Allocating Generation To Loads And Line Flows For Transmission Open Access

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Abstract — **This paper presents a new method for calculating the individual generators' shares in line flows, line losses and loads. The method is described and illustrated on active power flows, but it can be applied in the same way to reactive power flows. Starting from a power flow solution, the line flow matrix is formed. This matrix is used for identifying node types, tracing the power flow from generators downstream to loads, and to determine generators' participation factors to lines and loads. Neither exhaustive search nor matrix inversion is required. Hence, the method is claimed to be the least computationally demanding amongst all of the similar methods.**

Index Terms — **Power flow tracing, deregulation, open access transmission.**

I. Introduction

What fractions of the branch flows and losses are contributed by a particular generator? How much of the power output of a generator is used to supply a particular load? In a vertically integrated system, the answers to these and similar questions are of little importance.

However, in a competitive environment, such "usage allocation" questions must be answered clearly and unequivocally to ensure fairness and efficiency of the electric power market [1, 2]

This issue has been addressed in several works [1-10]. In [1], all power injections are translated into real and imaginary currents; contribution of each source to each sink is determined by tracing these currents. A method for determining contributions of individual active or reactive generations to branch complex loss, and contributions of individual complex generations to branch active and reactive power losses is presented in [2].

In [3], the topological generation and load distribution factors TGDF are derived based on power flow tracing methodology [4]. This method requires a matrix inversion and considers losses by introducing virtual nodes in every branch. This method was used as the basis of ex-ante transmission pricing method in [5].

New concepts such as domain, commons, and links were introduced in [6] and used to determine the contributions of generators in supplying a domain, i.e the set of buses supplied by the set of generators. The method performs a long search procedure to form the domains and commons, which has to done for active power and reactive power each time even if a slight change takes place in the power flow.

In [7], the graph theory is utilized to perform power flow tracing for systems without loop flows, and a method for dealing with loop flows was introduced in [8]. The method makes use of a number of matrices of large dimensions, which requires more time for building up and manipulation of these matrices. Moreover, it is essential for the method to determine the tracing sequence first, as most of calculations and even the bus ordering depend upon that sequence. The method has not described how to deal with systems with more than one sink node and/or more than one source node. The node generation distribution factors NGDF are calculated for active and reactive power flows separately in [9] based on a search algorithm, which determines the power flow directions. To overcome the time consuming feature of the search algorithm, a method based on matrix calculations is presented in [11] that analytically obtains the flow paths from sources to sinks. However, that method replaces each line by two lines; one carrying the power transfer and the other carrying losses; the direct result is to increase the dimensionality, solution time, and computation burden of the problem.

This paper presents a new method for determining the contributions of individual generators to loads and line power flows in an electrical power system. A straightforward procedure is described for downstream tracing of the power injected by generators to find out the proportions of the line flows contributed by each generator. With this information in hand, transmission losses caused by each generator and the power drawn by each load from each generator can be determined.

The proposed method has the following advantages: no exhaustive search is required; generator shares in line flows are calculated by using just one matrix; no matrix inversion required; no additional nodes are required to be added for handling losses.

The proposed flow tracing technique is applied for both active power and reactive power in the same manner. Hence, the generator share in the complex power flow in each line, and the losses incurred by it can all be accurately calculated.

The paper is organized as follows: fundamentals and main concepts of the proposed method are presented. The procedure for tracing the generator power is then introduced. Test cases on simple systems are presented along with comparisons to the most common methods. Application to the IEEE 30 bus test system is presented followed by a conclusion.

II. Fundamentals And Main Concepts

The proposed method is based on tracing the flow of power through the network starting from generators down stream to loads. This requires power flow solution, which is available either from off-line calculations or from the on-line state estimator. The state variables for the tracing process are the active power and reactive power flows at both ends of each line.

The system nodes are divided into four different categories based on the directions of power flows in the lines incident to it. The node types are source nodes, generation nodes, load nodes, and sink nodes.

Source node is a node with all the lines incident to it carrying outflows from it to other nodes. Sink node is a node with all of its incident lines carry inflows to it. Both of generation nodes and load nodes have some the incident lines carrying inflows to it, and some others carrying outflows from it. The net power injection at the generation node is positive while it is negative at the load node.

A power system has at least one source node, the node with the most leading voltage, and at least one sink node, the one with the most lagging voltage. However, there may be more than one sink node and more than one source node. Source and sink nodes represent the terminal nodes of a power system. Any feeding path starts with a source node and ends with a sink node. Intermediate nodes are a combination of both load and generation nodes. These nodes, unlike source and sink nodes, have some incident lines carrying inflows while others carrying outflows, Fig.1.

A. Participation factors

The participation factor of a node to the flow through a line is defined as follows:

$$A_{ij} = \frac{\text{power flow in line } j \text{ caused by generator at node } i}{\text{total power flow in line } j} \quad (1)$$

A_{ij} is positive if the line j carries outflow from node i, negative if it carries inflow, and zero if the node causes no flow in the line.

The power flow through the lines incident to a source node is totally contributed by that node, Fig.2.a Therefore, the participation factors of a source node to all of the lines incident to it are 1's. Sink nodes inject no power to the system, Fig 2.b. Therefore, its participation factors to all lines incident to it must be zeros.

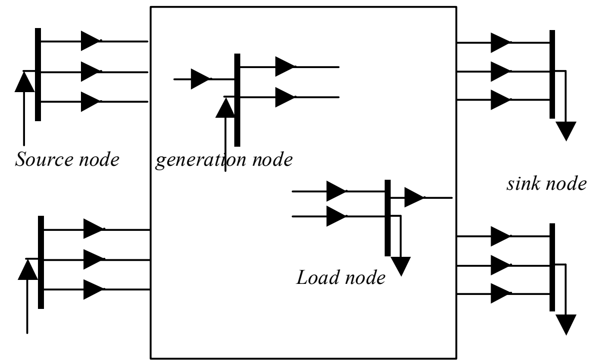

Fig.1 Types of power system nodes

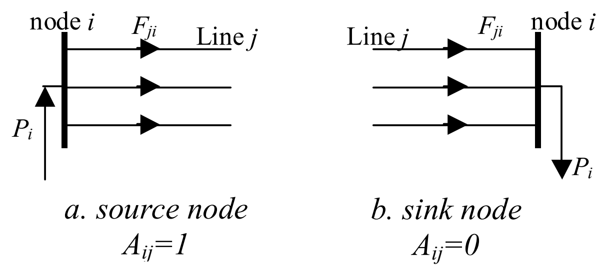

Fig. 2 Participation factors of source & sink nodes

Generation nodes and load nodes have branches carrying inflows and others carrying outflows besides the power injected or extracted at that node. Therefore, a mixing rule is required to determine the contribution of each input to a node into each output of the same node. The widely accepted proportional sharing principle [11] is used for this purpose.

The proportional sharing principle states that "the power flow reaching a bus from any power line splits among the lines evacuating power from the same bus proportionally to their power flows". Considering Fig.3, which shows a small part of a power system, applying the proportional sharing principle at node n gives the following participation factors.

$$A_{nc} = A_{nd} = \frac{p_n}{p_c + p_d}, \quad A_{na} = \frac{-p_a}{p_c + p_d}, \quad A_{nb} = \frac{-p_b}{p_c + p_d}$$

It should be noticed that:

- Participation factors of a generator at node to all of the lines carrying outflows of the same node are all equal.
- A line carrying inflow to a node contributes to the flow through the outflow lines in the same way a generator does. Therefore, such a line contributes to the flow through lines carrying outflows by the same ratio.

Now, if node i has participation factors A_{ia}, and A_{ib} to lines ***a***, and ***b*** respectively, power flows caused by node i through lines ***a***, ***b*** will be:

$$P_{ia} = P_i.A_{ia}, \qquad , P_{ib} = P_i.A_{ib}$$

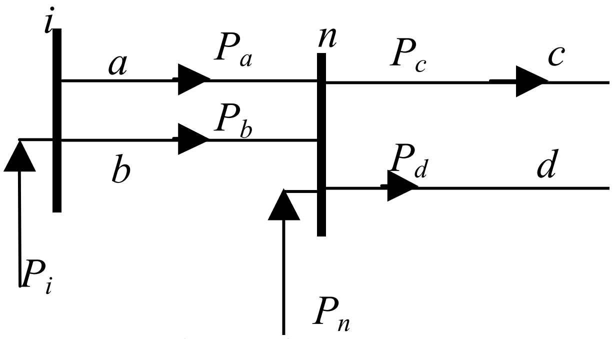

Fig.3 Part of a power system

The power flow caused by node i through line **c** will be:

$$P_{ic} = P_{ia} \,.\, A_{na} + P_{ib} \,.\, A_{nb}$$

Substituting for P_{ia} and P_{ib}, we get:

$$P_{ic} = P_i(A_{ia}\, A_{na} + A_{ib}\, A_{nb})$$

Participation factor of node i to line c can now be calculate as follows:

$$A_{ic} = A_{ia}\, A_{na} + A_{ib}\, A_{nb} \qquad (2)$$

It easy to find A_{id} in the same way, and to find out that it will be equal to A_{ic}. Eq. (2) can be written in a general form as follows:

$$A_{i,j} = \sum_{k\in\lambda_n} A_{i,k} . A_{n,k} \qquad , j \in \mu_n \qquad (3)$$

λ_n: the set of lines carrying inflows to node n

μ_n: the set of lines carrying outflows from node n

Eq.(3) can be applied in a recursive manner starting from source nodes, guided by directions of line power flows, down to sink nodes to determine participation factors of all generators to line flows. To avoid the need to exhaustive search of system lines each time a source or a generation node being processed, this work makes use of the line flow matrix, which is defined in the following section.

B. The line flow matrix

The line flow matrix, **F**, is formed with its rows corresponding to the system busses and columns corresponding to the system branches. Each column has all zero elements except the two corresponding to the end busses of the branch represented by that column. Each element has the value of the power entering or leaving the branch at the bus.

$$\mathbf{F} = [f_{ij}] \qquad i=1,N_B \quad j=1,N_L \qquad (4)$$

Where f_{ij} is the power extracted (outflow) or injected (inflow) by branch j at bus i. N_B is the number of busses and N_L is the number of lines. Outflows are considered positive whereas inflows are considered negative. f_{ij} is zero if line j is not incident to bus i.

The type of each node can be determined by examining the nonzero elements in the row corresponding to that node in the line flow matrix as follows.

Node (i) is a sink node if:

$$f_{ij} \le 0 \qquad , j = 1,\, N_L \qquad (5)$$

And is a load node if:

$$\sum_{j=1}^{N_L} f_{ij} \le 0 \qquad , \exists j : f_{ij} > 0 \qquad (6)$$

And is a generation node if:

$$\sum_{j=1}^{N_L} f_{ij} > 0 \qquad , \exists j : f_{ij} < 0 \qquad (7)$$

And is a source node if:

$$f_{ij} \ge 0 \qquad , j = 1,\, N_L \qquad (8)$$

C. Participation Factors Matrix

The participation factors matrix, **A**, is a matrix containing the participation factors of generators to line flows. It is initially formed from the line flow matrix as follows.

1. Source nodes: all of the power flow through lines incident to a source node are caused totally by that node only. Therefore each nonzero element in that row of **F** will be replaced by 1 in **A**.

$$A_{ij} = 1 \qquad \forall j : f_{ij} \ne 0 \qquad (9)$$

2. Sink nodes injects no power through any line. Therefore, all of the elements in that row must be zeros.

$$A_{ij} = 0 \qquad j = 1, \mathrm{N}_L \qquad (10)$$

3. For generation nodes,

$$A_{ij} = \begin{cases} \dfrac{P_i}{\sum_{m\in\alpha_P} f_{im}} & , f_{ij} > 0 \\ 0 & , f_{ij} = 0 \\ \dfrac{f_{ij}}{\sum_{m\in\alpha_P} f_{im}} & , f_{ij} < 0 \end{cases} \qquad (11)$$

α_P is the set of positive elements in the ith row,
P_i is the net power injected at bus *i*.

4. For load nodes,

$$A_{ij} = \begin{cases} 0 & , f_{ij} = 0 \\ \dfrac{f_{ij}}{\sum_{m \in \alpha N} |f_{im}|} & , f_{ij} < 0 \\ \alpha & , f_{ij} > 0 \end{cases} \qquad (12)$$

α_N is the set of negative elements in the ith row,
α is a very small positive number, 10^{-8} is used in this work.

III. The Tracing Procedure

The tracing procedure uses the elements of the matrix **A** to determine the power flow paths from generators down to loads and to determine the generator's participation factors to line flows along these paths. These paths are determined implicitly during the execution of the proposed procedure, hence there is no need for prior tracing of the power flow.

A positive element in a row corresponds to a line carrying outflow from that node to another. Negative elements in one row of **A**, define the lines carrying inflows to the node corresponding to that row.

Starting from a row with all positive elements, a source node, each negative element lying in the same column with one of the elements of that node defines the next node in the downstream direction on a feeding path to that node.

Negative elements of **A** will be eliminated, but they have two functions in the tracing process. The first is to determine the next node in the downstream direction. The second is to determine the participation factors of a source/ generation node to the flow through lines in the feeding path of that node.

The tracing procedure of the power contributed by a generator can be summarized as follows:

1. The procedure starts at a source node. Hence, rows with all positive elements have to be processed first..
2. For each nonzero element, A_{ij}, in the row corresponding to the generation node, each nonzero element corresponds a line fed by that generator, do the following:
   - check the element of column *j* containing A_{ij}. If there is no negative elements in that column, which means that this line connects that node directly to a sink node, go for the next A_{ij}, otherwise,
   - locate the row containing the negative element, *m*, that is the next node in the downstream direction. The negative element is thus A_{mj}
   - locate the positive elements in row *m*, these determine the lines following line *j* in the downstream direction of the feeding path of node *i*. If A_{mn} is positive, the element A_{in} has to be changed as follows:

   $$A_{in} = A_{in} + A_{ij} \cdot \left| A_{mj} \right| \quad , i = 1, N_B \quad i \neq m \qquad (13)$$

   and repeat that for all $A_{mn} > 0$

The negative sign of A_{mj} has to be removed when using (13), as the sign is used only to determine the directions. Therefore the modulus of A_{mj} is used rather than A_{mj}.

3. Repeat 2, but each time a row with all positive elements has to be picked up either a source node or a generation node with its negative elements already eliminated. If a number of nodes satisfy that condition, they can be processed in any order.
4. The process ends up when all negative elements of **A** have been eliminated. A final step is to set all the elements in rows corresponding to load nodes to zero.
5. The flow contributed by each generation in each branch can now be calculated as follows:

   $$\mathbf{T} = \mathbf{A}\ \mathbf{diag(Fj)} \qquad (14)$$

   Where: **T** is $N_B \times N_L$ matrix, T_{ij} is the flow contributed by a generator at node *i* through line *j*,
   diag(F_j) is a diagonal matrix with F_{jj} equals the power at the sending end of line j. Dimensions of **T** can be reduced to $N_G \times N_L$ by eliminating rows corresponding to load and sink nodes in **A**.
6. The power contributed by each generator to each bus can now be calculated as follows:

   $$\mathbf{P} = \mathbf{A}.\ \mathbf{F}^t \qquad (15)$$

   Where **P** is $N_B \times N_B$ matrix, $\mathbf{F}^t$ is the transpose of line flow matrix, and P_{ij} is the power contributed by the generator at node *i* to the power extracted at bus *j*. As in the case of **T**, dimensions of **P** can be reduced to $N_G \times N_B$ saving more than 70% of the memory requirements by just deleting zero rows corresponding to load and sink nodes from **A**.

IV. Test Cases

To test the new method, the systems used in [3], [7] and [9] are used to compare the proposed method with the available methods. Fig. 4 shows both of the power flow, Fig. 4.a, and the final result of power transfer between generators and loads, Fig. 4.b, for the system used in [7]. The reader is encouraged to read [7] to find out how much are the computations required and how many matrices are used.

The proposed method has been applied to the same simple system; the steps of calculations are shown below. It is clear that the amount of calculations is far less than that required for the method of [7].

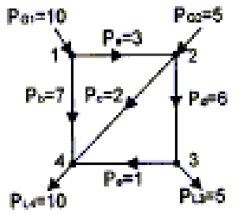

Fig 4.a Power flow

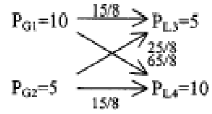

Fig. 4.b. Power Transfer between generators and loads

Fig. 4. Power flow and power transfer for 4 bus system

1.Form the line flow matrix

$$F = \begin{bmatrix} 3 & 7 & 0 & 0 & 0 \\ -3 & 0 & 2 & 6 & 0 \\ 0 & 0 & 0 & -6 & 1 \\ 0 & -7 & -2 & 0 & -1 \end{bmatrix}$$

2.Initial participation factors matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -\frac{3}{8} & 0 & \frac{5}{8} & \frac{5}{8} & 0 \\ 0 & 0 & 0 & -1 & 1e-8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3. eliminate the first negative element

$$A = \begin{bmatrix} 1 & 1 & \frac{3}{8} & \frac{3}{8} & 0 \\ 0 & 0 & \frac{5}{8} & \frac{5}{8} & 0 \\ 0 & 0 & 0 & -1 & 1e-8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4. eliminate the second negative element, and set all elements in load node rows to zero.

$$A = \begin{bmatrix} 1 & 1 & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ 0 & 0 & \frac{5}{8} & \frac{5}{8} & \frac{5}{8} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Generators' contributions to line flows. $\mathbf{T = A\ diag(F_j)}$

$$T = \begin{bmatrix} 3 & 7 & \frac{6}{8} & \frac{18}{8} & \frac{3}{8} \\ 0 & 0 & \frac{10}{8} & \frac{30}{8} & \frac{5}{8} \end{bmatrix}$$

6. Generators contributions to bus powers. $\mathbf{P = A\ F^t}$

$$P = \begin{bmatrix} 10 & 0 & -\frac{15}{8} & -\frac{65}{8} \\ 0 & 5 & -\frac{25}{8} & -\frac{15}{8} \end{bmatrix}$$

In P, step 6, positive elements means a power injected by a generator at a bus, while a negative element means power extracted by loads from a generator. It can be noticed that the results obtained using the proposed method for power transfer between generators and loads are the same as those obtained in [7].

The second test to the proposed method is carried out using the system used in [3] and [9]. Fig. 5 is taken from [3]; it shows the system with the active power flows. Table I is taken from [9]; it shows a comparison between the TGDF[3] and NGDF[9]. The small differences between the two methods are due to the treatment of losses in [3].

TABLE I.
TGDF AGAINST NGDF

	Generator 1		Generator 2	
Line	TGDF	NGDF	TGDF	NGDF
1-2	1.000	1.000	0.000	0.000
1-3	1.000	1.000	0.000	0.000
1-4	1.000	1.000	0.000	0.000
2-4	0.345	0.341	0.655	0.659
4-3	0.605	0.602	0.395	0.398

It is to be reminded that [3] introduces extra node on each line to account for losses and requires matrix inversion; the method of [9] utilizes a time consuming search algorithm. It is also to be said that TGDF and NGDF are nothing but we call the participation factors. Therefore, the participation factors are calculated using the proposed method and found to be as follows.

A=

1	1	1	0.341	0.6018
0	0	0	0.659	0.3982
0	0	0	0	0
0	0	0	0	0

It can be easily found out that the participation factors are exactly equal to NGDF; different from TGDF for the reason mentioned before. The real difference between the proposed method and all of the other method is the minimum computation requirements of the proposed method; of course not on expense of the accuracy or the versatility.

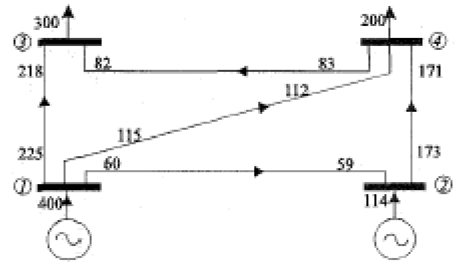

Fig. 5. Power flows of the system of [3].

V. APPLICATION TO THE IEEE 30 BUS SYSTEM

The proposed method has been applied to the IEEE 30 bus test system. Table II lists the active power transfers between generators and loads obtained by the proposed method.

TABLE II
POWER TRANSFERS OF THE IEEE 30 BUS SYSTEM

Bus	Power supplied by generator (MW)							
	1		2	8	11		13	
	New	[7]	New	New	New	[7]	New	[7]
3	2.40	2.4	0.00	0.00	0.00		0.00	
4	6.74	6.74	0.86	0.00	0.00		0.00	
5	50.72	50.77	18.78	0.13	0.00		0.00	
7	18.12	18.15	4.45	0.23	0.00		0.00	
10	2.77	2.78	0.68	0.04	2.32	2.31	0.00	
12	6.04	6.02	0.77	0.00	0.00		4.39	4.42
14	3.34	3.33	0.42	0.00	0.00		2.43	2.45
15	4.42	4.41	0.56	0.00	0.00		3.22	3.23
16	1.89	1.88	0.24	0.00	0.00		1.37	1.38
17	4.49	4.49	0.90	0.04	2.33	2.34	1.25	1.23
18	1.73	1.72	0.22	0.00	0.00		1.26	1.26
19	4.69	4.69	1.00	0.04	2.82	2.84	0.95	0.93
20	1.05	1.05	0.26	0.01	0.88	0.88	0.00	
21	8.35	8.39	2.05	0.11	6.99	6.97	0.00	
23	1.73	1.72	0.22	0.00	0.00		1.26	1.26
24	4.37	4.39	0.97	0.26	2.46	2.45	0.63	0.59
26	2.12	2.12	0.52	0.86	0.00		0.00	
29	1.46	1.45	0.36	0.59	0.00		0.00	
30	6.43	6.41	1.58	2.59	0.00		0.00	
Loss	5.70	5.72	1.02	0.11	0.14	0.14	0.15	0.16
Total	138.55	138.63	35.86	5.00	17.93	17.93	16.91	16.91

The power transfers of some of the generators obtained by the method of [7] are also listed. It is clear that the results obtained by the two methods are very close to each other. The minor differences between the two methods are a direct result of the way of treating losses in [7]; because the method of [7] was built originally for a lossless system; and losses are accounted for by considering them additional loads at either end of the line. The proposed method does not go for similar approximations; the line is treated as it is, since both ends of the line are considered in the tracing procedure and in calculating the participation factors.

VI. Conclusion

A new method for the determination of the contributions of individual generators to loads, line flows, and losses is introduced. The proposed method makes use of the power flow solution which is obtained either from off-line calculation or from the on-line output of the state estimator. The proposed method has the advantage of minimum computation requirements, as there is no need for search, matrix inversion or adding virtual nodes to account for losses. This makes it perfect for on line application to give real time price signals for both power producers and customers.

Transmission losses are implicitly accounted for since the power flow information at both ends of the line are used in the proposed power flow tracing algorithm. Such treatment implies that losses caused by partial flows are proportional to the partial flows. This is absolutely accurate if the of active power to reactive power is the same for all of the partial flows. It represents a good approximation for most cases. However, more consideration has to be paid for loss calculation to take into account the cross effects of active and reactive powers and their impact on line losses.

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