indx}) + G_{indx,1} e_1$$

and for q>1 (q=2, 3, …, p) are given through:

$$\alpha'_{i,q} = -\sum_{j\neq 1,i}\{G_{ij}\sum_{q=1}^{p-1}(e_{i,q} e_{j,p-q} + f_{i,q} f_{j,p-q}) + B_{ij}\sum_{q=1}^{p-1}(e_{j,q} f_{i,p-q} - e_{i,q} f_{j,p-q}) - G_{i,indx}\sum_{q=1}^{p-1}(f_{indx,q} f_{i,p-q} - e_{i,p-1})\} + B_{i,indx}\sum_{q=1}^{p-1}(f_{i,q} e_{indx,p-q} + f_{i,p-1})$$

$$\beta'_{i,q} = -\sum_{j\neq 1,i}\{G_{ij}\sum_{q=1}^{p-1}(e_{j,q} f_{i,p-q} - e_{i,q} f_{j,p-q}) - B_{ij}\sum_{q=1}^{p-1}(e_{i,q} e_{j,p-q} + f_{i,q} f_{j,p-q}) + G_{i,indx}\sum_{q=1}^{p-1}(e_{i,q} f_{indx,p-q} + f_{i,p-1})\} + B_{i,indx}\sum_{q=1}^{p-1}(f_{indx,q} f_{i,p-q} - e_{i,p-1})$$

$$\gamma_q = -\sum_{j\neq 1,i}\{G_{indx,j}\sum_{q=1}^{p-1}(f_{i,q} f_{j,p-q} - e_{j,p-1}) + B_{indx,j}\sum_{q=1}^{p-1}(e_{j,q} f_{indx,p-q} + f_{j,p-1})\} - G_{indx,indx}\sum_{q=1}^{p-1} f_{indx,q} f_{indx,p-q}$$

Finally, $\beta'_{indx,q}$ shall be calculated via:

$$\beta'_{indx,q} = -\sum_{j\neq 1,i}\{G_{indx,j}\sum_{q=1}^{p-1}(e_{j,q} f_{indx,p-q} + f_{j,p-1}) - B_{indx,j}\sum_{q=1}^{p-1}(f_{indx,q} f_{j,p-q} - e_{j,p-1})\} + B_{indx,indx}\sum_{q=1}^{p-1} f_{indx,q} f_{indx,p-q}$$

Proof for theorems mentioned above, is simply done by substituting variables described in expressions (17) and (18) and (25) to (27), into rectangular form of load flow equations, i.e. (3) and (4), and then set the coefficients correspondent to perturbation parameter's powers from 1 to p, equal to zero.

B. Proposed Method Algorithm

To evaluate voltage stability of a power system, respected to results achieved from theorem 1 stated at section 4-1, we started from base working point, increasing load amount by initiating perturbation parameter ε_λ. Adjacent to voltage stability boundary, matrix J_0 as expressed in (20), is inclined to singularity. Therefore, to find the voltage stability boundary accurately, we change perturbation parameter from ε_λ to ε_{eindx} (indx is critical bus), just preceding the SNB point. We shall calculate boundary of voltage stability with respect to the results taken from theorem 2 and subtracting real part of voltage at critical bus. Proposed method flowchart is illustrated in Figure3. So, it is necessary for two significant questions to be inspected in advance:

1. How the critical bus is recognized?
2. When is perturbation parameter supposed to be changed from ε_λ to ε_{eindx}?

The bus with the highest ratio of voltage drop to load variation is recognized as critical bus, hence we use relation (30) to discriminate the critical bus.

$$indx = \{i : \left|\frac{\Delta V_i}{\Delta\lambda}\right| = MAX, \quad 2 \le i \le 2n_1 + n_2 + 1\} \tag{30}$$

In proposed method, perturbation parameter is changed from ε_λ to ε_{eindx} according to slope of V-λ curve ($\Delta v_{indx}/\Delta\lambda$). The flowchart for the proposed method is shown in figure 4. As it can be seen, z^* is such a slope of V-λ curve, in which the perturbation parameter is changed from ε_λ to ε_{eindx}.

Z_{cr} is such a slope (infinity) of V-λ curve which is correspondent to the highest value of load parameter (λ_{cr}), where after λ tends to be decrease.

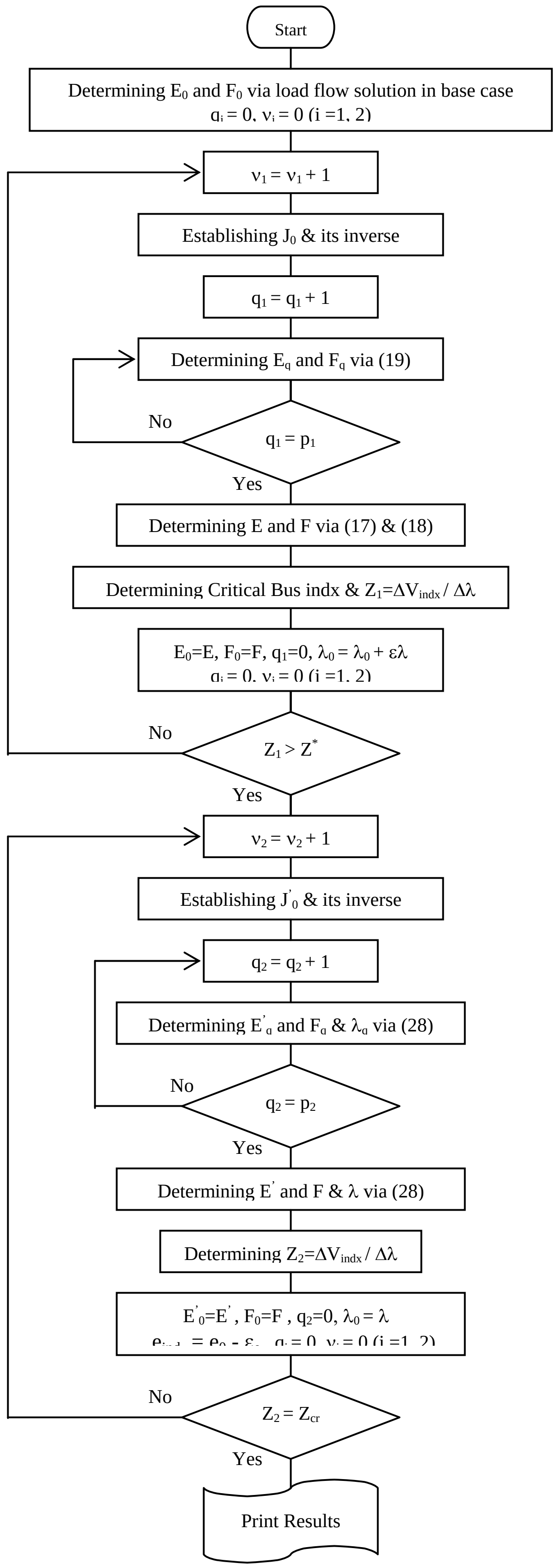

Fig.3. Proposed method Flowchart

V. IEEE STANDARDS SYSTEM STUDY

By using the proposed method in this paper, voltage stability of power networks with IEEE standard bus14, 30 and 57 are evaluated [17]. Limitation considerations on reactive power generation for buss used in voltage control process are also respected in simulations. In Table 1, results for bus voltages of a network complying IEEE-14 BUS standard are presented for due one per-unit increase in load related to the load at the base case. Two different cases are simulated and studied for 1 per-unit load increase through the proposed method for tracking solutions of a network containing fourteen buss.

Perturbation parameter (ε_λ) is considered 0.25 for the first case, and 0.1 for the other. In each case, system results for three values of p_1 (i.e. 5, 6 and 7) are separately available through the table entries. CPF method for continuation parameter $\Delta\lambda$=0.2 are also available in Table 1.

By using proposed method, solutions are achieved in 4 iterations in case ε_λ = 0.25, and in 10 iterations in case ε_λ = 0.1. This can be reasoned because of the fact that according to the algorithm indicated in section 2-4, while each iteration only one inversion operation on matrix is executed. Also in CPF method, the solutions are achieved after four iterations, at each of iterations, four to eight (once in predicting stage, three to seven times in corrector stage) matrix inversion operations are executed. Therefore, quickness of the proposed method is higher than the CPF method. According to the results gathered in Table 1, it can be found out that the relative error of the results belonging to the new method in proportion to the CPF, is highly ignorable, and can be even more decreased by easily increasing p_1. Meanwhile, one of the problems commonly occurred in CPF method is the fact that there is a convergence condition at corrector stage i.e. an appropriate step size at predicting stage has to be selected. In comparison, proposed method does not possess such a problem because of being one-stage of the algorithm, and the only condition for convergence is that perturbation parameter be less than unit (ε<1). It means that for higher ε (and still less than unit), it is needed to consider the higher powers of p_1, in order to achieve the results with lesser approximation, however, approaching nearer to the SNB point, and to determine it accurately, perturbation parameter has to be lesser

In Table 2, results for bus voltages of a network complying IEEE-14 BUS standard are presented in saddle-node-bifurcation point. Three different cases are simulated and studied for the proposed method, where perturbation parameter (ε_λ) is considered 0.1 for the first time, and after 1 per-unit increase in load, is decreased to 0.05. After that the load had grown to 1.3 per-unit, perturbation parameter is changed from ε_λ to ε_{e14}. The new value of ε_{e14} is 0.05 in

$$REP\ \ Case\ 1 = 100 \times \frac{1.4997\ - 1.5187}{1.5187} = -1.25\,\%$$

$$REP\ \ Case\ 2 = 100 \times \frac{1.5141\ - 1.5187}{1.5187} = -0.3\%$$

$$REP\ \ Case\ 3 = 100 \times \frac{1.5181\ - 1.5187}{1.5187} = -0.04\,\% \approx 0$$

As it can be seen, error amount tends to be zero in third case, consequently it shall be possible to calculate more exact solution through proposed new method by proper selection of perturbation parameter ε_{eindx}.

TABLE I

BUS VOLTAGES OF IEEE-14 BUS FOR DUE 1 PER-UNIT INCREASE IN LOAD RELATED TO THE BASE LOAD PATTERN

	New Method (ε_λ=0.25)			New Method (ε_λ=0.1)			CPF
BUS #	p_1=5	p_1=6	p_1=7	p_1=5	p_1=6	p_1=7	$\Delta\lambda$=0.2
1	1.06<0	1.06<0	1.06<0	1.06<0	1.06<0	1.06<0	1.06<0
2	0.9823<-06.691	0.9823<-06.691	0.9823<-06.691	0.9823<-06.691	0.9823<-06.691	0.9823<-06.691	0.9823<-06.691
3	0.8635<-19.146	0.8635<-19.146	0.8635<-19.146	0.8635<-19.146	0.8635<-19.146	0.8635<-19.146	0.8635<-19.146
4	0.9010<-15.126	0.9010<-15.126	09010<-15.126	0.9011<-15.126	0.9010<-15.126	0.9010<-15.126	0.9010<-15.126
5	0.9224<-12.669	0.9224<-12.669	0.9224<-12.669	0.9224<-12.669	0.9224<-12.669	0.9224<-12.669	0.9224<-12.669
6	0.8372<-24.286	0.8372<-24.286	0.8372<-24.286	0.8372<-24.286	0.8372<-24.286	0.8372<-24.286	0.8372<-24.286
7	0.8663<-21.769	0.8663<-21.769	0.8663<21.769	0.8663<-21.769	0.8663<-21.769	0.8663<-21.769	0.8663<-21.769
8	0.9127<-21.769	0.9127<-21.769	0.9127<-21.769	0.9127<-21.769	0.9127<-21.769	0.9127<-21.769	0.9217<-21.769
9	0.8344<-25.454	0.8344<-25.454	0.8344<-25.454	0.8344<-25.453	0.8344<-25.454	0.8344<-25.454	0.8344<-25.454
10	0.8218<-25.905	0.8218<-25.905	0.8218<-25.905	0.8218<-25.905	0.8218<-25.905	0.8218<-25.905	0.8218<-25.905
11	0.8234<-25.405	0.8234<-25.405	0.8234<-25.405	0.8234<-25.404	0.8234<-25.405	0.8234<-25.405	0.8234<-25.405
12	0.8114<-26.200	0.8114<-26.201	0.8114<-26.200	0.8114<-26.200	0.8114<-26.200	0.8114<-26.201	0.8114<-26.201
13	0.8042<-26.368	0.8042<-26.368	0.8042<-26.368	0.8042<-26.368	0.8042<-26.368	0.8042<-26.368	0.8042<-26.368
14	**0.7865<-28.139**	**0.7865<-28.139**	**0.7865<-28.139**	**0.7865<-28.138**	**0.7865<-28.139**	**0.7865<-28.139**	**0.7865<-28.139**

TABLE II

BUS VOLTAGES OF IEEE-14 BUS FOR DUE 1 PER-UNIT INCREASE IN LOAD RELATED TO THE BASE LOAD PATTERN

		New Method (ε_{e14}=0.05) λ_{cr}=1.4997		New Method (ε_{e14}=0.01) λ_{cr}=1.5141		New Method * λ_{cr}=1.5181	CPF** λ_{cr}=1.5187
BUS #	Base Case	p_2=3	p_2=5	p_2=3	p_2=5	p_2=3	$\Delta\lambda$=0.1
1	1.06<0	1.06<0	1.06<0	1.06<0	1.06<0	1.06<0	1.06<0
2	1.0389<-04.910	0.8906<-07.876	0.8906<-07.876	0.8817<-07.950	0.8817<-07.950	0.8819<-07.960	0.8816<-07.961
3	0.9768<-12.497	0.6802<-27.837	0.6802<-28.836	0.6616<-28.788	0.6616<-28.788	0.6619<-28.805	0.6613<-28.835
4	1.0041<-10.297	0.7250<-20.383	0.725<-20.379	0.7081<-20.896	0.7081<-20.896	0.7086<-20.912	0.7081<-20.927
5	1.0145<-08.783	0.7641<-16.400	0.7641<-16.400	0.7489<-16.732	0.7489<-16.732	0.7495<-16.750	0.7490<-16.759
6	0.9657<-15.215	0.5985<-40.070	0.5985<-40.069	0.5780<-41.958	0.5780<-41.958	0.5793<-41.909	0.5786<-41.971
7	0.9934<-14.061	0.6344<-33.497	0.6344<-33.497	0.6138<-34.846	0.6138<-34.846	0.6150<-34.834	0.6144<-34.877
8	1.0343<-14.061	0.6952<-33.497	0.6952<-33.497	0.6763<-34.846	0.6763<-34.846	0.6774<-34.834	0.6768<-34.877
9	0.9756<-16.054	0.5732<-41.990	0.5732<-41.989	0.5512<-44.015	0.5512<-44.015	0.5528<-43.966	0.5520<-44.032
10	0.9658<-16.261	0.5556<-43.217	0.5556<-43.216	0.5328<-45.354	0.5328<-45.354	0.5344<-45.298	0.5336<-45.368
11	0.9621<-15.914	0.5667<-42.310	0.5667<-42.313	0.5445<-44.375	0.5445<-44.357	0.5460<-44.319	0.5453<-44.387
12	0.9509<-16.241	0.5521<-44.589	0.5521<-44.588	0.5304<-46.870	0.5304<-46.870	0.5319<-46.801	0.5312<-46.876
13	0.9474<-16.357	0.5378<-44.952	0.5378<-44.951	0.5164<-47.335	0.5164<-47.335	0.5180<-47.268	0.5173<-47.347
14* **	**0.9426<-17.364**	**0.4970<-49.520**	**0.4969<-49.519**	**0.4815<-52.578**	**0.4815<-52.578**	**0.4839<-52.494**	**0.4833<-52.594**

*ε_{e14}=0.001 **$\Delta|v_{14}|$ =0.001 ***Critical Bus

In Table 3, Boundary for voltage stability, say base case separation from SNB point achieved by proposed new method and CPF is compared for the networks complying IEEE-14, IEEE-30 and IEEE-57 BUS standards.

TABLE III

SNB POINT PROPERTY (λ_{CR}) IN IEEE-BUS STANDARDS

IEEE Case Study	λ_{cr} (Proposed Method)	λ_{cr} (CPF Method)
14 BUS	1.5181	1.5787
30 BUS	1.2879	1.2870
57 BUS	3.4836	3.4814

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The V-λ curve achieved by proposed method for critical bus in networks complying IEEE-57 BUS standard is plotted in Figure 4, as well as 10 points belonging to the lower part of the curve are indicated in Table 4 for more illustration. Also, curves achieved by proposed method for voltage control bus (2nd and 3rd) in the same network is plotted in Figure 5. It shows that, all parts of V-λ curve can be determined by using the method proposed in this paper.

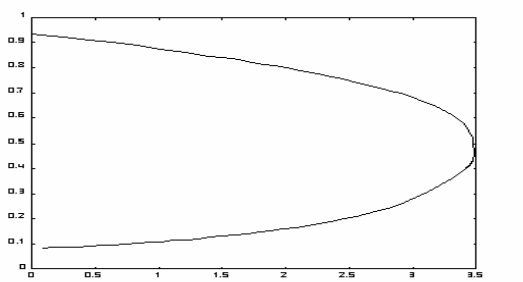

Fig.4. V-λ Curve for critical Bus(31st) in network complying IEEE- 57 BUS standard

TABLE IV

TEN POINTS OF LOWER PART OF V-λ CURVE IN IEEE-57 BUS STANDARDS (* SNB POINT)

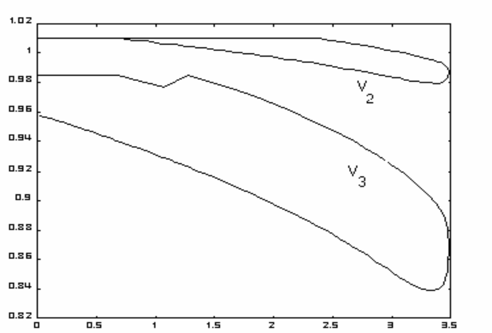

Figure 5- V-λ Curve for Voltage Control Bus(31st) in network complying IEEE- 57 BUS standard

VI. CONCLUSION

In this paper, a new method for evaluation of voltage stability phenomena in power systems and so determination of voltage stability boundaries is developed. It is represented that the method proposed at the paper has a good accuracy, so it can be used in plotting V-λ curves. Respecting the fact that only one stage of matrix inversion operation is done during each iteration, the new proposed method has very more quickness in comparison with continuation power flow method. Additionally, saddle-node-bifurcation point of the power system could be determined with proper accuracy and high calculation speed by appropriate selecting the perturbation parameter in new method

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