Cholesky Factors Based Wavelet Transform Domain LMF Algorithm

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***Abstract*— This paper presents a new wavelet transform domain least mean fourth (LMF) algorithm. The algorithm exploits the special sparse structure of the wavelet transform of wide classes of correlation matrices and their Cholesky factors in order to compute a whitening transformation of the input data in the wavelet domain and minimize computational complexity. This method explicitly computes a sparse estimate of the wavelet domain correlation matrix of the input process. It then computes the Cholesky factor of that matrix and uses its inverse to whiten the input. The proposed algorithm has faster convergence rate than that of wavelet transform domain least mean square (LMS) algorithm.**

I. INTRODUCTION

The LMS algorithm [1] provides a solution to the optimal Weiner Filter criterion minimizing the mean square value of the error in a stochastic approximation sense. LMS belongs to the gradient type algorithmic schemes, thus inheriting their low computational complexity and their slow convergence, especially on highly correlated signals like speech. The reason is that LMS algorithm is directly dependent on the correlation matrix $\mathbf{R}$ of the input vector $\mathbf{x}_n$. Therefore, when the eigenvalue spread of $\mathbf{R}$ is large, LMS algorithm experiences a *gradient noise amplification* [1].

To improve the convergence speed of the LMS algorithm, the input vector can be transformed so that the input correlation matrix in the transformed domain has a lower eigenvalue spread (i.e. close to 1). In particular, if $\mathbf{R}$ is nonsingular, we can prewhiten the input to the LMS adaptive filter by premultiplying $\mathbf{x}_n$ by the matrix $\mathbf{L}^{-1}$, where $\mathbf{L}$ is the lower triangular Cholesky factor of $\mathbf{R}$, i.e., $\mathbf{R} = \mathbf{L}\mathbf{L}^T$. This approach is equivalent to using a Newton-LMS type algorithm [2] and requires $O(N^2)$ flops, making it computationally expensive. In case $\mathbf{R}$ is singular, whitening cannot be used, and an alternative procedure such as the one described in [3] can be used.

The increase in computational complexity due to prewhitening may be alleviated if $\mathbf{R}$ has some special sparse structure. In particular, if $\mathbf{R}$ is diagonal, this increase is minimal. This observation has motivated the development of a variety of transform domain algorithms (cf. [4] and the references cited therein) [5], [6], [7], wherein one hopes to obtain a near diagonal input correlation matrix in the transform domain by a proper choice of the transform. A Newton-LMS type filter realized in the transform domain then leads to an improved convergence speed with a much smaller computational burden. For example, the DFT (implemented using fast Fourier transform (FFT) algorithms) or the DCT nearly diagonalize a large (large filter lengths) Hermitian symmetric and Toeplitz correlation matrix because such a matrix can be approximated by a circulant matrix [8]. The cost of such a transformation is a minimal $O[Nlog(N)]$ flops. Subband adaptive filters with improved convergence and low computational burden have also been suggested [9]. In [10], a new wavelet transform domain least mean square (LMS) algorithm was proposed which exploited the special sparse structure of the wavelet transform of wide classes of correlation matrices and their Cholesky factors in order to compute a whitening transformation of the input data in the wavelet domain and minimize computational complexity. It is well known that least mean fourth (LMF) algorithm [11] and its normalized version (NLMF) algorithm [12] have better performance in non Gaussian environments. This gives motivation towards wavelet transform domain LMF algorithm.

In this paper, a wavelet transform domain least mean fourth (LMF) algorithm is proposed which uses the special sparse structures of the input correlation matrices and their Cholesky factors in the discrete wavelet transform (DWT) domain. This approach explicitly computes a sparse estimate of the wavelet domain input correlation matrix and its Cholesky factor. It then uses the inverse of the Cholesky factor to whiten the input.

This paper is organized as follows: in Section II, the DWT computation for finite sequences is briefly discussed. In Section III, the proposed wavelet domain LMF algorithm is developed. Simulation results are presented in Section IV while results are concluded in section V.

II. THE DISCRETE WAVELET TRANSFORM

Although the discretized continuous wavelet transform enables the computation of the continuous wavelet transform (CWT) by computers, it is not a true discrete transform. As a matter of fact, the wavelet series is simply a sampled version of the CWT, and the information it provides is highly redundant as far as the reconstruction of the signal is concerned. This redundancy, on the other hand, requires a significant amount of computation time and resources. The discrete wavelet transform (DWT), on the other hand, provides sufficient information both for analysis and synthesis of the

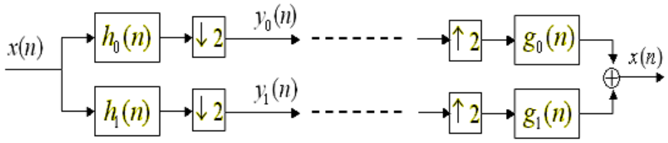

Fig. 1. Wavelet Transform Based Analysis and Synthesis Filter Banks.

original signal, with a significant reduction in the computation time. The DWT is considerably easier to implement when compared to the CWT.

The discrete wavelet transform of a square integrable function (finite energy signal) $s(t)$ is defined as

$$C(j,k) = \int_R s(t)\psi_{j,k}(t)dt \qquad (j,k) \in Z^2,\ s \in L^2(R) \quad (1)$$

where $C(j,k)$ are the discrete wavelet transform coefficient. $\psi_{j,k}(t)$ are the wavelet expansion functions or the wavelet basis functions. These are related to the original mother wavelet function denoted as $\psi(.)$ and are given as follows.

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k), \quad (2)$$

where j and k are the dilation and translation parameters, respectively.

In this work, discrete nonredundant M-band wavelet decompositions [13] is used as shown in Figure 1. Wavelet transforms have been treated in considerable detail, and wavelet decompositions reconstruction constraints have been related to perfect reconstruction filter (PRF) banks [14].

III. Wavelet Transform Domain LMF Algorithm

In this section, we describe a new wavelet transform domain LMF algorithm based on a fast technique for whitening the input data. This technique rely on the DWT domain sparse structure of wide classes of input correlation matrices and their Cholesky factors. This approach explicitly computes a sparse estimate of the input correlation matrix and its Cholesky factor in the DWT domain. It then uses the inverse of this Cholesky factor to whiten the input.

If $\mathbf{y}_n$ denotes the $N \times N$ discrete wavelet transform of input $\mathbf{x}_n$, then $\mathbf{y}_n$ can be written as:

$$\mathbf{y}_n = \mathbf{Q}\mathbf{x}_n, \quad (3)$$

where $\mathbf{Q}$ represents the unitary transformation for discrete wavelets. Similarly wavelet transformed domain weight vector $\mathbf{w}_n$ is given by:

$$\mathbf{w}_n = \mathbf{Q}\mathbf{h}_n, \quad (4)$$

where $\mathbf{h}_n$ is the time domain weight vector of the adaptive filter. The wavelet transform domain adaptive FIR filter is shown in Figure 2. The update of wavelet transformed domain weight vector $\mathbf{w}_n$ in the proposed algorithm is given by:

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \mu\hat{\mathbf{g}}_n, \quad (5)$$

where, the vector $\hat{\mathbf{g}}_n$ is obtained by solving

$$\hat{\mathbf{R}}_{yn}\hat{\mathbf{g}}_n = -2e_n^3\mathbf{y}_n, \quad (6)$$

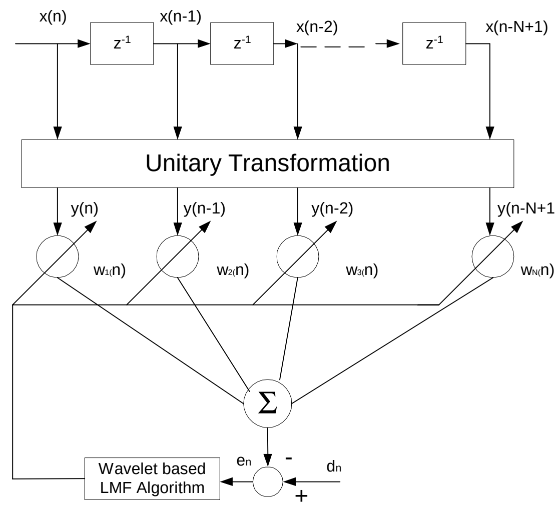

Fig. 2. Transformed Domain Adaptive FIR Filter.

where $\hat{\mathbf{R}}_{yn}$ is the sparse estimate of the correlation matrix $\mathbf{R}_{yn}$ of wavelet transformed input $\mathbf{y}_n$, and e_n is the output error defined as:

$$e_n = d_n - \mathbf{w}_n^T\mathbf{y}_n, \quad (7)$$

where d_n is the desired response. In the case of system identification scenario, d_n is given by:

$$d_n = \mathbf{c}_n^T\mathbf{x}_n + \xi_n, \quad (8)$$

where $\mathbf{c}_n$ and ξ_n represent unknown system and the additive noise, respectively. Figure 3 depicts this clearly. In this work,

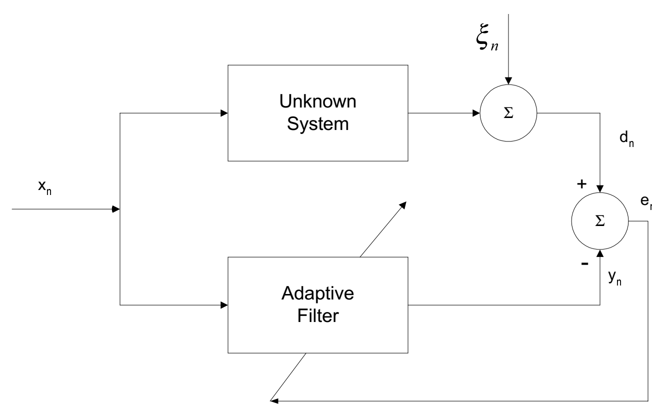

Fig. 3. System Identification Scenario.

$\hat{\mathbf{R}}_{yn}$ is computed by estimating only the expected nonnegligible entries of $\mathbf{R}_{yn}$. All other entries of $\mathbf{R}_{yn}$ are set to zero. For this purpose, the sparse Cholesky factorization of $\hat{\mathbf{R}}_{yn}$ is used. Thus rewriting equ (6) in terms of the Cholesky factors as follows:

$$\hat{\mathbf{L}}_{yn}\hat{\mathbf{D}}_{yn}\hat{\mathbf{L}}_n^T\hat{\mathbf{g}}_n = -2e_n^3\mathbf{y}_n. \quad (9)$$

The above equation can be solved in three steps as follows:

$$\hat{\mathbf{L}}_{yn}\mathbf{a} \quad = \quad -2e_n^3\mathbf{y}_n, \quad (10)$$

$$\hat{\mathbf{D}}_{yn}\tilde{\mathbf{a}} \quad = \quad \mathbf{a}, \qquad (11)$$

and

$$\hat{\mathbf{L}}_n^T\hat{\mathbf{g}}_n \quad = \quad \tilde{\mathbf{a}}. \qquad (12)$$

Since $\hat{\mathbf{D}}_{yn}$ is a diagonal matrix, evaluation of $\hat{\mathbf{g}}_n$ using this approach has very low complexity.

A. Summary of the Algorithm

Following are the steps of implementation of the proposed algorithm:

1) Compute the discrete wavelet transform of $\mathbf{x}_n$ (i.e., $\mathbf{y}_n$).
2) Maintain and update $\hat{\mathbf{R}}_{yn}$, which is the sparse matrix approximate of the DWT domain correlation matrix.
3) Compute the approximate Cholesky factors $\hat{\mathbf{L}}_{yn}$ and $\hat{\mathbf{D}}_{yn}$ of $\hat{\mathbf{R}}_{yn}$.
4) Finding the inverse of $\hat{\mathbf{R}}_{yn}$.
5) Update the transformed weight vector using equations (5) and (6).

IV. SIMULATION RESULTS

In this section, the results of the computer simulations are presented which are made to investigate the performance behaviors of the proposed LMF algorithm. These results are compared with the results of wavelet domain LMS algorithm in unknown system identification problem which shows better performance of the proposed algorithm in terms of convergence speed. The performance measure considered is the normalized weight error norm ($10log_{10}\|\mathbf{w}_n - \mathbf{c}\|^2/\|\mathbf{c}\|^2$). where $\mathbf{c}$ is the vector representing the unknown system.

We have chosen "Harr" wavelets for computation of the DWT of the input sequence. The unknown systems to be identified has FIR model given by $[0.5,\ 1]^T$. The observation noise is uncorrelated with the input sequence. The noise added has zero mean while signal to noise ratio (SNR) used is 20 dB. The length of the adaptive filter is chosen equal to the length of the unknown system. The performance measure is analyzed in both gaussian and uniform environment. The results are obtained by averaging over 50 independent runs.

In Figures 4, and 5, it is shown that the proposed algorithm has achieved the same noise floor in a smaller number of iterations as compared to the wavelet domain LMS algorithm. In both environments, the proposed algorithm achieved the same steady state approximately in 500 iterations earlier than the wavelet domain LMS algorithm. The stability of the proposed algorithm is compared with the wavelet domain LMS algorithm when there is a sudden change in the environment.

In Figures 6, and 7, it is shown that the proposed algorithm has a better ability to recover from a sudden change in the environment. In both Gaussian and uniform environments, the proposed algorithm has recovered from sudden change faster than that of the wavelet domain LMS algorithm.

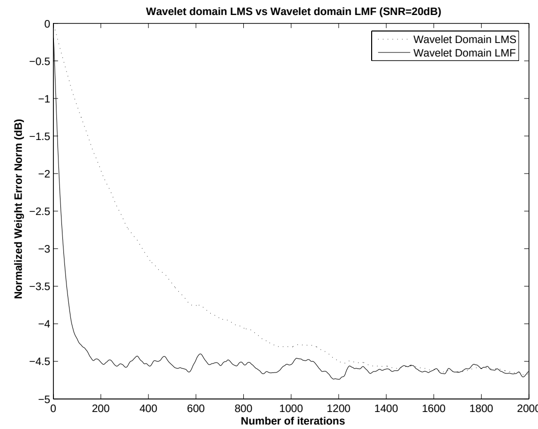

Fig. 4. Comparison of the convergence speed in Gaussian environment.

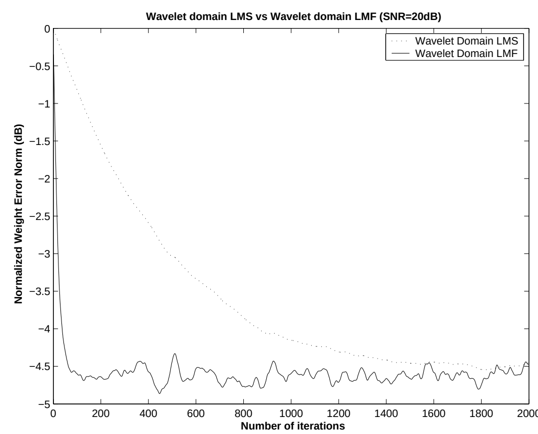

Fig. 5. Comparison of the convergence speed in uniform environment.

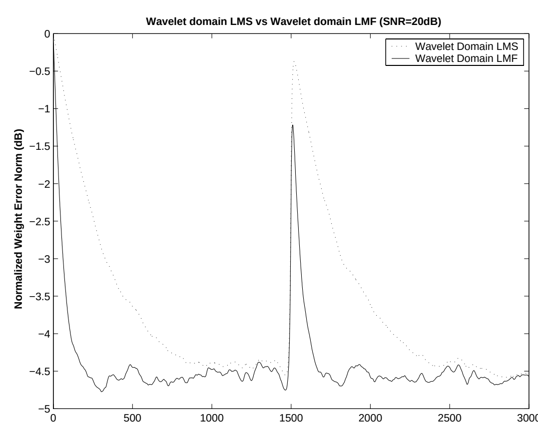

Fig. 6. Comparison of the recovery ability for a sudden change in the Gaussian environment.

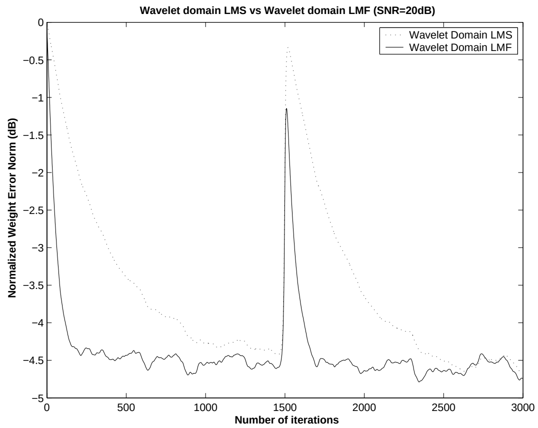

Fig. 7. Comparison of the recovery ability for a sudden change in the uniform environment.

V. Conclusion

A new wavelet transform domain least mean fourth (LMF) algorithm is proposed in this work. The algorithm exploits the special sparse structure of the wavelet transform of wide classes of correlation matrices and their Cholesky factors in order to compute a whitening transformation of the input data in the wavelet domain and minimize computational complexity. This method explicitly computes a sparse estimate of the wavelet domain correlation matrix of the input process. It then computes the Cholesky factor of that matrix and uses its inverse to whiten the input. The proposed algorithm has faster convergence rate and better recovery ability from a sudden change in the environment than that of wavelet transform domain least mean square (LMS) algorithm.

Acknowledgements

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