Design and Analysis Of Second Order IIR Notch Filters With Double Frequency Initialization

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***Abstract* — This paper presents a new design method for notch IIR filters. The designed filters offer better transient frequency response for a given number of samples when compared to zero and projection initialization. Fine-tuning the positions of the poles to maximize the 3dB bandwidth of the transient frequency response for a given number of samples achieves the improvement. Second order filters were successfully designed for processing different numbers of samples and their performances were studied by considering their frequency responses.**

***Index Terms* — IIR filters, Initialization, Notch, Clutter.**

I. Introduction

Digital IIR notch filters are usually designed for steady state operation (i.e. processing a large number of input samples). In some applications, such as electrocardiogram (ECG), only a limited number of measured samples are available for processing. One typical example is to eliminate the power line disturbance in the recording of electrocardiogram [1]. Conventional IIR Notch filters with a limited number of samples will suffer from the transient effect which will result in degrading their rejection characteristics. Their frequency responses will be a function of the number of processed samples. These responses are called the transient frequency responses and they are different from the steady state responses.

One way of improving the transient frequency response of IIR filters is by initializing their internal memories (i.e. the output of the delay elements) with values other than zero. This requires a processor which is called the initialization processor to calculate the initial values from the first received sample in real time. These values will be the steady state values of the filter for a given input signal. The input signal may be approximated by a step function and this method is called zero frequency initialization [2]. The initialization processor will improve the transient step response (zero frequency response) of the filter irrespective of the number of the processed samples provided that the filter has one steady state zero at zero frequency. However, the initialization process will affect the transient frequency response at frequencies other than zero. These effects were studied for a variety of IIR filters [3].

On the other hand if the input signal can be approximated by a single tone sine wave then another processor exists for improving the transient rejection response of the IIR filter, provided that the filter has at least one z-plane zero at that particular frequency [4]. The processor requires two channels (i.e. in-phase and quadrature-phase) and calculates the steady state values of the internal memories of the filter from the first input sample of the two channels. In this case the initialization processor requires complex multipliers. For example, the initialization of a second order filter will result in forcing the filter to reach the steady state at either the positive or negative notch frequency. This method will improve the transient rejection characteristic of the IIR filter at the notch frequency but will force the resultant transient frequency response to be asymmetrical.

Other methods have been proposed in the literature to improve the transient performance of IIR filters, however they are suitable only for batch processing. Projection initialization is based on removing the transient component from the output signal [5]. Using this condition, the values of the internal memories can be calculated. This method is used in Ultrasound colour flow imaging for clutter filtering. Another sort of projection was suggested to suppress the transient in notch IIR filters [6]. Exponential initialization which is described in [7], suggests initializing an IIR filter to suppress the transient from a complex exponential with frequency equal to the estimated mean frequency. Also regression filters were suggested to remove clutter in colour flow imaging. A regression filter calculates the best least square fit of the signal to a set of the curve forms modelling the clutter signal, and subtracts this clutter approximation from the original signal [8].

Recently a new initialization processor was introduced which can work at two frequencies in order to keep the resultant transient frequency response symmetrical [9]. This was achieved by breaking the second order real filter into the product of two first order complex sections and initializing the internal memories of each section with its steady state values. This resulted in forcing the filter to reach its steady state frequency response at these two frequencies by using the first received input sample of the quadrature channels.

The initialization technique was initially used as a post-design method to improve the transient frequency response of steady state IIR filters such as Chebychev and Elliptic filters. However, it can be used to design filters with optimum transient performances [10]. This paper deals with the design of new filters with an initialization processor which can force the steady state response at two frequencies while optimizing the transient response at other frequencies. These filters will be called throughout this paper "transient filters". The optimization criteria used in the design method is the maximization of the 3dB bandwidth of the transient frequency response.

II. DESIGN METHOD

A new design method for IIR notch filters with initialization is introduced in this section. The ideal notch filter is characterized by having unity gain at all frequencies except the notch frequency where the gain is zero as shown in Fig. 1.

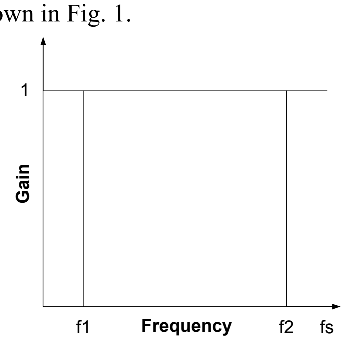

Fig. 1. An ideal notch filter

The initialization technique described in [9] is incorporated in our design method. The transfer function of a real second order IIR filter is given in the following equation:

$$H(z) = \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{1 - b_1 z^{-1} - b_2 z^{-2}}, \qquad (2)$$

where a_1, a_2 are the feed-forward and b_1, b_2 are the feed-back coefficients of the filter.

The z-transfer function of this filter can be rewritten as:

$$H(z) = \frac{1 - (a_r + ja_i)z^{-1}}{1 - (b_r + jb_i)z^{-1}} \times \frac{1 - (a_r - ja_i)z^{-1}}{1 - (b_r - jb_i)z^{-1}}, \qquad (3)$$

where $a_r = \cos(2\pi f_1 T)$, $a_i = \sin(2\pi f_1 T)$ *and* f_1 is the notch frequency. The constants b_r and b_i are the real and imaginary parts of the complex conjugate pair of poles. Accordingly, a second order IIR notch filter can be realized by using first order sections. The transfer function will be the product of only two complex first-order sections. The filter should have either complex or real transmission zeros, but they must be located on the unit circle. For example a second order filter will have the following zeros:

$$z_1 = e^{j2\pi f_1 T}, z_2 = z_1^{-1} \qquad (4)$$

where T is the sampling interval $(1/f_s)$

The initialization processor calculates the steady state values of the internal memories for each section from the first received input sample and feeds them to the associated section. Each first order section will have zero transient frequency response at that particular frequency. The output of each section is fed to a multiplier so that the filter will have two zeros in the transient frequency response irrespective of the number of processed samples.

In the proposed method, the feedback coefficients of the IIR filter are designed to maximize the 3dB bandwidth of the transient frequency response. To illustrate the basic principle behind this new design method, a second order real notch filter is used, but the same method can be used to design higher order filters.

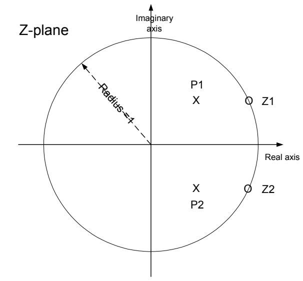

Fig. 2. The second order filter specifications

Fig. 2 shows the requirements of the second order filter. The filter must have two zeros at $z = z_1$ and $z = z_2 = z_1^{-1}$. The positions of the zeros of this filter are fixed on the unit circle. The only degree of freedom that can be used during the design process is the positions of the poles. This filter has a conjugate pair of poles because it is a real filter. The position of these poles, which will maximize the 3dB bandwidth of the transient frequency response, is determined by using a search algorithm as illustrated in Fig. 3. The search is carried out in two passes:

1. In the first pass, the area within the unit circle is searched with a coarse resolution *(*ΔRe_{Coarse}, ΔIm_{Coarse}) to find the position of the pole that maximizes the 3dB bandwidth.
2. In the second pass, the results of the first pass, i.e. the positions of the poles, are taken as seeds for the fine search within the unit circle. This is done by searching a square with a predefined width (S_W) and resolution *(*ΔRe_{Fine}, ΔIm_{Fine}) to find the best position of the poles according to the optimization criteria.

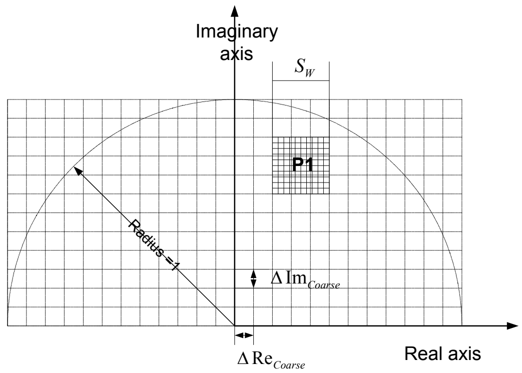

Fig. 3. The search algorithm to find the positions of the poles.

Using this method, second order filters were designed for different number of samples. Fig. 4 shows the transient frequency response for a filter which was designed for 7 samples. The 3dB bandwidth for this filter is 0.7303 f_s.

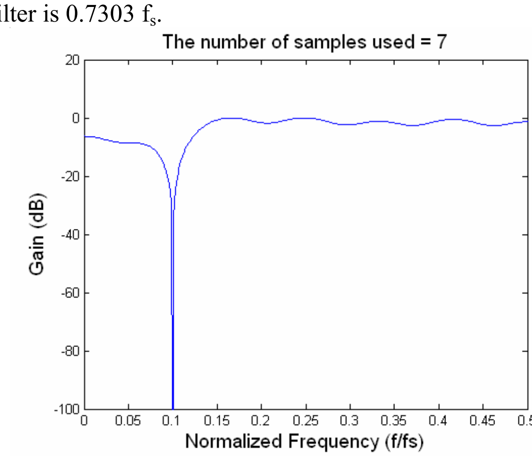

Fig. 4. The transient frequency response of a second order filter. The zeros are located at z = 0.1f_s and z = 0.9f_s and the poles are located at z = 0.576 ± j 0.592.

III. Analysis and Discussion

In order to study the performance of filters designed by this new method and to compare them with existing initialization methods, a steady state filter that maximizes the 3 dB bandwidth should be designed. For this, the same search algorithm described in Section II was used to design the steady state filters that maximize the 3 dB bandwidth.

The performance of one filter will be studied. This filter has transmission zeros at z = 0.1fs and z = 0.9fs. The transfer functions of this filter is described in the following equation:

$$H_1(z)=\frac{1-(0.8090+j0.5878)\ z^{-1}}{1-(0.80+j0.5820)\ z^{-1}}\times\frac{1-(0.8090+j0.5878)\ z^{-1}}{1-(0.80-j0.582)\ z^{-1}} \qquad (5)$$

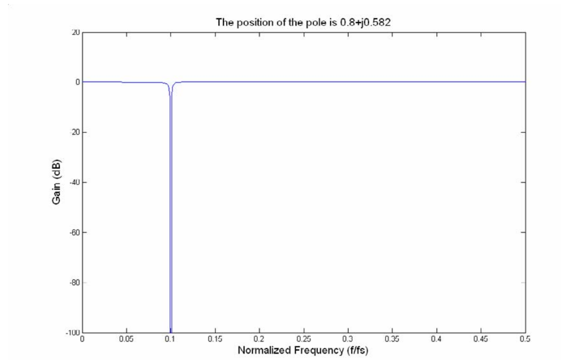

Fig. 5. The steady state frequency response of the filter described in $H_1(z)$ and its 3dB bandwidth is 0.9940 f_s.

In the following sections, the transient frequency responses of the new designed filters will be compared with

– steady state frequency responses without initialization,
– steady state frequency response with double frequency initialization as described in Section II and in [9],
– transient frequency response without initialisation, and
– steady state frequency response with projection initialization.

A. New Filter

Using the same locations of zeros as in $H_1(z)$, a new filter was designed for 7 samples. The transfer function of the new filter is described in the following equation:

$$H_2(z)=\frac{1-(0.8090+j0.5878)\ z^{-1}}{1-(0.576+j0.592)\ z^{-1}}\times\frac{1-(0.8090+j0.5878)\ z^{-1}}{1-(0.576-j0.592)\ z^{-1}} \qquad (7)$$

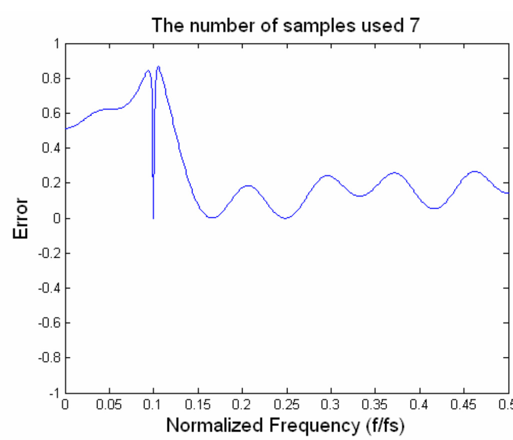

Fig. 6. The error signal between the steady state frequency response of $H_1(z)$ and the transient frequency response of the new filter described in $H_2(z)$. The 3dB bandwidth of the new filter is 0.7303 f_s.

It is clear from Fig. 6 that the transient frequency response will have the steady state frequency response at the notch frequency. However, some ripples are introduced in the pass band as a penalty for this.

B. Transient Frequency Response Errors with Double Frequency Initialization

Fig. 7 shows that the transient frequency responses will have the steady state frequency response at the notch frequency. However, the ripples introduced in the pass band have higher amplitude when compared to the ones in the new designed filters.

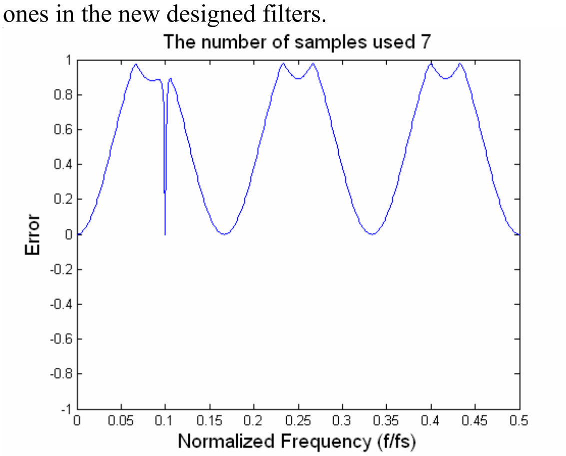

Fig. 7. The error signal between the steady state frequency response of $H_1(z)$ and its transient frequency response using double frequency initialisation. The 3dB bandwidth of the initialised filter is 0.3427 f_s.

C. Transient Frequency Response Errors without Initialization

It is clear from Fig. 8 that the transient frequency responses will not have the steady state frequency response at the notch frequency. However, there will almost be no ripples in the pass band.

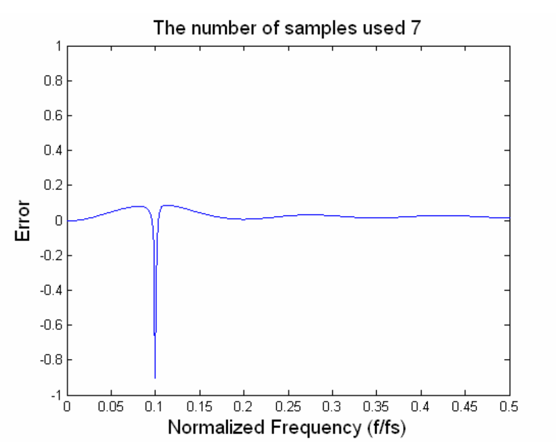

Fig. 8. The error signal between the steady state frequency response of $H_1(z)$ and its transient frequency response without initialisation. The 3dB bandwidth without initialisation is 1 f_s.

D. Transient Frequency Response Errors with Projection Initialization

It is clear from Fig. 9 that the transient frequency response with projection initialization will have the steady state frequency response at the notch frequency. However, some ripples will be introduced in the pass band. Note that the error signal in this figure is almost the same as the error signals in Fig. 6.

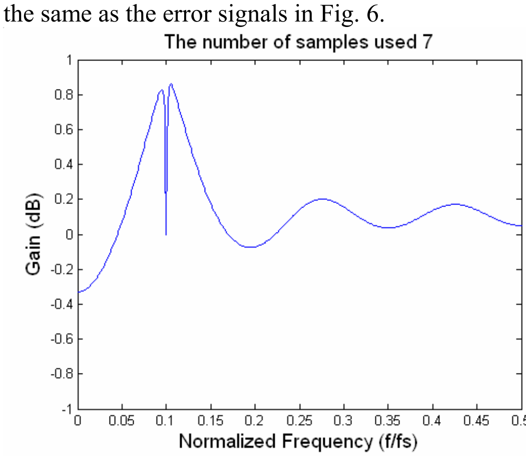

Fig. 9. The error signal between the steady state frequency response of $H_1(z)$ and its transient frequency response using projection initialisation. The 3dB bandwidth of the projection initialised filter is 0.3546f_s.

E. Frequency Responses

In figures 10 and 11 the following symbols are used:

- FResp1: The transient frequency response of the new designed filter
- FResp2: The transient frequency response of the steady state filter without initialization.
- FResp3: The steady state frequency response.
- FResp4: The transient frequency response of the steady state filter using projection initialization.

Fig. 10 shows a comparison between the transient frequency response without initialization, the steady state frequency response, the transient frequency response of the new designed filter, and the transient frequency response using projection initialization. Since the new designed filter was optimized for 7 samples, the projection initialization was also done with 7 samples. It is clear from this figure that the transient frequency response of the new designed filter is better than the transient frequency response of the steady state filter with or without initialization. The rejection characteristics in transient frequency response of the filter with zero initialization is completely removed, see the dotted line in Fig. 10. Only initialization will force the transient frequency response to have zero at notch frequency, however ripples are introduced in pass band. The rejection characteristics of the new filter are the same as the steady state one. Note that some ripples will be introduced in the pass band, but these ripples are almost the same as the ripples in the projection initialized transient frequency response.

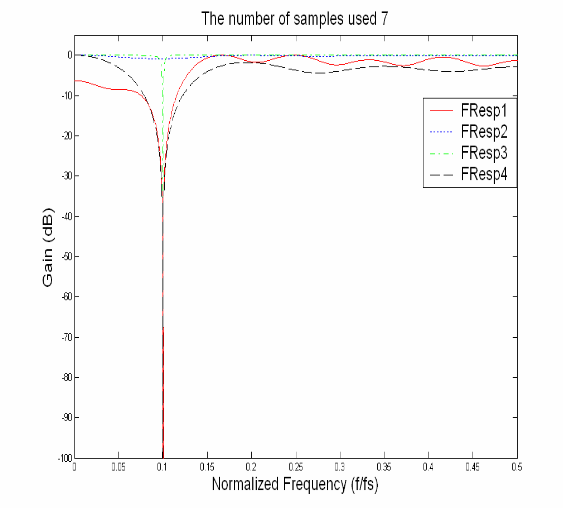

Fig. 10. A comparison between frequency responses for 7 samples.

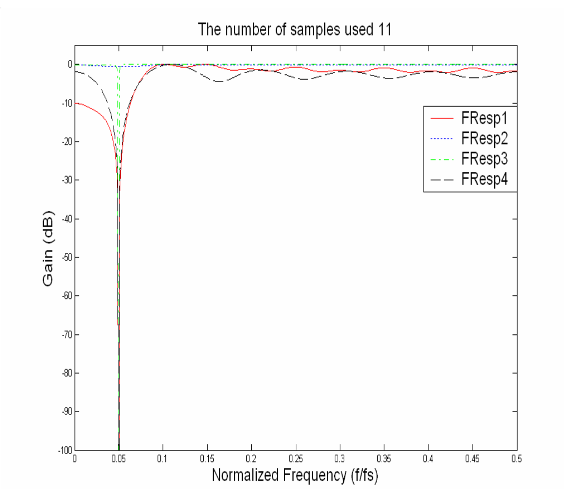

Fig. 11. A comparison between frequency responses for 11 samples. The zeros are located at z=0.05 fs and z=0.95 fs.

By comparing Fig. 11 and Fig. 10, it is clear that the transient frequency response of the new designed filter is either the same or better as the transient frequency response with projection initialization. Note that in projection initialization, all of the samples must be available in order to perform filtering operation: however the new designed filters can perform filtering in real time using the first received sample.

IV. Conclusions

A new technique for designing IIR notch filters with maximum bandwidth using double frequency initialization was presented. The technique is suitable for designing second order IIR notch filters with a complex conjugate pair of zeros. Using this method, optimum second order filters can be successfully designed for any specific number of samples. The frequency response of the new designed filter has better characteristics when compared to zero and projection initialization. Furthermore, the new designed filters can be used in real-time while the projection initialization method is only suitable for batch processing.

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