Design of IIR Filter Using Model Order Reduction Techniques

Mohammed Mujahid Ulla Faiz (260258)

Department of Electrical Engineering

Contents

Abstract

A new technique proposed by B.S. Chen [1] for IIR filter design based on SVD of the Hankel matrix, balanced realization and all-pass functions is simulated. Here IIR filter is obtained via an Optimal Hankel-norm Approximation. The error between the optimal filter with order r and the desired filter is found to be equal to the $(r+1)\,th$ singular value of the Hankel matrix.

The designed low-pass filter is given to illustrate the proposed design algorithm[1]. The LPF is tested using seismic data obtained from seismic sensors.

1 Introduction

A wide variety of methods for IIR filter design, such as the Pade approximation, the Chebyshev approximation, Recursive filter design etc suffer from problems like:

- No linear phase in the pass band of the filter.
- The resulting design is not always stable.
- The resulting filter is minimally sensitive to parameter variations and round off noise.

Hence a new filter design was proposed by B.S. Chen based on the SVD of the Hankel matrix formed from the modified impulse response of the desired filter. Here the state components, which are weekly coupled to both input and output are discarded to reduce the model.

The major objective of this proposed design is to minimize the error between the order reduced filter and the desired one in the Hankel-norm sense

2 Digital Filters

In the filter design process we find the coefficients that closely approximate the desired frequency response specifications i.e., we derive the transfer function H(z)

IIR filters have the following advantages over the FIR filter:

- FIR filters have high order though they are always Stable and have Linear Phase.
- Computational Complexity in implementing FIR.
- IIR filter design requires less memory because it involves fewer parameters and requires less power consumption.

3 Model Order Reduction (MOR)

You have an internally complex dynamical system and you want to reduce its complexity, preserving input-output behavior. In addition MOR greatly reduces simulation time.

MOR ensures good approximation of the original system by the reduced system in various aspects like stability, frequency responses etc.

Design of both stable and linear phase IIR filters is very difficult without MOR.

Ex: Balanced Model Order Reduction (BMR), Optimal Hankel-norm Approximation (OHA) etc.

4 Optimal Hankel-norm Approximation (OHA)

The Hankel-norm of H(z) is defined as:

$$\|H\|_H = \overline{\sigma}\left(\Phi\left(H\right)\right) \tag{1}$$

Where H(z) is a stable transfer function and $\overline{\sigma}$ denotes the largest singular value of $\Phi\left(H\right)$, the Hankel matrix.

We need to find a filter $H^1\left(Z\right)$ with order r such that

$$min_{H^1(Z)} \left\|H\left(Z\right) - H^1\left(z\right)\right\| \tag{2}$$

To solve this problem it is necessary to find a balanced realization (A, B, C) of H(z) with corresponding

$$\sum = diag\left(\sigma_1, \sigma_2, ..., \sigma_r, \sigma_{r+2}, ..., \sigma_m, \sigma_{r+1}\right) \tag{3}$$

Where $\sum$ is the diagonal matrix consisting of the nonzero singular values of $\Phi\left(H\right)$ with the ordering

$$\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_r \geq \sigma_{r+1} \geq \sigma_{r+2} \geq ... \geq \sigma_m > 0 \tag{4}$$

If the filter is in a balanced state-space realization The SVD of $\Phi\left(H\right)$ will be

$$\Phi\left(H\right) = U \sum V^T = \Omega_o \Omega_c \tag{5}$$

Where $\Omega_o = U\sqrt{\sum}$ and $\Omega_c = \sqrt{\sum} V^T$ denote the observability and the controllability matrices respectively so that

$$\Omega_o^T \Omega_o = \Omega_c \Omega_c^T \tag{6}$$

i.e., when the IIR filter is realised in a balanced state-space form, the observability grammian is equal to the controllability grammian.

The IIR filter using OHA is given by

$$\boxed{H_o^1\left(Z\right) = \hat{C}\left(zI - \hat{A}\right)^{-1}\hat{B}} \tag{7}$$

5 Simulation Results

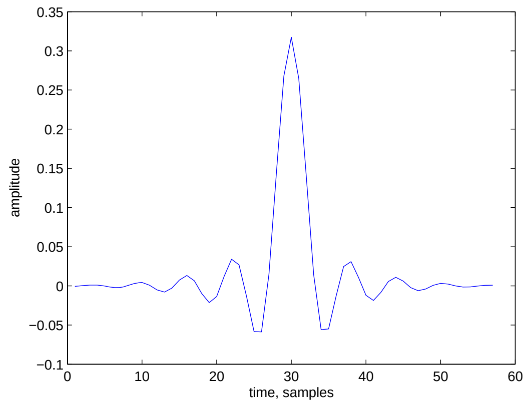

Figure 1: The window shifted-truncated impulse response of FIR with order = 57

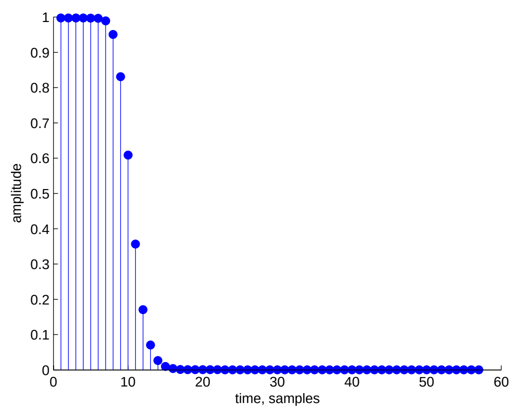

Figure 2: The singular values of the Hankel Matrix of FIR with order = 57

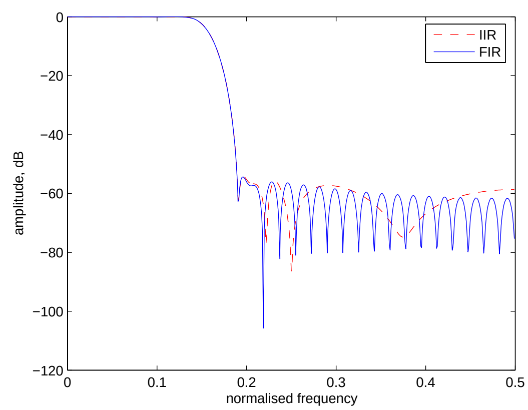

Figure 3: The overall frequency responses of IIR with order = 20 and FIR with order = 57

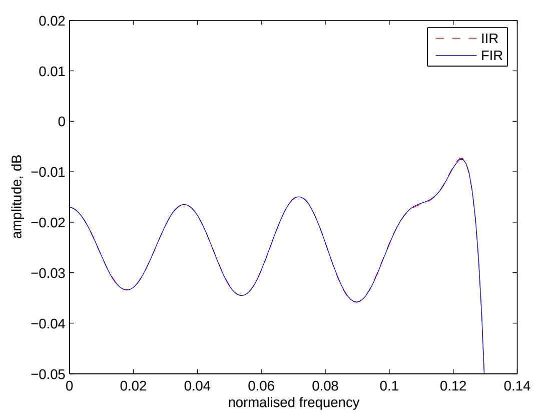

Figure 4: The passband frequency responses of IIR with order = 20 and FIR with order = 57

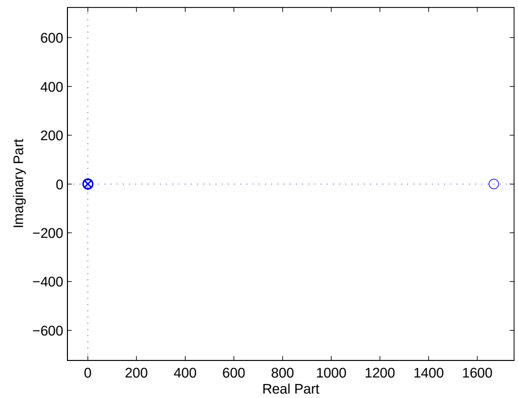

Figure 5: Pole-zero plot of IIR with order = 20

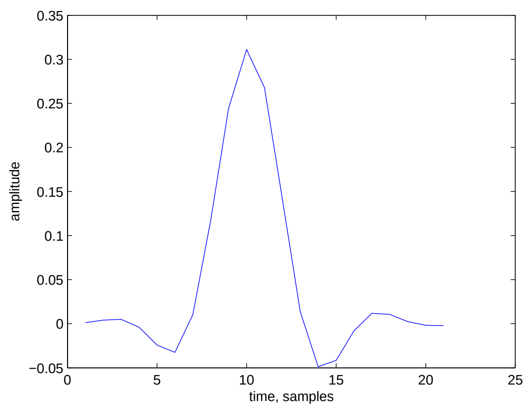

Figure 6: The window shifted-truncated impulse response of FIR with order = 21

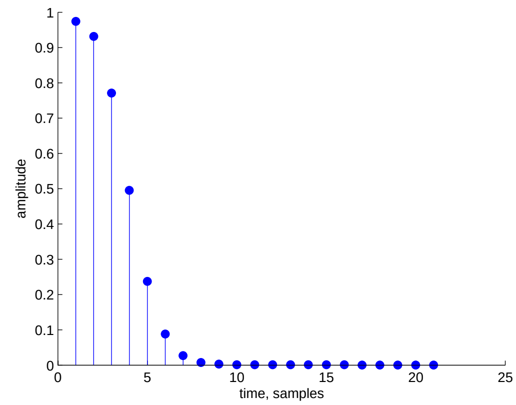

Figure 7: The singular values of the Hankel Matrix of FIR with order = 21

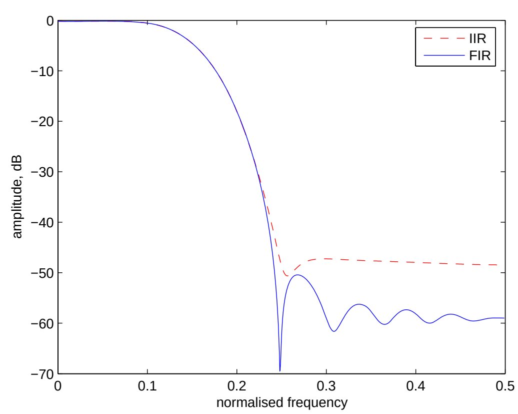

Figure 8: The overall frequency responses of IIR with order = 8 and FIR with order = 21

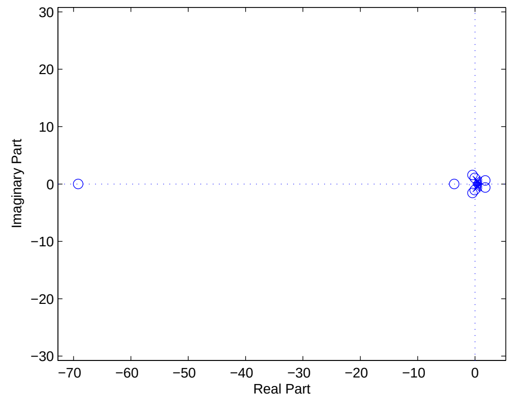

Figure 9: Pole-zero plot of IIR with order = 8

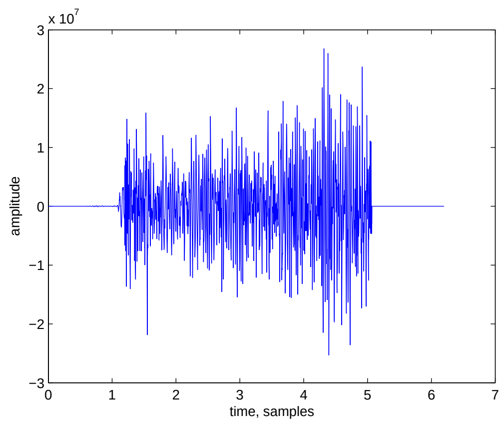

Figure 10: trace 1 before filtering in time domain

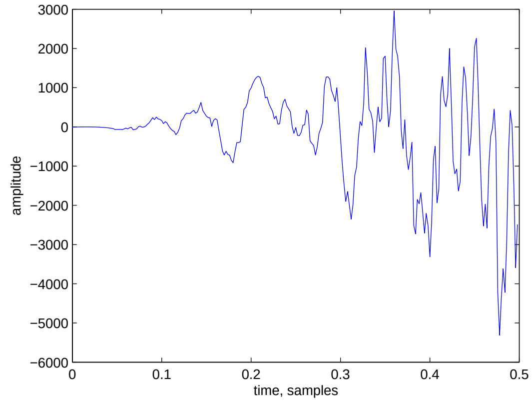

Figure 11: few samples of trace 1 before filtering in time domain

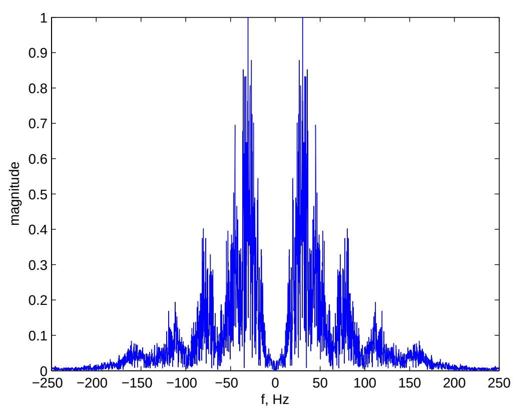

Figure 12: trace 1 before filtering in frequency domain

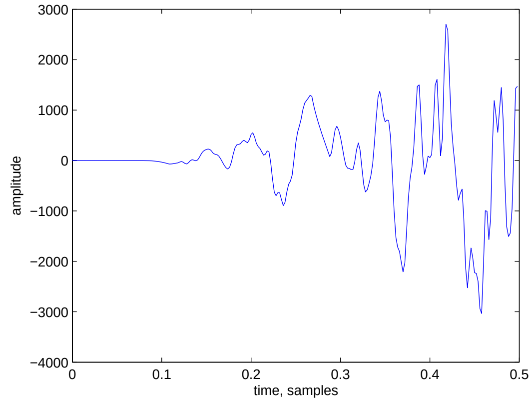

Figure 13: trace 1 after FIR filtering with order = 57 in time domain

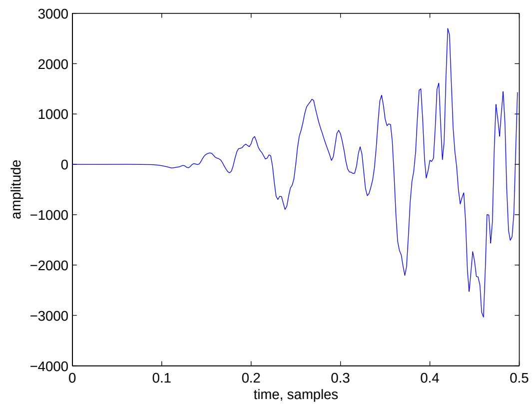

Figure 14: trace 1 after IIR filtering with order = 20 in time domain

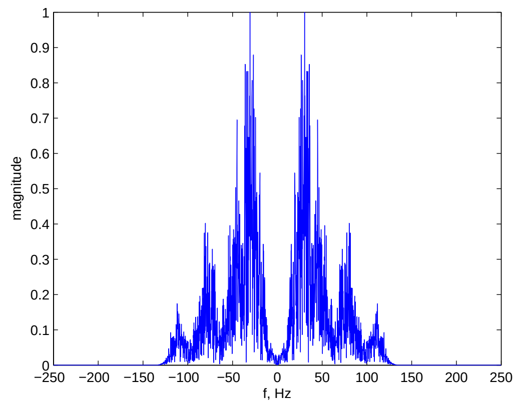

Figure 15: trace 1 after FIR filtering with order = 57 in frequency domain

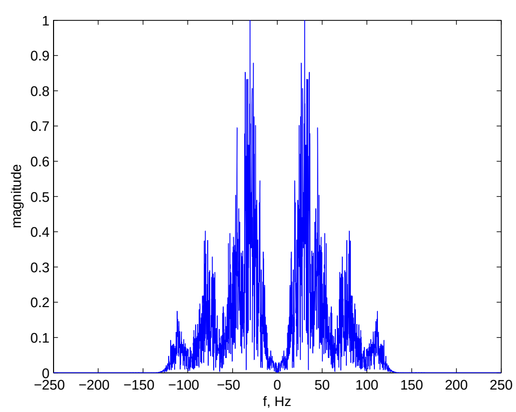

Figure 16: trace 1 after IIR filtering with order = 20 in frequency domain

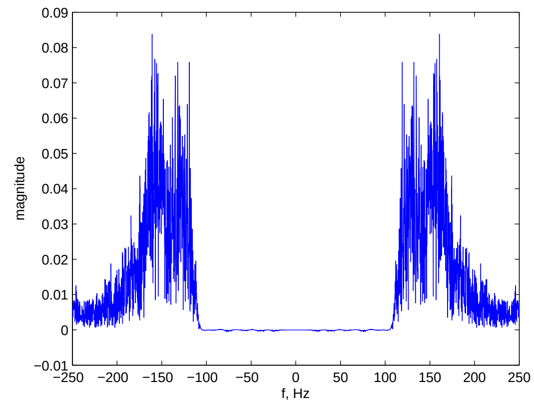

Figure 17: error signal in FIR filter with order = 57 in frequency domain

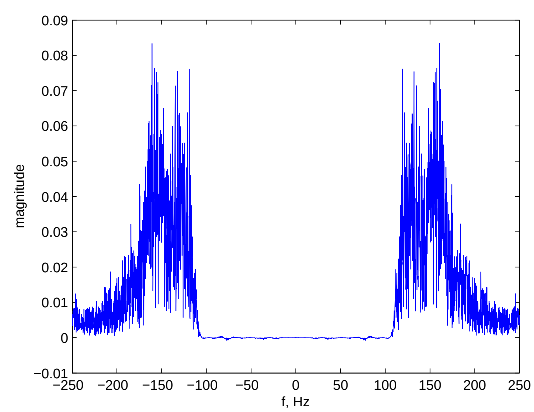

Figure 18: error signal in IIR filter with order = 20 in frequency domain

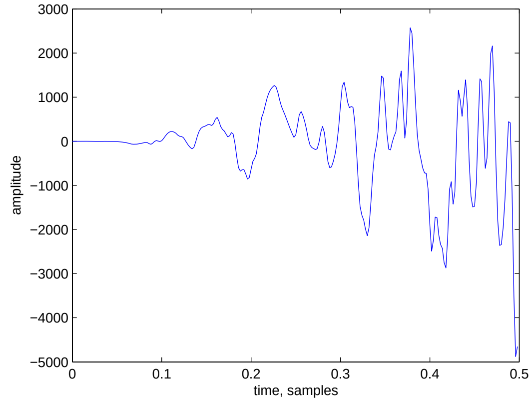

Figure 19: trace 1 after FIR filtering with order = 21 in time domain

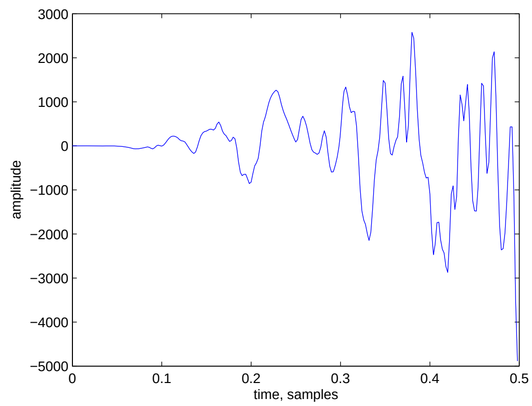

Figure 20: trace 1 after IIR filtering with order = 8 in time domain

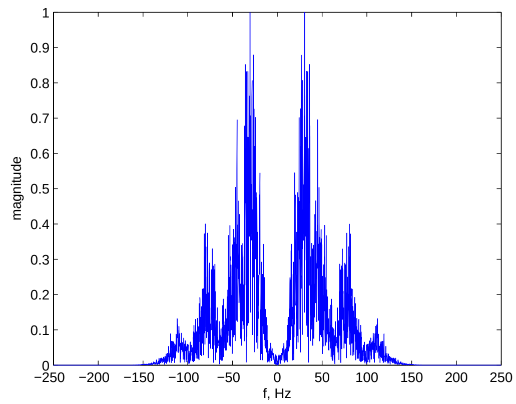

Figure 21: trace 1 after FIR filtering with order = 21 in frequency domain

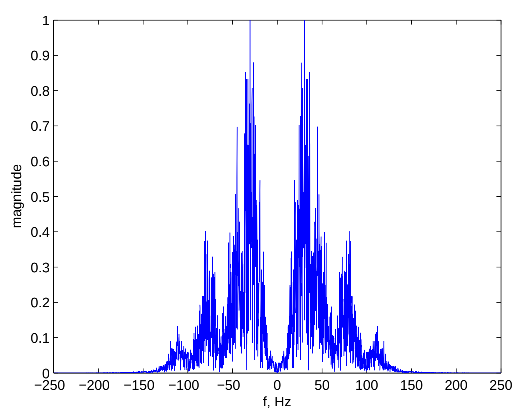

Figure 22: trace 1 after IIR filtering with order = 8 in frequency domain

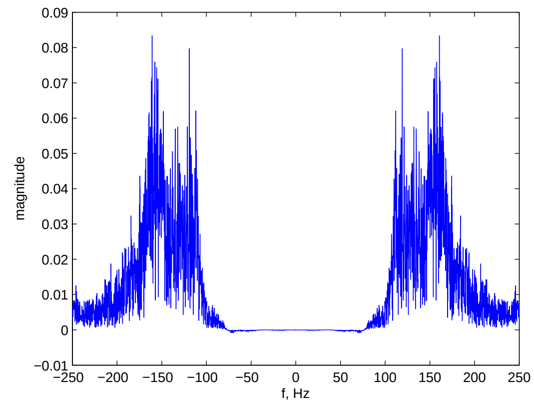

Figure 23: error signal in FIR filter with order = 21 in frequency domain

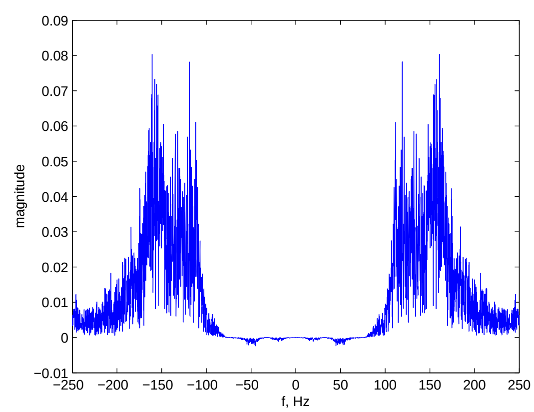

Figure 24: error signal in IIR filter with order = 8 in frequency domain

6 Conclusion

The advantages of IIR filter design using Optimal Hankel-norm Approximation are summarized as follows:

- The IIR filter is optimal in the Hankel-norm sense.
- Possible to predict the error between the IIR filter's response and FIR one, which is σ_{r+1}

  Where r is the filter order
- The performance of the IIR filter closely approximates that of FIR filter.
- The resulting design is always stable i.e., there is no need to modify the unstable poles of the filter.

The IIR LPF was tested by passing a trace obtained from a seismic sensor through it. The cut-off frequency of the filter was chosen as 60 Hz and the sampling frequency was chosen as 500 Hz.

The error in the Hankel-norm sense is found to be $\sigma_{20} = 0.0095$ for order 20(IIR) and $\sigma_8 = 0.0593$ for order 8(IIR) respectively. From this we conclude that decrease in the filter order increases the error and vice versa.

References

[1] B.S. Chen, S.C. Peng, B.W. Chiou, "IIR filter design via optimal Hankel-norm approximation"

[2] B.S. Chen, S.C. Peng, B.W. Chiou, "Minimum sensitivity IIR filter design using principal component approach"