Radiation of Slotted Circular or Elliptical Antenna Coated With Isorefractive Metamaterials Layer

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ABSRACT

Exact solution to the electromagnetic radiation from a conducting slotted circular or elliptical cylinder coated by an Isorefractive metamaterials is derived by using the separation of variables technique. The fields inside and outside the dielectric layer are expressed in terms of Mathieu functions. No matrix inversion is required after obtaining expressions for the radiated field expansion coefficients by imposing the appropriate boundary conditions. Numerical results are plotted for the radiation pattern, aperture conductance and antenna gain. The results show that slotted antenna coated with Isorefractive metamaterials has more directive beam and lower side-lobes compared to that coated with conventional dielectric or metamaterials.

1. INTRODUCTION

Radiation properties of an axially slotted antenna are very important in communications and airplane industries. Numerous authors in the literature have investigated the radiation by dielectric coated slotted circular and elliptical cylinders. For example, Hurd [1] studied the radiation pattern of a dielectric axially slotted cylinder. The external admittance of an axial slot on a dielectric coated metal cylinder was investigated by Knop [2]. Shafai [3] obtained the radiation properties of an axial slotted antenna coated with a homogenous material. Wong [4,5] investigated the radiation properties of slotted cylinder of elliptical cross section while Richmond [6] studied the radiation from an axial slot antenna on a dielectric coated elliptic cylinder. The analysis is later extended to the radiation by axial slots on a dielectric coated nonconfocal conducting elliptic cylinder by [7]. Hussein and Hamid [8] studied the radiation by N axially slotted cylinder of elliptical cross section coated with a lossy dielectric material. Recently, Hamid investigated the radiation characteristics of slotted circular or elliptical cylinder coated by lossy and lossless metamaterials [9-10].
Lately, materials posses both lossy and lossles metamaterials have gained considerable attention by many researches [11-15].

New artificial class of materials with interesting electromagnetic properties have been recently introduced [16]. Two media separated by an interface are called isorefractive if they have the same refractive index, such a relation is maintained when the permittivity and permeability of the two media obey

$$m_1 e_1 = m_2 e_2 \qquad (1)$$

So that the propagation constant k and the wavelength l are the same in both media

$$k = \frac{2p}{l} = w\sqrt{e_i m_i}, \; i = 1,2 \qquad (2)$$

In addition, the two media have different intrinsic impedances

$$Z_i = \sqrt{\frac{m_i}{e_i}}, \quad i = 1, 2 \qquad (3)$$

In this paper, a theoretical analysis based on boundary value solution for the case of antenna radiation by an axial slot on a conducting circular or elliptical cylinder coated by a isorefractive metamaterials is presented. The fields inside and outside the isorefractive metamaterials coating are expressed in terms of radial and angular Mathieu functions. The isorefractive metamaterials elliptic layer allows an exact solution as the boundary conditions lead to one-to-one matching between the field modes on either side of the interface, and thus no need for matrix inversion since new expressions are obtained for the radiated field expansion coefficients. Numerical results are presented for the radiation pattern, aperture conductance and antenna gain against the coating thickness as well as compared with uncoated, conventionally dielectric and metamaterials coated antenna.

2. FORMULATION

Fig. 1 illustrates the geometry of the problem. The structure is assumed to be infinite along the *z*-axis. The symbols a_c and b_c correspond to the semi-major and semi-minor axes of the conducting cylinder, respectively, while the symbols a and b are semi-major and semi-minor axes of the dielectric coating. The elliptical coordinate system

(u,v,z) is defined in terms of the Cartesian coordinate system (x,y,z) by $x = F\cosh(u)\cos(n)$ and $y = F\sinh(u)\sin(n)$, where F is the semifocal length of the elliptical cross section.

The electric fields outside the dielectric layer (region I) for $(x > x_1)$ and inside the dielectric layer (region II) for $x < x_1$ can be expressed in terms of Mathieu functions as follows

$$E_z^I = \sum_{m=0}^{\infty} C_{em} R_{em}^{(4)}(c,x) S_{em}(c,h) + \sum_{m=1}^{\infty} C_{om} R_{om}^{(4)}(c,x) S_{om}(c,h) \tag{4}$$

$$E_z^{II} = \sum_{m=0}^{\infty} [A_{em} R_{em}^{(1)}(c,x) + B_{em} R_{em}^{(2)}(c,x)] S_{em}(c,h) + \sum_{m=1}^{\infty} [A_{om} R_{om}^{(1)}(c,x) + B_{om} R_{om}^{(2)}(c,x)] S_{om}(c,h) \tag{5}$$

where $A_{\substack{em\\om}}, B_{\substack{em\\om}}$ and $C_{\substack{em\\om}}$ are the unknown expansion coefficients, $R^{(1)}_{\substack{em\\om}}, R^{(2)}_{\substack{em\\om}}$ and $R^{(4)}_{\substack{em\\om}}$ are the even and odd modified Mathieu functions of the first, second and fourth kind, respectively. It should be noted that, $x = \cosh u$, $h = \cos n$ and $c = kF$. The magnetic field components inside and outside the dielectric layer can be obtained using Maxwell's equations as

$$H_v^I = \frac{-j}{w m_1 h} \left\{ \sum_{m=0}^{\infty} C_{em} R_{em}^{(4)'}(c,x) S_{em}(c,h) + \sum_{m=1}^{\infty} C_{om} R_{om}^{(4)'}(c,x) S_{om}(c,h) \right\} \tag{6}$$

$$H_v^{II} = \frac{-j}{w m_2 h} \left\{ \sum_{m=0}^{\infty} [A_{em} R_{em}^{(1)'}(c,x) + B_{em} R_{em}^{(2)'}(c,x)] S_{em}(c,h) + \sum_{m=1}^{\infty} [A_{om} R_{om}^{(1)'}(c,x) + B_{om} R_{om}^{(2)'}(c,x)] S_{om}(c,h) \right\} \tag{7}$$

where $h = F\sqrt{(\cosh^2 u - \cos^2 n}$. The prime in equations (6) and (7) denotes derivative with respect to u while m_1 and m_2 are the permeabilities of regions 1 and 2, respectively.

We require E_z to be continuous $(E_z^I = E_z^{II})$ a cross the interface at $x = x_1$. Applying the orthogonality property of the angular Mathiue functions , this leads to

$$[A_{em} R_{em}^{(1)}(c,x_1) + B_{em} R_{em}^{(2)}(c,x_1)] N_{em}(c) = C_{em} R_{em}^{(4)}(c,x_1) \tag{8}$$

$$[A_{om} R_{om}^{(1)}(c,x_1) + B_{om} R_{om}^{(2)}(c,x_1)] N_{om}(c) = C_{om} R_{om}^{(4)}(c,x_1) \tag{9}$$

where

$$N_{\substack{em\\om}}(c) = \int_0^{2p} [S_{\substack{em\\om}}(c,h)]^2 \, dv \tag{10}$$

Continuity of the tangential magnetic field components at $x = x_1$ require that

$$\frac{1}{m_2}[A_{em} R_{em}^{(1)'}(c,x_1) + B_{em} R_{em}^{(2)'}(c,x_1)] N_{em}(c) = \frac{1}{m_1} C_{em} R_{em}^{(4)'}(c,x_1) \tag{11}$$

$$\frac{1}{m_2}[A_{om} R_{om}^{(1)'}(c,x_1) + B_{om} R_{om}^{(2)'}(c,x_1)] N_{om}(c) = \frac{1}{m_1} C_{om} R_{om}^{(4)'}(c,x_1) \tag{12}$$

In region (II), the tangential electric field on the conducting surface $(x = x_c)$ must vanish except at the slot location. This leads to

$$\sum_{m=0}^{\infty} [A_{em} R_{em}^{(1)}(c,x_c) + B_{em} R_{em}^{(2)}(c,x_c)] S_{em}(c,h) + \sum_{m=1}^{\infty} [A_{om} R_{om}^{(1)}(c,x_c) + B_{om} R_{om}^{(2)}(c,x_c)] S_{om}(c,h) = \begin{cases} F(v) & n_1 < n < n_2 \\ 0, & \text{elsewhere.} \end{cases} \tag{13}$$

Multiplying both sides of (13) by $S_{en}(c,h)$ and integrating over $0 < v < 2p$, we obtain

$$[A_{en} R_{en}^{(1)}(c,x_c) + B_{en} R_{en}^{(2)}(c,x_c)] N_{en}(c) = F_{en} = \int_{v_1}^{v_2} F(v) S_{en}(c,h) dn \tag{14}$$

A similar equation may be obtained for the odd solution by following the steps in deriving equation (14), this leads to

$$[A_{on} R_{on}^{(1)}(c,x_c) + B_{on} R_{on}^{(2)}(c,x_c)] N_{on}(c) = F_{on} = \int_{v_1}^{v_2} F(v) S_{on}(c,h) dv \tag{15}$$

For the integrals in equations (14) and (15) to be evaluated, we express the field at the slot location as [6-8]

$$F(n) = E_0 \cos[p(n_0 - n)/(2a)] \tag{16}$$

where

$$n_0 = (n_1 + n_2)/2 \tag{17}$$

$$a = (n_2 - n_1)/2 \tag{18}$$

The Mathieu angular functions are expressed in terms of Fourier series as [17]

$$S_{en}(c,h) = \sum_k D_e^k(c,n) \cos(kn) \tag{19}$$

$$S_{on}(c,h) = \sum_k D_o^k(c,n) \sin(kn) \tag{20}$$

The summations in (19) and (20) extend over even values of k if n is even, and over odd values of k if n is odd. $D^k_{\substack{e\\o}}$ are the Fourier series coefficients. Substituting equations (16)-(20) into right side of equations (14) and (15), one obtains an expression for F_{en} and F_{on} as

$$F_{en} = E_O \sum_k D_e^k(c,n) \int_{n_1}^{n_2} \cos[p(n - n_0)/(2a)] \cos(kn) dn \tag{21}$$

$$F_{on} = E_O \sum_k D_o^k(c,n) \int_{n_1}^{n_2} \cos[p(n - n_0)/(2a)] \sin(kn) dn \tag{22}$$

Solving for $B_{\substack{en\\on}}$ from equations (14) and (15) and using the result in equations (8)-(9) and (11)-(12) with the elimination of $A_{\substack{en\\on}}$, we obtain expressions for the radiated field expansion coefficients $C_{\substack{en\\on}}$ as follows

$$C_{\substack{em\\om}} = \frac{F_{\substack{em\\om}}}{N_{\substack{em\\om}}(c)} \left[\frac{P_{\substack{em\\om}}(x_1, x_c) \dfrac{R^{(2)}_{\substack{em\\om}}(c, x_1)}{R^{(2)}_{\substack{em\\om}}(c, x_c)} - \dfrac{R'^{(2)}_{\substack{em\\om}}(c, x_1)}{R^{(2)}_{\substack{em\\om}}(c, x_c)}}{P_{\substack{em\\om}}(x_1, x_c) R^{(4)}_{\substack{em\\om}}(c, x_1) - \dfrac{m_2}{m_1} R'^{(4)}_{\substack{em\\om}}(c, x_1)} \right] \quad (23)$$

where

$$P_{\substack{em\\om}}(x_1, x_c) = \frac{R'^{(1)}_{\substack{em\\om}}(c, x_1) R^{(2)}_{\substack{em\\om}}(c, x_c) - R^{(1)}_{\substack{em\\om}}(c, x_c) R'^{(2)}_{\substack{em\\om}}(c, x_1)}{R^{(1)}_{\substack{em\\om}}(c, x_1) R^{(2)}_{\substack{em\\om}}(c, x_c) - R^{(1)}_{\substack{em\\om}}(c, x_c) R^{(2)}_{\substack{em\\om}}(c, x_1)} \quad (24)$$

3. NUMERICAL RESULTS

Once the unknown expansion coefficients are obtained, quantities of interest such as far field radiation pattern, antenna gain and aperture conductance can be computed. The far field expression of the antenna can be expressed as

$$E^I_z(r, f) = \sqrt{\frac{j}{kr}} e^{-jkr} \left[\sum_{m=0}^{\infty} j^m C_{em} S_{em}(c, \cos f) + \sum_{m=1}^{\infty} j^m C_{om} S_{om}(c, \cos f) \right] \quad (25)$$

where r and f denote the polar coordinates in the circular cylindrical system. The antenna gain may be expressed as

$$G(f) = \frac{1}{Z_1 kr} \left[\left| \sum_{m=0}^{\infty} j^m C_{em} S_{em}(c, \cos f) \right|^2 + \left| \sum_{m=1}^{\infty} j^m C_{om} S_{om}(c, \cos f) \right|^2 \right]$$

(26)

where Z_1 is the intrinsic impedance of region I (in this case it is taken to be free space). The aperture conductance per unit length of the slot antenna is defined as

$$G_a = 2pr \frac{S_{av}}{|E_0|^2} \quad (27)$$

where S_{av} is the average power density averaged over an imaginary cylinder of radius r and given by

$$S_{av} = \frac{1}{2pZ_1 kr} \left[\sum_{m=0}^{\infty} |C_{em}| N_{em}(c) + \sum_{m=1}^{\infty} |C_{om}| N_{om}(c) \right] \quad (28)$$

The geometrical parameters used in obtaining the numerical results are $a_c = l$, $b_c = l/2$, $b = b_c + t$, where t is the coating thickness, $n_0 = 90^o$ and $a = 2.8657^o$. Figure 2 shows the radiation pattern numerical results (gain versus f) obtained for uncoated antenna presented by solid line for comparison ($e_r = 1, m_r = 1$), coated with conventional dielectric material presented by solid line with circles ($e_r = 4$, $m_r = 1$), coated with metamaterials presented by circles ($e_r = -4, m_r = -1$), and coated with isorefractive metamaterials presented by crosses ($e_r = -4, m_r = -1/4$) for dielectric thickness $t = 0.25\,l$. It can be seen that the isorefractive metamaterials coating makes the beam sharper, more directive with lower side-lobs of 50 dB when compared to the conventional dielectric material coating. The metam aterials coating also makes the beam more directive but with higher side-lobes when compared with the isorefractive metamaterials coating. This is may be due to the fact that the radiated field in the isorefractive metamaterials layer dose not suffer reflection since the wave number is the same in all regions.

The antenna gain versus dielectric coating thickness (t=0.15, 0.2 and 0.3 l) at $f = 90^o$ is plotted in Fig. 3 for the same geometrical parameters as in Fig. 2. One may notice that by increasing the thickness of the isorefractive metamaterials coating from t=0.15 l to t=0.3 l enhances the gain at $f = 90^o$ along with a reduction of sidelobes from 40 to 60 dB. On the other hand, it was shown earlier [6-7] that by increasing the thickness of the conventional dielectric coating results in a reduction of the main beam along with an increase in the side-lobes. Fig. 4 is similar to 3 except for a circular slotted antenna. It can be seen that the circular antenna has a similar behavior at $f = 90^o$ as the elliptic antenna.

The gain versus electrical coating thickness for a single slotted elliptical antenna with the same geometrical parameters used in Fig. 2 is displayed in Fig. 5. The gain is evaluated at f =90^o since the slot is centered at v =90^o where the gain is expected to be maximum. For very small coating thickness, the conventional coating, metamaterials and isorefractive metamaterials have the same effect on the gain. As t becomes greater than 0.075 l , the metamaterials and isorefractive metamareials coating make the antenna gain higher in values when compared with the conventional coating. It can also be seen the isorefractive metamaterials have higher gain than the metamaterials coating for thickness greater than 0.1 l . Further, the presence of surface waves in the case of conventional coating starts to disappear in the case of metamaterials and isorefractive metamaterials coating for higher values of electrical thickness.

4. CONCLUSIONS

Exact radiation from a slot on a conducting circular or elliptical cylinder coated with isorefractive metamaterials concentric layer was obtained using the boundary value technique. No matrix inversion is required and with the application of the boundary conditions has lead to a one-to-one matching between the field modes on either side of the interface, and thus expressions were obtained for the radiation expansion coefficients. Finally, the isorefractive metamaterials coating may be used to enhance the gain and reduce the side-lobes of slotted antenna over a certain coating range..

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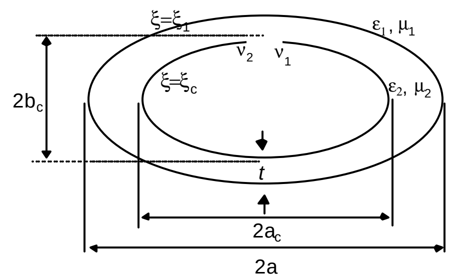

Fig. 1 Geometry of axially Slotted antenna on elliptic cylinder with dielectric coating.

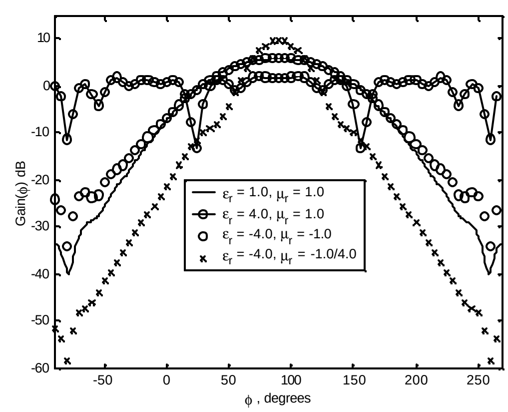

Fig. 2 Radiation pattern for an axially slotted elliptic cylinder coated with different types of dielectric materials and coating thickness $t = 0.25 l$.

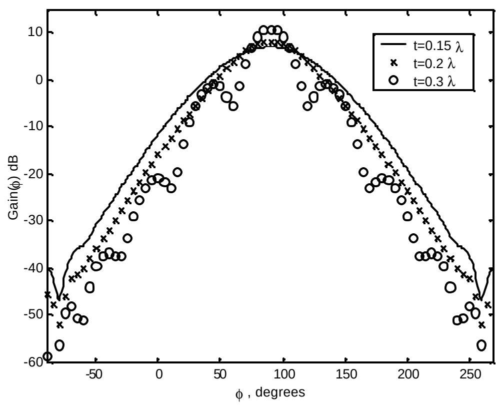

Fig. 3 Radiation pattern for an axially slotted elliptic cylinder coated with isorefractive metamaterials ($e_r = -4.0$ and $m_r = -1/4$) and various coating thickness.

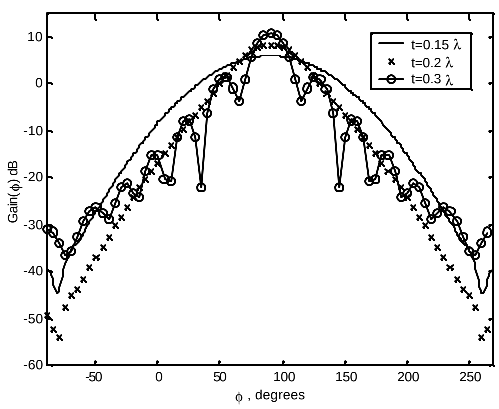

Fig. 4 Radiation pattern for an axially slotted circular cylinder coated with isorefractive metamaterials ($e_r = -4.0$ and $m_r = -1/4$) and various coating thickness.

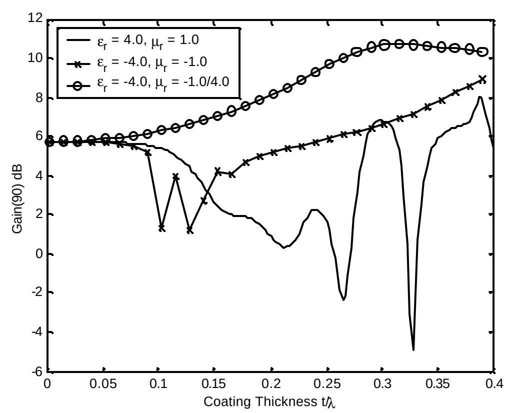

Fig. 5 : Gain at versus coating thickness for an axially slotted elliptic cylinder coated with conventional, metamaterilas and isorefractive metamaterials.