Control of a Bioreactor Using Static and Dynamic Sliding Mode Controllers

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Abstract - The paper deals with the control of a bioreactor. Three sliding mode control schemes are proposed for the bioreactor benchmark problem [1]. The first two controllers are static sliding mode controllers; the third controller is a dynamic sliding mode controller. The three controllers guarantee the asymptotic convergence of the states of the system to their desired values. Simulation results are given to illustrate the effectiveness of the proposed controllers. It is found that the proposed control schemes work well for the benchmark bioreactor problem. Also, the simulation results indicate that the proposed schemes are robust to variations in the parameters of the plant.

Index Terms: Sliding Mode Control, Bioreactor.

I. INTRODUCTION

In general chemical processes are highly nonlinear systems which are usually difficult to control. An example of such processes is the bioreactor which is considered to be a challenging control problem. This is the case because of its nonlinearity and the fact that small changes in the parameters of the system can cause the bioreactor to become unstable.

The bioreactor is a tank containing water, nutrients, and biological cells; nutrients and cells are introduced into the tank where they mix. The volume in the tank is maintained at a constant level by removing tank contents at a rate equal to the incoming rate; this rate is called the flow rate and is the variable by which the bioreactor is controlled. The bioreactor control problem is to maintain the amount of cells at a desired concentration level [1].

Several researchers have worked on the control of the bioreactor; for example see [2]-[10]. Doerschuk and Sarrafian [2] compared conventional controllers to radial based function controllers for the benchmark bioreactor control problem. Doerschuk *et. al.* [3] used a hybrid neural network/conventional controller to control the benchmark bioreactor problem. Abu Zitar and Hassoun [4] designed a neuro-controller for the bioreactor; the controller is trained with rules extracted by a genetic assisted reinforcement learning system. Agrawal and Lim [5] analyzed various control schemes for continuous bioreactors. Gorinevsky [6] used indirect adaptive control with affine radial basis function to control the bioreactor. Zhao and Skogestad [7] compared several control configurations for continuous bioreactors. Efe and Kaynak [8] used classical and dynamical neural networks to identify a nonlinear bioreactor plant; Efe *et. al.* [9] proposed a neural network assisted nonlinear controller for a bioreactor. Derdiyok and Levent [10] used a conventional sliding mode controller to control the benchmark bioreactor.

In this paper, we use static as well as dynamic sliding mode control schemes to control the benchmark bioreactor. Sliding mode controllers are chosen because of their robustness to changes in the parameters of the system. The proposed controllers guarantee the asymptotic stability of the states of the system.

II. DYNAMIC MODEL OF THE BIOREACTOR

The bioreactor benchmark problem set by Ungar [1] consists of a tank containing water, nutrients and biological cells. Nutrients and cells mix in the tank. The tank's volume is maintained at a constant level through the removal of the contents of the tank at a rate which is equal to the incoming flow rate.

The dynamic model of the system can be written as:

$$\begin{aligned} \dot{c}_1(t) &= -c_1(t)u(t) + c_1(t)(1-c_2(t))e^{c_2/\gamma} \\ \dot{c}_2(t) &= -c_2(t)u(t) + c_1(t)(1-c_2(t))e^{c_2/\gamma}\frac{1+\beta}{1+\beta-c_2(t)} \end{aligned} \tag{1}$$

where,

$c_1(t)$: the normalized cell concentration;

$c_2(t)$: the normalized amount of nutrient per unit volume;

β : the growth rate parameter;

γ : the nutrient inhibition parameter;

$u(t)$: the normalized flow rate (control input).

Note that the state variables $c_1(t)$ and $c_2(t)$ are constrained such that:

$$0 < c_1(t) < 1 \quad (2)$$
$$0 < c_2(t) < 1 \quad (3)$$

The flow rate $u(t)$ is constrained such that:

$$0 < u(t) < 2 \quad (4)$$

The objective of the bioreactor control problem is to maintain the cell concentration $c_1(t)$ at a desired cell concentration level c_{1d}.

Define $\mathbf{c}$ and $\mathbf{x}$ such that $\mathbf{c} = [c_1 \;\; c_2]^T$ and $\mathbf{x} = [x_1 \;\; x_2]^T$. Let the transformation $\mathbf{x} = T(\mathbf{c})$ be such that:

$$x_1 = \ln(c_1) - \ln(c_2) = \ln \frac{c_1}{c_2} \quad (5)$$
$$x_2 = (1 - c_2)\left(1 - \frac{c_1}{c_2} p\right) e^{c_2/\gamma} \quad (6)$$

where

$$p = \frac{1 + \beta}{1 + \beta - c_2} \quad (7)$$

Hence, we can write the dynamic model of the bioreactor given by (1) as:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f + gu \quad (8)$$

where,

$$f = c_1(1 - c_2)pe^{2c_2/\gamma} \times \Big(-1 + \frac{c_1}{c_2} p + \frac{1}{\gamma}(1 - c_2)(1 - \frac{c_1}{c_2} p) + (1 - c_2)(1 + \beta)\frac{c_1(1 + \beta - 2c_2)}{c_2^2(1 + \beta - c_2)^2}\Big) - (1 - c_2)^2 \frac{c_1}{c_2} pe^{2c_2/\gamma} \quad (9)$$

$$g = -c_2 e^{c_2/\gamma}\Big(-1 + \frac{c_1}{c_2} p + \frac{1}{\gamma}(1 - c_2)(1 - \frac{c_1}{c_2} p) + (1 - c_2)(1 + \beta)\frac{c_1(1 + \beta - 2c_2)}{c_2^2(1 + \beta - c_2)^2}\Big) + (1 - c_2)\frac{c_1}{c_2} pe^{c_2/\gamma} \quad (10)$$

Define the errors e_1 and e_2 such that:

$$e_1 = x_1 - x_{1d}$$
$$e_2 = x_2 - x_{2d} \quad (11)$$

where x_{1d} and x_{2d} are the desired values of x_1 and x_2, respectively. It can be checked that $x_{1d} = \ln\left(\frac{c_{1d}}{c_{2d}}\right)$ and $x_{2d} = 0$ where c_{1d} is the desired cell concentration and c_{2d} is the normalized amount of nutrient per unit volume at steady state.

Remark 1: The amount of nutrient per unit volume at steady state is related to the desired cell concentration through the following equation:

$$c_{2d}(1 + \beta - c_{2d}) = c_{1d}(1 + \beta) \quad (12)$$

III. DESIGN OF THE FIRST STATIC SLIDING MODE CONTROLLER

Let α be a positive scalar. Define the sliding surface σ such that:

$$\sigma = \dot{e}_1 + \alpha e_1$$
$$= x_2 + \alpha(\ln \frac{c_1}{c_2} - \ln \frac{c_{1d}}{c_{2d}}) = x_2 + \alpha \ln \frac{c_1 c_{2d}}{c_2 c_{1d}}$$
$$= (1 - c_2)\left(1 - \frac{c_1 p}{c_2}\right) e^{c_2/\gamma} + \alpha \ln \frac{c_1 c_{2d}}{c_2 c_{1d}} \quad (13)$$

Let W be a positive scalar.

Proposition 1: The discontinuous sliding mode controller:

$$u = \frac{1}{g}\left[-f - \alpha(1 - c_2)\left(1 - \frac{c_1 p}{c_2}\right) e^{c_2/\gamma} - W sign(\sigma)\right] \quad (14)$$

guarantees the asymptotic convergence of $c_1(t)$ and $c_2(t)$ to their desired values.

Proof:

Taking the derivative of (13) with respect to time and using (8) and (14), we obtain:

$$\dot{\sigma} = \ddot{e}_1 + \alpha \dot{e}_1$$
$$= \dot{x}_2 + \alpha x_2$$
$$= f + gu + \alpha(1 - c_2)\left(1 - \frac{c_1 p}{c_2}\right) e^{c_2/\gamma}$$
$$= -W sign(\sigma) \quad (15)$$

The trajectories associated with the unforced discontinuous dynamics $\dot{\sigma} = -W sign(\sigma)$ exhibit a finite time reachability to zero from any given initial condition provided that the constant W is positive. On the sliding surface $\sigma = 0$, the system dynamics are determined by the first order differential equation $\dot{e}_1 = -\alpha e_1$. Since α is a positive scalar, it can be concluded that $e_1(t)$ and $\dot{e}_1(t)$will converge to zero asymptotically. Hence, $x_1(t)$ and $x_2(t)$ will converge to their desired values asymptotically.

Since $x_1(t)$ and $x_2(t)$ converge to their desired values asymptotically, then $c_1(t)$ and $c_2(t)$ will converge to their desired values asymptotically.

Therefore, controller scheme (14) guarantees the asymptotic convergence of the normalized cell concentration and the normalized amount of nutrient per unit volume to their desired values.

IV. DESIGN OF THE SECOND STATIC SLIDING MODE CONTROLLER

The controller proposed in the previous section suffers from the chattering problem. To eliminate chattering, we propose to use another control scheme.

Let the function S be such:

$$S = e_1^{exp(-e_1\ tanh\ e_1)} = e_1^r \tag{16}$$

The derivative of S with respect to time is:

$$\dot{S} = e_1^r[r\log(e_1)(-e_1(1-\tanh^2 e_1) - \tanh e_1) + \frac{r}{e_1}]\dot{e}_1 \tag{17}$$

Define the sliding surface σ such that:

$$\begin{aligned}\sigma &= \dot{e}_1 + \alpha e_1 + \mu S \qquad (18)\\ &= (1-c_2)\left(1-\frac{c_1 p}{c_2}\right)e^{c_2/\gamma} + \alpha \ln\frac{c_1 c_{2d}}{c_2 c_{1d}} + \mu S\end{aligned}$$

with α and μ being positive scalars. Also, let K_s be a positive scalar.

Proposition 2: The continuous sliding mode controller:

$$\begin{aligned}u &= \frac{1}{g}\big[-f - \alpha(1-c_2)\big(1-\frac{c_1}{c_2}p\big)e^{c_2/\gamma}\\ &\quad -\mu\dot{S} - K_s tanh(\sigma)\big] \qquad (19)\end{aligned}$$

guarantees the asymptotic convergence of $c_1(t)$ and $c_2(t)$ to their desired values.

Proof:

Taking the derivative of (18) with respect to time and using (8) and (19), we obtain:

$$\begin{aligned}\dot{\sigma} &= \ddot{e}_1 + \alpha\dot{e}_1 + \mu\dot{S}\\ &= \dot{x}_2 + \alpha x_2 + \mu\dot{S}\\ &= f + gu + \alpha(1-c_2)\left(1-\frac{c_1 p}{c_2}\right)e^{c_2/\gamma} + \mu\dot{S}\\ &= -K_s tanh(\sigma) \qquad (20)\end{aligned}$$

Let the Lyapunov function candidate V be such that:

$$V = \frac{1}{2}\sigma^2 \tag{21}$$

Taking the derivative of V with respect to time and using (20), it follows that:

$$\begin{aligned}\dot{V} &= \sigma\dot{\sigma}\\ &= -K_s\ \sigma\ tanh(\sigma) \qquad (22)\end{aligned}$$

Since K_s is chosen to be positive, it follows that $\dot{V} < 0$ for $\sigma \neq 0$ and $\dot{V} = 0$ for $\sigma = 0$.

Therefore, it can be concluded that the dynamics

$$\dot{\sigma} = -K_s\ tanh(\sigma) \tag{23}$$

exhibit a finite time reachability to zero from any given initial condition provided that the constant K_s is positive.

On the sliding surface, the system dynamics are determined by the following first order differential equation $\dot{e}_1 = -\alpha e_1 - \mu S$. It can be shown that $e_1(t)$ and $\dot{e}_1(t)$ will converge to zero asymptotically as α and μ are positive scalars. Hence, $x_1(t)$ and $x_2(t)$ will converge to their desired values asymptotically. Thus $c_1(t)$ and $c_2(t)$ will converge to their desired values asymptotically.

Therefore, controller scheme (19) guarantees the asymptotic convergence of the normalized cell concentration and the normalized amount of nutrient per unit volume to their desired values.

V. DESIGN OF THE DYNAMIC SLIDING MODE CONTROLLER

In the previous section, the hyperbolic function is used to reduce the chattering of the static sliding mode controller. Another method to reduce the chattering is to use a dynamic sliding mode controller.

Sira-Ramirez *et al.* [11] proposed the use of a robust redundant feedback controller, based on dynamical sliding mode control, for nonlinear systems for which a smooth feedback control policy is available.

Motivated by the work of Sira-Ramirez *et al.* [11], a dynamic sliding mode controller is now designed for the bioreactor. Recall that the model of the bioreactor can be written as,

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f + gu \qquad (24)\end{aligned}$$

Let α_1 and α_2 be properly chosen positive design parameters.

It is easy to show that the feedback linearization controller,

$$\begin{aligned}u &= -\frac{1}{g}\left(f + \alpha_1 x_2 + \alpha_2(x_1 - x_{1d})\right) \qquad (25)\\ &= -\frac{1}{g}\left(f + \alpha_1(1-c_2)\left(1-\frac{c_1}{c_2}p\right)e^{c_2/\gamma} + \alpha_2 \ln\frac{c_1 c_{2d}}{c_2 c_{1d}}\right)\end{aligned}$$

guarantees the asymptotic convergence of x_1 and x_2 to their desired values as $t \to \infty$. Hence controller (25) guarantees the asymptotic convergence of c_1 and c_2 to their desired values as $t \to \infty$.

However, the feedback linearization controller (25) is not robust. Hence a dynamic sliding mode controller is proposed. This controller contains the same terms as the feedback linearization controller (25) plus an additional term which is used for robustness purposes.

Let the input-dependent switching surface $s(\mathbf{x}, u)$ be such that,

$$s(\mathbf{x}, u) = u + \frac{1}{g}\left(f + \alpha_1 x_2 + \alpha_2(x_1 - x_{1d})\right) \tag{26}$$

Let W be a positive scalar.

Proposition 3: The dynamic sliding mode control scheme:

$$\begin{aligned} u &= -\frac{1}{g}\Big(f + \alpha_1(1-c_2)\big(1 - \frac{c_1}{c_2}p\big)e^{c_2/\gamma} \\ &\quad + \alpha_2 \ln\frac{c_1 c_{2d}}{c_2 c_{1d}}\Big) + v \end{aligned} \tag{27}$$

with,

$$\dot{v} = -W\,sign\Big(u + \frac{1}{g}\big(f + \alpha_1 x_2 + \alpha_2(x_1 - x_{1d})\big)\Big) \tag{28}$$

when applied to the bioreactor system, guarantees the asymptotic convergence of $c_1(t)$ and $c_2(t)$ to their desired values as $t \to \infty$.

Proof:

It can be easily checked from (27)-(28) that $s(\mathbf{x}, u)\dot{s}(\mathbf{x}, u) < 0$. The trajectories associated with the discontinuous dynamics (28) exhibit a finite time reachability to zero from any given initial condition provided that the constant gain W is chosen to be strictly positive constant. Since $s(\mathbf{x}, u)$ is driven to zero in finite time, the output $y = x_1$ on the sliding surface $(s(\mathbf{x}, u) = 0)$ is governed after such finite time by the second order differential equation $\ddot{x}_1 + \alpha_1\dot{x}_1 + \alpha_2(x_1 - x_{1d}) = 0$. Thus, $x_1(t)$ will converge to its desired value x_{1d} as $t \to \infty$ because α_1 and α_2 are positive scalars. Also, $x_2(t)$ will converge to its desired value x_{2d}

Thus, it can be concluded that the dynamic sliding mode controller (27)-(28) when applied to the bioreactor system guarantees the asymptotic convergence of $x_1(t)$ and $x_2(t)$ to their desired values as $t \to \infty$. Thus $c_1(t)$ and $c_2(t)$ will asymptotically converge to their desired values as $t \to \infty$.

VI. SIMULATION RESULTS

The controllers designed in sections 3, 4 and 5 are simulated using the MATLAB software. The controllers are constructed using the Simulink platform. The parameters of the bioreactor system are $\beta = 0.02$ and $\gamma = 0.48$.

The controller is tested for three cases [2].

Case 1: The system starts at a stable initial point and it ends at a stable point.

Case 2: The system starts at an unstable initial point and it ends at a stable point.

Case 3: The system starts at an unstable initial point and it ends at an unstable point.

Figures 1-3 show the simulation results for the three different cases when the first static sliding mode controller is used. it can be seen from the figures that $c_1(t)$ and $c_2(t)$ converge to their desired values in about 2.5 sec for the three cases.

Figures 4-6 show the simulation results for the three different cases when the second static sliding mode controller is used. it can be seen from the figures that $c_1(t)$ and $c_2(t)$ converge to their desired values in about 2.5 sec for the three cases.

Figures 7-9 show the simulation results for the three different cases when the third dynamic sliding mode controller is used. it can be seen from the figures that $c_1(t)$ and $c_2(t)$ converge to their desired values in about 4 sec for the three cases. Note that a failure is induced at t=5 sec in the smooth controller; it is clear from the graph that the redundant controller ensures that the states of the system still converge to their desired values.

Moreover, the performances of the three controllers are simulated when the parameters β and γ are perturbed by 5% and 50%. The simulations results are not shown because of space limitations. However, it is found that the performances of the system are quite good. These results are expected as it is well known that sliding mode control techniques are robust to changes in the parameters of the system.

VII. CONCLUSION

Three sliding mode control schemes are proposed for the bioreactor benchmark problem. At first a static sliding mode controller is proposed. Simulations results show that this controller works well even when the parameters of the system are not known exactly. However, the first controller suffers from chattering in the control signal. To rectify this problem, a second static sliding mode controller is introduced. The simulation results indicate that this controller works well and it is robust to changes in the parameters of the bioreactor. Another technique to reduce the chattering is through the use of a dynamic sliding mode controller. Hence, a dynamic sliding mode controller is proposed for the bioreactor. Simulation results indicate that this controller works very well and it is robust to changes in the parameters of the system. Future research will address the issues associated with the implementation of the proposed sliding mode controllers.

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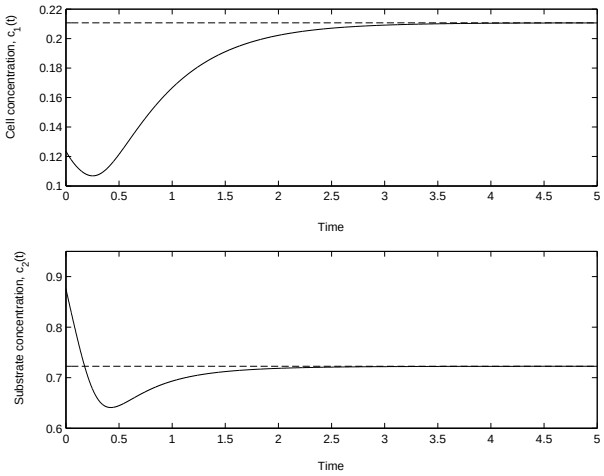

Figure 1: The states $c_1(t)$ and $c_2(t)$ versus time when the first controller is used, case 1

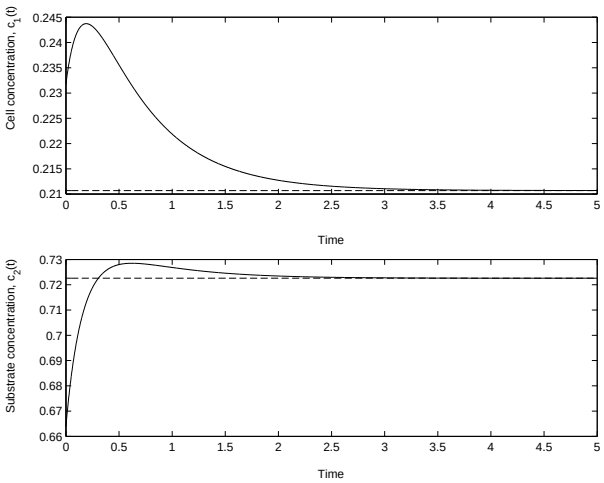

Figure 2: The states $c_1(t)$ and $c_2(t)$ versus time when the first controller is used, case 2

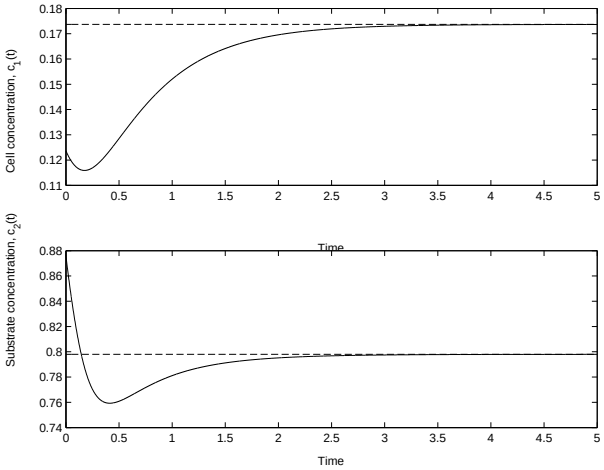

Figure 3: The states $c_1(t)$ and $c_2(t)$ versus time when the first controller is used, case 3

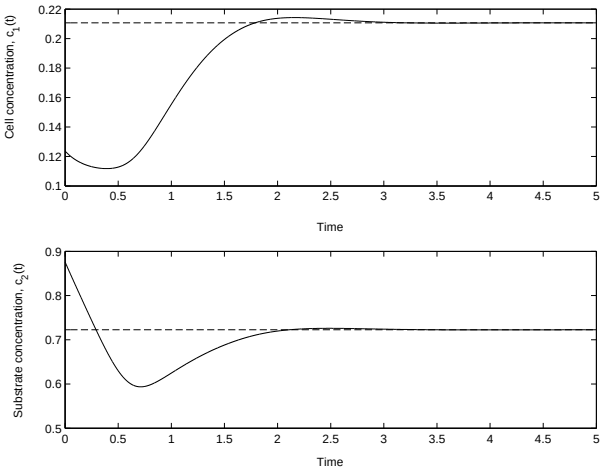

Figure 4: The states $c_1(t)$ and $c_2(t)$ versus time when the second controller is used, case 1

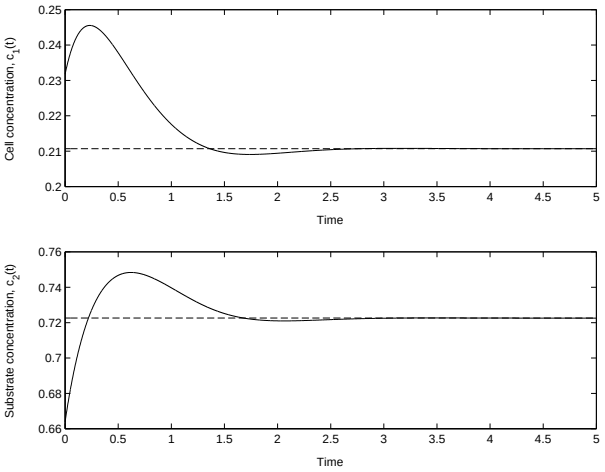

Figure 5: The states $c_1(t)$ and $c_2(t)$ versus time when the second controller is used, case 2

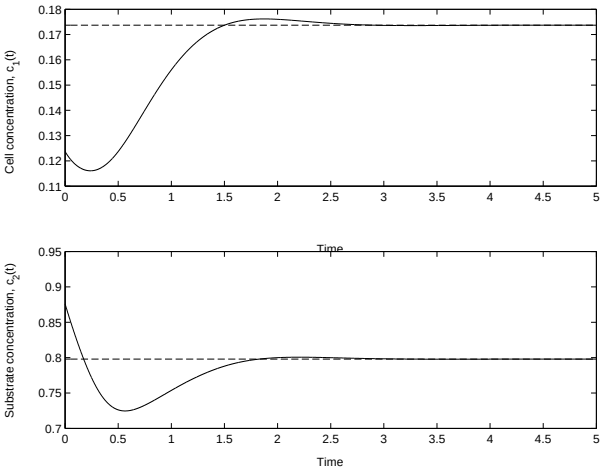

Figure 6: The states $c_1(t)$ and $c_2(t)$ versus time when the second controller is used, case 3

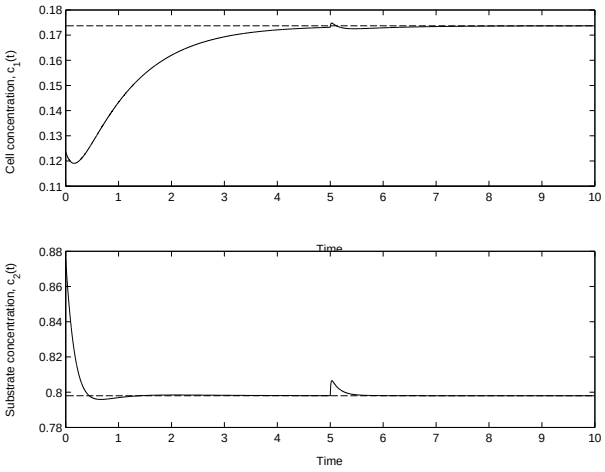

Figure 7: The states $c_1(t)$ and $c_2(t)$ versus time when the third controller is used, case 1

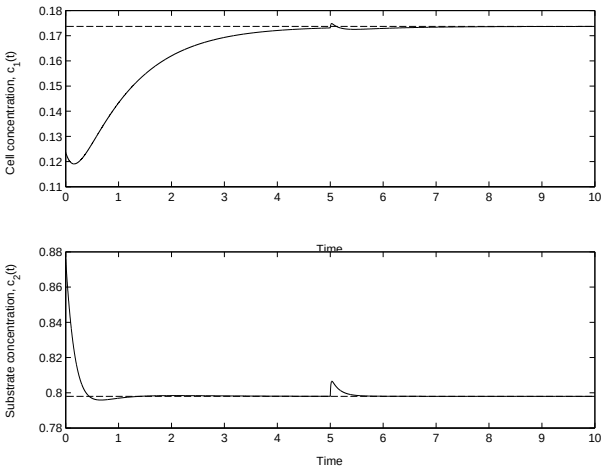

Figure 8: The states $c_1(t)$ and $c_2(t)$ versus time when the third controller is used, case 2

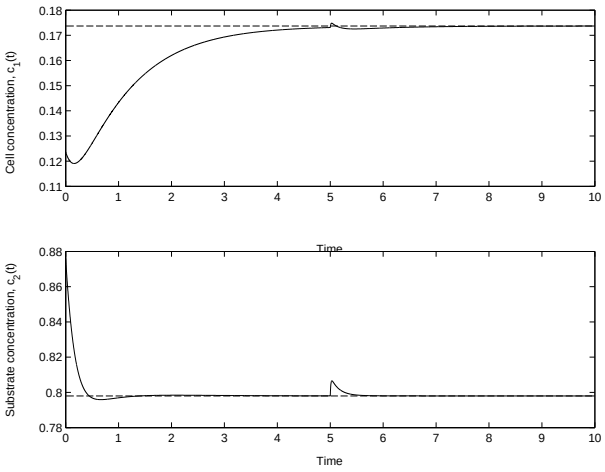

Figure 9: The states $c_1(t)$ and $c_2(t)$ versus time when the third controller is used, case 3