A New Generalized Approach for Performance Evaluation of Communication Systems with Intentional/Non-Intentional Jamming

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Abstract— **In this work, a new generalized approach is proposed to evaluate the probability of error performance of communication systems in the presence of interference. The interference can be intentional such as multitone jamming, or non-intentional such as multiple access interference or co-channel interference. As an example, the new approach is applied to compute the exact probability of error for a frequency hopping spread spectrum system with noncoherently demodulated M-ary amplitude shift keying signal in the presence of multitone jamming and white Gaussian noise.**

Index Terms* — *Multitone jamming, Frequency hopping, Multiple access interference.

I. INTRODUCTION

Evaluation of the probability of error for communication systems in the presence of intentional or non-intentional interference has been intensively considered in the literature [1] [2]. Most of the obtained results are just an approximations [2], or bounds [3]. Recently, Maghsoodi *et.al* derived a generalized formula to compute the probability density function (pdf) of the resultant amplitude of the sum of n randomly phased sinusoidal signals. In this work we apply the results of [4] to derive the exact symbol error rate (SER) of frequency hopping spread spectrum systems (FHSS) using noncoherent M-*ary* amplitude shift keying (MASK) in the presence of multitone jamming and narrow band Gaussian noise. the generalization of this approach is also presented.

In frequency hopping (FH) systems, the spectrum of the transmitted signal is spread over a much larger bandwidth by continuously hopping the transmitted signal frequency over a large number of orthogonal frequency bands. This process is pivotal when the transmitter is trying to avoid an intentional interference process that is usually called jamming. The jammer ultimate goal is to hit the transmitted symbol by injecting the spectrum with a signal that is capable of destroying the transmitted symbol. The jamming signal usually consists of a number of tones transmitted according to certain criteria. On the other hand, the party who is trying to send the data has to search for a strategy to avoid the jamming signal and to reduce the damage when the transmitted symbol is hit. The common strategy to avoid the jamming signal is to employ FH techniques where the transmitter continuously switches its carrier frequency. Reducing the damage when a symbol is jammed can achieved by using modulation schemes with high jamming rejection properties.

Independent multitone jamming (MTJ) is a technique adapted by the jammers where a jammed frequency hopping band may be hit with as few as one tone to as many as n tones, and the numbers of jamming tones in different jammed FH bands may not be the same. There is another kind of MTJ in FH spread-spectrum systems, so called band MTJ. In band MTJ, each jammed FH band contains the same number of jammer tones. The term worst-case MTJ is used to refer to the case where there is at most one jamming tone in one FH band [5].

Due to the difficulty of maintaining phase coherence in FH systems, using noncoherent detection was the most attractive solution proposed. Noncoherently detected M-*ary* frequency shift keying (MFSK) was the modulation scheme that received the most attention by the researchers. Hence, several papers have been published on the analyses of the error probability performance of slow, orthogonal, frequency-hopped MFSK system with noncoherent receivers under independent MTJ [6]–[7].

Recently, a new bandwidth-efficient noncoherent modulation scheme was proposed for FH multiple access (FH-MA) networks, namely M-*ary* amplitude shift keying (MASK) [2] and [8]. The high bandwidth efficiency of MASK has enabled an increase to the number of frequency bands q that the transmitter can hop in, thus reducing the probability of hit by other users. Using MASK in MTJ environment will reduce the probability of hit by the jammer due to the increment of q. Or, the jammer has to distribute its power over a larger number of tones which will decrease the amount of power devoted for each jamming tone. However, the MASK sensitivity to MTJ has to be investigated to determine its jamming rejection capabilities.

In this work we derive the exact symbol error rate (SER) of FH systems using noncoherent MASK for worst-case MTJ and AWGN. Towards this end, the pdf of the general formula for the amplitude of the sum of $n+1$ sinusoidal signals with an arbitrary phase is exploited to compute the desired SER of the FH-MASK system.

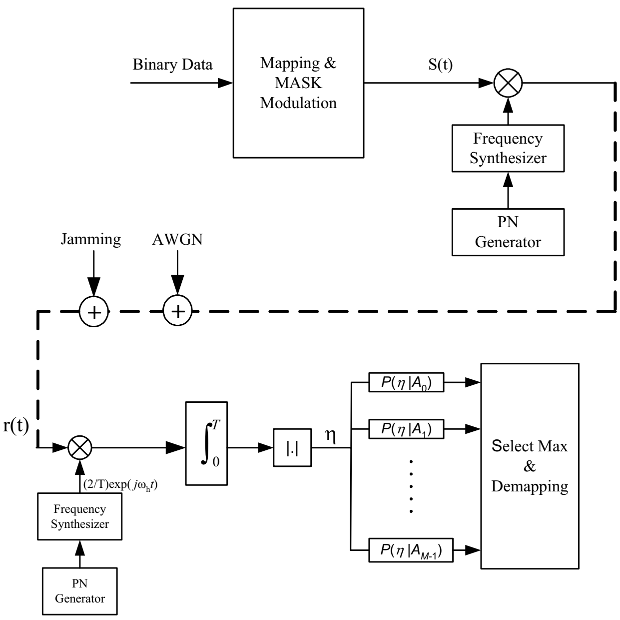

Fig. 1. FHSS-MASK system block diagram

II. System Model

The block diagram of the FH-MASK system is shown in Fig. 1. The conventional MASK modulator maps k binary bits into one out of $M = 2^k$ possible symbols. The modulation process includes Gray coding and frequency upconversion to an intermediate frequency f_h. The binary data rate $R_b = 1/T_b$, where T_b is the bit duration, the symbol rate $R_s = R_b/k = 1/(kT_s)$ where T_s is the symbol duration. The hopping rate $f_H = R_s$, which corresponds to slow FH. The output of the modulator is then mixed with a frequency generated by the frequency synthesizer which is controlled by a pseudo-noise (PN) code generator.

At the receiver front end, the received signal consists of the desired signal corrupted by AWGN and MTJ. This signal is dehopped by a frequency synthesizer which is also controlled by a PN code generator. We assume that the frequency synthesizer of the receiver is in perfect synchronization with that of the transmitter. The received signal after dehopping can be expressed as

$$r(t) = A_i \cos(2\pi f_h t + \phi_d) + \alpha_j A_J \cos(2\pi f_h t + \phi_J) + n(t) \tag{1}$$

where α is the jamming factor, $j = 0, 1$, $\alpha_1 = 1$ and $\alpha_0 = 0$, A_i is the desired signal amplitude $i \in \{1, 2, ..., M\}$, ϕ_s and ϕ_J are the random phases of the desired and jamming signals respectively, and A_J is the jamming signal amplitude. The jamming signal frequency is assumed to coincide exactly with one of the hopping frequencies [5]. Since the FH band may be jammed by only a single tone, this corresponds to worst-case jamming.

A slow FH system that transmits one MASK symbol during each hop is considered in this paper. The carrier frequency of the transmitter is hopped pseudo randomly between q non overlapping frequency slots, each has bandwidth BW_{slot}. Hence, the total spread spectrum bandwidth $BW_{SS} = qBW_{slot}$. The MTJ is assumed to have Q equal power interfering tones with total power of P_{JT}. The power of each jamming tone is $P_J = P_{JT}/Q$. The probability that a particular frequency being affected by the jamming tone is $P(\alpha_1) = Q/q$. The effective signal to jamming ratio (SJR) is usually used to compare FH systems independently of the number of FH bands q, and independently of the spread spectrum bandwidth BW_{ss} [5]

$$\text{SJR} = \frac{E_b}{P_{JT}/BW_{SS}} = \frac{E_b}{P_J}\frac{q}{Q}BW_{slot} \tag{2}$$

The dehopped signal is fed to a maximum likelihood (ML) noncoherent MASK demodulator that consists of M branches of a quadrature receivers configured as energy detectors [9]. The envelop (η) of every symbol is used to compute M likelihood values using the conditional pdf $P(\eta|A_i)$, the decision is made in favor of the branch with maximum likelihood value. An equivalent configuration of the ML detector is to compare η to a set of $M-1$ thresholds, Fig. 1. The thresholds are optimum if they are computed as the intersections of the conditional pdfs $P(\eta|A_i)$. The thresholds are optimum in the sense that they reduce the SER. The optimum thresholds should be computed and loaded to the receiver based on the system E_b/N_0. Such configuration reduces the computational power required by the receiver since the optimum thresholds are changed only when E_b/N_0 is changed. This configuration also enables for further simplification when suboptimal fixed thresholds are used [10].

III. The Sinusoidal Addition Theorem and the Amplitude Density

Given that the transmitted signal was jammed by n independent tones, the received signal in AWGN channel can be expressed as

$$r(t) = A_i \cos(\omega_d t + \phi_d) + \sum_{i=1}^{n} A_J \cos(\omega_i t + \phi_i) + n(t) \tag{3}$$

The first term in (3) represents the desired signal component with amplitude A_i and phase ϕ_d. The second term is the interference produced intentionally by the jammer. It is usually assumed that the jamming tones have equal amplitudes. The third term is a two sided AWGN with power spectral density $N_0/2$. The set of phases ϕ_d, ϕ_1, ..., ϕ_n are independent random variables distributed uniformly over $[-\pi, \pi]$. The carriers frequencies ω_d, ω_1, ..., ω_n are usually considered to be equal [5]. This assumption is needed since the interfering signals may not have exactly the same frequency as the reference signal.

To analytically evaluate the performance of communication systems in such environments it is necessary to find the statistical properties of the received signal $r(t)$ given in (3) and the statistical properties of the decision variable

η. Based on the sinusoidal addition theorem (SAT), the sum of $n+1$ sinusoidal signals can be expressed as

$$\sum_{i=1}^{n+1} A_i \cos(\omega t + \phi_i) = B_{n+1} \cos(\omega t + \theta_{n+1}) \quad (4)$$

where B_{n+1} is the amplitude and θ_{n+1} is the phase of the new cosine signal, $\theta_{n+1} \in [-\pi, \pi]$. For worst-case jamming $n = 1$, ϕ_d and ϕ_1 are independent and uniformly distributed over $[-\pi, \pi]$, thus, since the amplitudes of the desired and jamming signals are A_i and A_J respectively, the pdf $f_{B_2}(b_2)$ becomes [4],

$$f_{B_2}(b_2) = \frac{2b_2}{\pi\sqrt{4A_i^2A_J^2 - (b_2^2 - A_i^2 - A_J)^2}} \quad (5)$$

where $||A_i| - |A_J|| < b_2 < |A_i| + |A_J|$ and zero otherwise.

IV. Probability of Error

The average probability of error, symbol error rate (SER), can be calculated by averaging the probability of error over all possible symbols and jamming conditions. Thus,

$$P_e = \sum_{i=1}^{M}\sum_{j=0}^{1} (P_e|A_i, \alpha_j)\, P(A_i, \alpha_j) \quad (6)$$

since A_i and α_j are independent, then (6) can be expanded to

$$\begin{aligned} P_e \;=\; & \sum_{i=1}^{M} (P_e|A_i, \alpha_1)\, P(A_i)\, P(\alpha_1) + \\ & (P_e|A_i, \alpha_0)\, P(A_i)\, P(\alpha_0) \end{aligned} \quad (7)$$

The evaluation of (7) requires the knowledge of the conditional pdf's $P(\eta|A_i, \alpha_j)$. The derivation of these pdfs and the computation of the corresponding error probability is performed in the following two subsections.

A. SER for Unjammed Symbols

For α_0 (i.e. no jamming) the conditional pdf $P(\eta|A_i, \alpha_0) = P(\eta|A_i)$, which is the well known Ricean distribution [9],

$$P(\eta|A_i, \alpha_0) = \frac{\eta}{\sigma^2} \exp\left(-\frac{\eta^2 + A_i^2 T_s/2}{2\sigma^2}\right) I_0\left(\frac{\eta A_i T_s}{\sqrt{2}\sigma^2}\right) \quad (8)$$

where σ^2 is the noise variance, and I_0 is the modified Bessel function of the first kind and zeroth order. The decision circuit will make an error for a symbol with an amplitude A_i if the noise pushes η out of the range specified by the thresholds γ_i and γ_{i+1}. The probability of such event for equiprobable symbols is given by

$$\begin{aligned} P_e &= M - \sum_{i=0}^{M-1} \int_{\gamma_i}^{\gamma_{i+1}} P(\eta|A_i) d\eta \\ &= M - \left[\begin{array}{l} \int_0^{\gamma_1} P(\eta|A_0) d\eta + \\ \sum_{i=1}^{M-2} \int_{\gamma_i}^{\gamma_{i+1}} P(\eta|A_i) d\eta + \\ \int_{\gamma_{M-1}}^{\infty} P(\eta|A_{M-1}) d\eta \end{array} \right] \end{aligned} \quad (9)$$

where $\gamma_0 = 0$, $\gamma_M = \infty$, the rest of the optimum thresholds, $\gamma_1, ..., \gamma_{M-1}$, can be computed as the the intersections of all adjacent pdfs $P(\eta|A_0)$ and $P(\eta|A_1)$, $P(\eta|A_1)$ and $P(\eta|A_2)$, and so on. These thresholds are optimum in the sense that they minimize P_e. Notice that the integrals in (9) represent the probability of making a correct decision P_c. If we set $A_0 = 0$, the distribution of $P(\eta|A_0 = 0)$ becomes Rayleigh [9],

$$P(\eta|A_0, \alpha_0) = \frac{\eta}{\sigma^2} \exp\left(-\frac{\eta^2}{2\sigma^2}\right) \quad (10)$$

Evaluating the first integral in (9) gives

$$Pc_1 = \int_0^{\gamma_1} P(\eta|A_0 = 0) d\eta = 1 - \exp\left(-\frac{\gamma_1^2}{2\sigma^2}\right) \quad (11)$$

The second and the third integrals in (9) have no closed-form solutions however they can be expressed in terms of the Marcum Q-function,

$$\begin{aligned} Pc_2 &= \sum_{i=1}^{M-2} \int_{\gamma_i}^{\gamma_{i+1}} P(\eta|A_i) d\eta + \int_{\gamma_{M-1}}^{\infty} P(\eta|A_{M-1}) d\eta \\ &= \sum_{i=1}^{M-2} \left[Q\left(\sqrt{\frac{A_i^2 T_s}{2\sigma^2}}, \frac{\gamma_i}{\sigma}\right) - Q\left(\sqrt{\frac{A_i^2 T_s}{2\sigma^2}}, \frac{\gamma_{i+1}}{\sigma}\right) \right] \\ &+ Q\left(\sqrt{\frac{A_{M-1}^2 T_s}{2\sigma^2}}, \frac{\gamma_{M-1}}{\sigma}\right) \end{aligned} \quad (12)$$

For uniformly spaced amplitudes, $A_i = i \times d$, where $i \in \{0, 1, ..., M-1\}$, d is the difference between adjacent amplitudes. The energy difference between adjacent symbols can be expressed as

$$E_d \triangleq \frac{d^2 T_s}{2} = \frac{6E_s}{2M^2 - 3M + 1} = \chi E_s \quad (13)$$

where E_s is the average symbol energy.

$$\begin{aligned} Pc_2 \;=\; & \\ & \sum_{i=1}^{M-2} Q\left(i\sqrt{\chi\frac{2E_s}{N_0}}, \frac{\gamma_{i+1}}{\sigma}\right) - Q\left(i\sqrt{\chi\frac{2E_s}{N_0}}, \frac{\gamma_i}{\sigma}\right) \\ & + Q\left((M-1)\sqrt{\chi\frac{2E_s}{N_0}}, \frac{\gamma_{M-1}}{\sigma}\right) \end{aligned} \quad (14)$$

The optimum thresholds are usually replaced with a fixed suboptimal thresholds for high E_b/N_0 values [10].

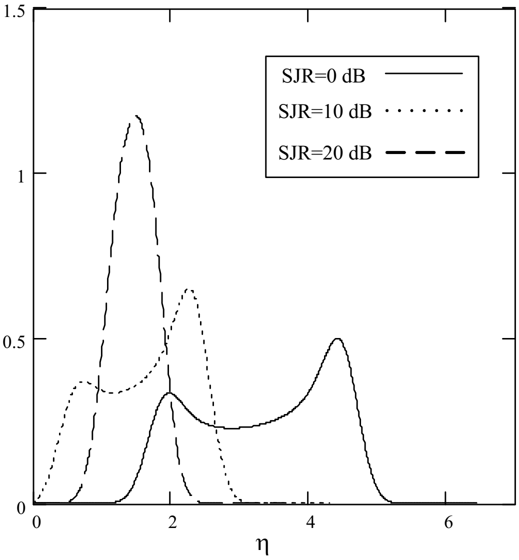

Fig. 2. The pdf of η for various SJRs.

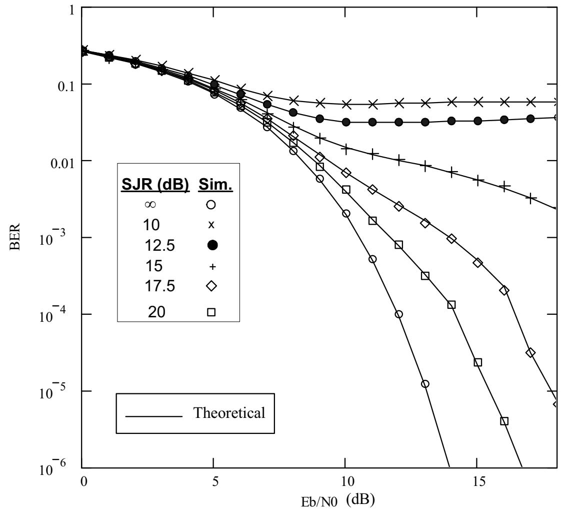

Fig. 3. Analytical and simulated BER performance of FH-MASK for various jamming conditions, $M = 2$, $q/Q = 0.1$.

B. SER for Jammed Symbols

When the reference signal is jammed, the received signal is given by (1) where the jamming factor is α_1. Using the SAT, the received signal $r(t)$ can be represented by a single sinusoid with a random amplitude b_2. The pdf of η in this case is conditionally Ricean as given by (8) with the amplitude A_i is replaced by a random variable b_2 which depends on A_i,

$$P(\eta|A_i, b_2) = \frac{\eta}{\sigma^2} \exp\left(-\frac{\eta^2 + b_2^2 T_s/2}{2\sigma^2}\right) I_0\left(\frac{\eta b_2 T_s}{\sqrt{2}\sigma^2}\right) \tag{15}$$

Since b_2 is random with a pdf that is given in (5), the unconditional pdf can be computed by integrating $P(\eta|A_i, b_2)$ multiplied by the pdf of b_2 given in (5) for all possible values of b_2. The only exceptional case is for the symbol $A_0 = 0$, the conditional pdf $P(\eta|A_0, b_2)$ can by represented by (8) with A_i replaced by A_J. Therefore, the unconditional pdf given α_1 and $A_i \neq 0$ can be expressed as

$$P(\eta|A_i) = \int_{||A_i|-|A_J||}^{|A_i|+|A_J|} P(\eta|A_i, b_2) f_{B_2}(b_2)\, db_2 \tag{16}$$

The pdf described by (16) is shown in Fig. 2 for different SJRs. Since $P(\eta|A_i, \alpha_1)$ is now available, the SER given that the symbol was hit can be evaluated using (9).

It should be pointed out that the thresholds used when the symbol is jammed are the same thresholds used when the symbol is not jammed. Such approach is typical in the absence of channel side information (CSI). If the CSI is available, the SER can be reduced by using a new set of threshold that is computed based on (16).

The same approach for computing the pdf of the decision variables can be generalized for any value of n since the interfering signals can be represented by a single sinusoid according to the SAT. The pdf of the resultant amplitude can be derived for any value of n [4]. In the case of dispersive channels, the pdf can be computed following the same procedure, however, the derivation of the amplitudes pdf should be carried out for random instead of constant amplitudes.

V. Numerical Results

In this section, the SER performance of noncoherent FH-MASK as a function of E_b/N_0 and SJR is presented. The SJR defined in Section II as $SJR = \frac{E_b}{P_J}\frac{q}{Q}BW_{slot}$, where $E_b = P_s T_s/\log_2(M)$, P_s and T_s are the symbol average power and period respectively. For fair comparison, P_s was fix at one, hence the power of the ith symbol becomes $P_i = \frac{6(i-1)P_s}{2M^2-3M+1}$, $i = 0, 1, ..., M-1$.

The BER performance for $M = 2$ as a function of E_b/N_0 is shown in Fig. 3. The SJR was increased from 10 to 20 dB in 2.5 dB steps. While the BER decreases as E_b/N_0 increases for $SJR \geq 15$ dB as expected, the case is not the same for SJR of 10 and 12.5 dB as depicted in Fig. 3. This behavior can be understood using (16) and Fig. 4 where the pdf of η is shown for a jammed MASK symbol. The figure shows the pdf of η at E_b/N_0 10 and 15 dB, and it also shows the optimum thresholds at E_b/N_0 of 10 dB which are approximately equal to the thresholds at E_b/N_0 of 15 dB. The probability of making a correct decision is the area under the pdf between γ_1 and γ_2, which is larger for E_b/N_0 of 10 dB. The same discussion applies for higher values of M as depicted in Fig. 5 and Fig. 6 respectively

At high SJR, the value of η for any symbol A_i without noise does not exceed the specified thresholds specified to detect that symbol. In this case, the receiver will make

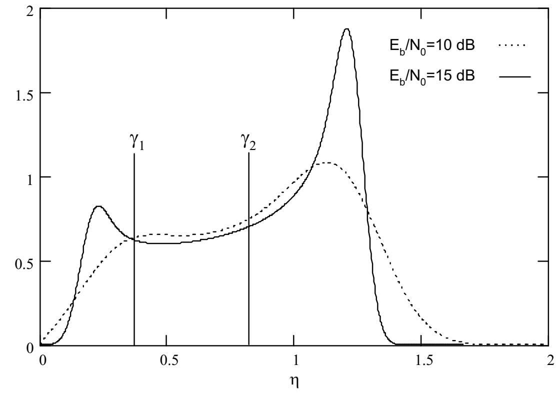

Fig. 4. The pdf of a jammed MASK symbol, $M = 4$, $i = 1$, and $SJR = 10$ dB.

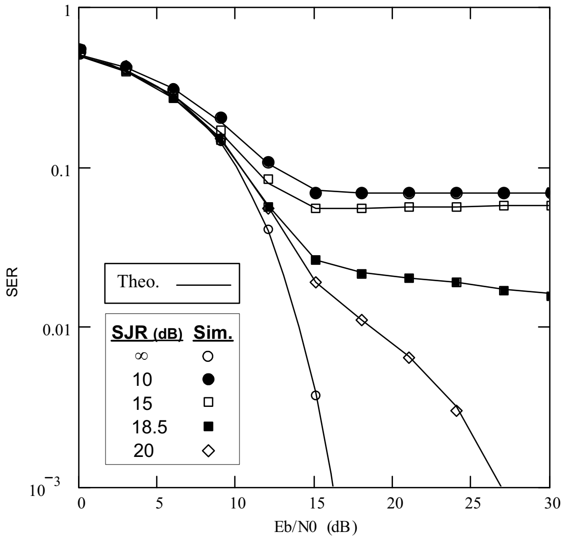

Fig. 5. Analytical and simulated SER performance of FH-MASK for various jamming conditions, $M = 4$, $q/Q = 0.1$.

a correct decision even if the transmitted symbol was jammed. This explains the continuous reduction in the SER as E_b/N_0 is increased for $SJR \gtrapprox$ 15, 18.5, and 20 dB for $M =$ 2, 4, and 8 respectively. It also explains the existence of the error floor since in the absence of noise η can till exceed the thresholds boundaries.

The SER as a function of SJR for various values of M and E_b/N_0 is depicted in Fig. 7. This figure shows that two error floors are introduced, the first error floor appears at low SJR, the second appears at high SJR. At low SJR the pdf of η exceeds the thresholds specified by the receiver even with the absence of noise as shown in Fig. 4. Increasing the SJR will shift the pdf of η to the left and at the same time it will reduce its width $||A_i| - |A_J|| < b_2 < |A_i| + |A_J|$. The effect of the left shift is limited as long as the width of the pdf significantly exceeds the

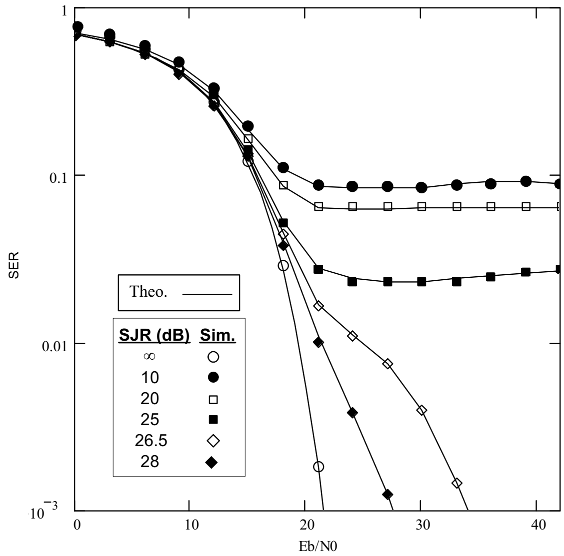

Fig. 6. Analytical and simulated SER performance of FH-MASK for various jamming conditions, $M = 8$, $q/Q = 0.1$.

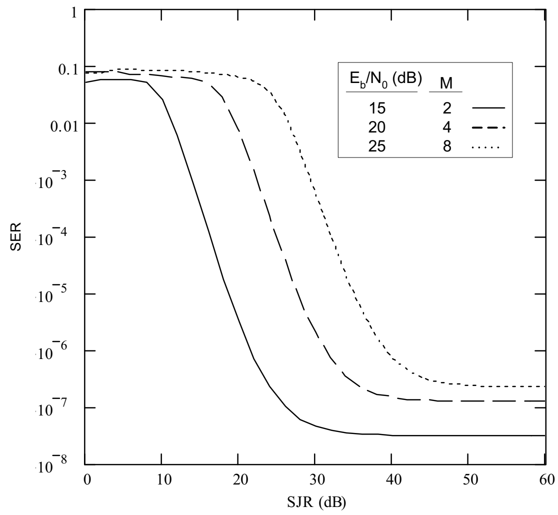

Fig. 7. Analytical SER performance of FH-MASK as a function of SJR for various values of E_b/N_0, $q/Q = 0.1$.

specified thresholds. Therefore, no improvement in the SER is expected unless the SJR is increased beyond a value that is adequate to substantially reduce the width of the pdf of η which explains the first error floor appearance. The second error floor appears when the SJR is high enough to make the jammer effect negligible, in this case the SER approaches its value at the designated E_b/N_0 without jamming.

VI. Conclusion

In this work, a new approach was proposed as an efficient technique to evaluate the probability of error performance for communication systems with interference. The simplicity and flexibility of the new approach was demonstrated via the evaluation of the SER of a noncoherently detected MASK signals in FH networks with multitone jamming. The proposed approach has substantially simplified the derivation of the pdf of the decision variables.

VII. Future Work

In future work, the generalized approach will be used to evaluate the performance of various communication systems with multiple simultaneous interfering signals under different communication channels.

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