Spatial Sequence Estimation Based Decoding Algorithm for V-BLAST

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***Abstract*--In this paper we introduce a novel detection algorithm for V-BLAST. The original V-BLAST detector is based on a serial interference cancellation (SIC) algorithm. Although the algorithm is relatively simple to implement, it does not achieve the performance predicted by theoretical analysis. The nulling operation reduces the receive diversity and the serial processing results in unequal diversity advantage for different layers. Also the error propagation severely degrades the performance. A detection algorithm for V-BLAST based on sequence estimation is proposed. The concept of maximum likelihood sequence estimation (MLSE) is applied to combat spatial interference. State reduction techniques are also considered for practical implementation. Simulation results show that the proposed algorithm outperforms interference cancellation based algorithms.**
Keywords--V-BLAST, MLSE, RSSE, sequence estimation, tail-biting trellis.

I. INTRODUCTION

In [1],[2] it was shown that rich-scattering flat fading wireless channels have huge theoretical capacity. Using multiple antennas both at the transmitter and receiver is a viable way of achieving this capacity. One of the multiple input multiple output (MIMO) techniques that received considerable attention in recent times is Vertical – Bell-Labs LAyered Space-Time Architecture (V-BLAST) [3],[4]. In V-BLAST, by spatially multiplexing transmitted symbols over multiple antennas, high spectral efficiency is obtained. The original V-BLAST detector is based on a serial interference cancellation (SIC) algorithm. Although the algorithm is practical and relatively simple to implement, it fails to achieve the performance predicted by theoretical analysis. The nulling operation reduces the receive diversity advantage and the serial processing results in unequal diversity advantage for different layers. In addition, cancellation operation results in error propagation which severely degrades the performance. Parallel interference cancellation (PIC) processing has also been applied to V-BLAST. Theoretically, it has the potential to provide full receive diversity. However, in practice, due to error propagation it never achieves that [5]. The optimum detection algorithm for V-BLAST is Maximum likelihood (ML) which is impractical due to its huge exponential complexity. However, with the introduction of sphere decoding [6], ML performance could be attained with cubic complexity on average at high SNR.

In this paper, we introduce a novel detection algorithm for V-BLAST which is based on sequence estimation. It is well known that maximum likelihood sequence estimation (MLSE) is the optimum detector in an inter-symbol interference (ISI) environment. However the concept of MLSE is not limited to interference over time and can be extended to combat spatial interference. One such scenario is the multi-user detection (MUD). In this paper we will show that the concept of sequence estimation can be applied to MIMO systems like V-BLAST. In V-BLAST, the overall diversity level is limited by the diversity level achieved by the first detected layer [7]. We will show that, by applying sequence estimation to V-BLAST detection, the receive diversity level of the first detected layer can be increased. This in turn improves the overall performance.

The remainder of this paper is organized as follow. The system model and V-BLAST are briefly described in Section 2. Spatial sequence estimation based detection algorithm for V-BLAST is introduced in Section 3. State reduction technique and its application in the context of the proposed algorithm are discussed in Section 4 and 5, respectively. Finally, conclusions are drawn in Section 6.

II. SYSTEM MODEL

We consider a MIMO system that has N_T transmitting and N_R receiving antennas and is denoted by a (N_T, N_R) system. The transmitters are assumed synchronized. The single input data stream is demultiplexed into N_T parallel layers. All the layers are transmitted through its respective antennas simultaneously. The channel is assumed to be Rayleigh flat fading. Each fade coefficient is a complex Gaussian random variable with zero mean and 0.5 variance per dimension. The channel state information (CSI) is assumed perfectly known at the receiver. However, the transmitter has no knowledge of the channel. Let h_{mn} be the path gain from the transmitting antenna n to the receiving antenna m, with $n = 1, 2, \ldots, N_T$ and $m = 1, 2, \ldots, N_R$. The channel matrix is defined as

$$\mathbf{H}_{N_R \times N_T} = \begin{bmatrix} h_{1,1} & h_{1,1} & \cdots & h_{1,N_T} \\ h_{2,1} & h_{1,1} & \cdots & h_{2,N_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R,1} & h_{N_R,1} & \cdots & h_{N_R,N_T} \end{bmatrix} \quad (1)$$

At each receiving antenna, the received signal is the superposition of N_T transmitted signals corrupted by Rayleigh fading and noise. The $N_R \times 1$ received signal vector, $\mathbf{r}$, can be written as

$$\mathbf{r} = \mathbf{Ha} + \mathbf{v} \quad (2)$$

where $\mathbf{v}$ is the $N_R \times 1$ noise vector with independent complex Gaussian variables of zero mean and $N_0/2$ variance per dimension. The $N_T \times 1$ transmitted symbol vector $\mathbf{a}$ is given by

$$\mathbf{a} = \left([s[1], s[2], \ldots, s[N_T]]\right)^T \quad (3)$$

where $(\bullet)^T$ is the transpose of the original quantity, $s[k]$ is the symbol transmitted from antenna k. All the symbols are drawn from the signal set $\mathcal{A}$ with cardinality $M = |\mathcal{A}|$.

The original V-BLAST detector is based on a serial interference nulling/cancellation algorithm. Nulling is done by linearly weighting the received vector to meet some performance criteria, i.e. zero-forcing (ZF). Once a layer is detected, it is used to cancel the interference by subtracting from the received vector. Since the detected layers are used to cancel interference, order of detection is a critical issue in the detection process. The ZF V-BLAST algorithm with ordering is summarized here [3]:

initialization:

$$\begin{aligned} & i = 1 \\ & \mathbf{G}_1 = \mathbf{H}^\dagger \\ & k_1 = \arg\min_j \left\|(\mathbf{G}_1)_j\right\|^2 \end{aligned} \quad (4)$$

recursion:

$$\begin{aligned} & \mathbf{w}_{k_i} = (\mathbf{G}_i)_{k_i} \\ & y_{k_i} = \mathbf{w}_{k_i}^T \mathbf{r}_i \\ & \hat{s}_{k_i} = Q(y_{k_i}) \\ & \mathbf{r}_{i+1} = \mathbf{r}_i - \hat{s}_{k_i} (\mathbf{H})_{k_i} \\ & \mathbf{G}_{i+1} = \mathbf{H}_{\bar{k}_i}^\dagger \\ & k_{i+1} = \arg\min_{j \notin \{k_1, \ldots k_i\}} \left\|(\mathbf{G}_{i+1})_j\right\|^2 \\ & i = i + 1 \end{aligned} \quad (5)$$

III. SPATIAL SEQUENCE ESTIMATION (SSE) BASED DETECTION ALGORITHM FOR V-BLAST

In this algorithm, a trellis over space is built for different groups of transmit antennas. Here a trellis is a compact way of visualizing all possible sequences of spatial states. For example, let's consider a group of L transmit antennas with $L \le N_T$. The state at the k-th stage, σ_k, describes all possible values taken on by $\mu = L - 1$ transmit antennas. Drawing an analogy to the ISI channel, μ corresponds to the memory of the channel while L is the length of the channel. The first stage of the trellis may be initialized as

$$\sigma_1 \triangleq \left(s[\mu], \ldots, s[2], s[1]\right), \quad 0 < \mu < N_T \quad (6)$$

The subsequent stages can be derived according to

$$\sigma_{k+1} \triangleq \left(s[t], \sigma_k[1:\mu-1]\right), \quad 1 \le k < N_T \quad (7)$$

where

$$t = \text{mod}(\mu + k - 1, N_T + 1) + \left\lfloor (\mu + k - 1)/(N_T + 1) \right\rfloor \quad (8)$$

For successive stages, transmit antennas are grouped in such a way that there is a valid transition between states. Like in any other trellis, for a valid transition from state σ_k to σ_{k+1}, the first $\mu - 1$ elements of state σ_k should match the last $\mu - 1$ elements of state σ_{k+1}. This is ensured by (7). An example trellis is depicted in Fig. 1. Note that the trellis starts and terminates at the same state description, i.e. the trellis wraps around upon itself. This type of trellis is called *tail-biting* trellis.

Once the trellis is formulated, the received signal component corresponding to each stage is constructed by employing group interference nulling technique [8]. Group interference nulling technique can maintain the desired group of signals while suppressing the effect of interfering group of signals. At any stage, all the transmitting antennas except those present in the group are nulled out.

To illustrate group interference nulling method, let us assume that the first stage, σ_1, is formed according to (5). Since transmitting antennas $1, 2, \ldots, L$ are present in the group, the remaining transmit antennas $L+1, L+2, \ldots, N_T$ are nulled out to calculate the corresponding received vector and modified channel matrix of the first stage.

Let $\mathcal{N}(\sigma_1)$ be the null space of $\mathbf{\Lambda}(\sigma_1)$ where

$$\mathbf{\Lambda}(\sigma_1)_{N_R \times N_T - L} = \begin{bmatrix} h_{1,L+1} & h_{1,L+2} & \cdots & h_{1,N_T} \\ h_{2,L+1} & h_{2,L+2} & \cdots & h_{2,N_T} \\ \cdots & \vdots & \ddots & \vdots \\ h_{N_R,L+1} & h_{N_R,L+2} & \cdots & h_{N_R,N_T} \end{bmatrix} \quad (9)$$

Since the dimension of $\mathcal{N}(\sigma_1)$ is $(N_R - N_T + L)$, an $(N_R - N_T + L) \times N_R$ matrix, $\mathbf{\Theta}(\sigma_1)$, whose rows form a set or orthonormal vectors in $\mathcal{N}(\sigma_1)$ can be obtained. By left multiplying (2) by $\mathbf{\Theta}(\sigma_1)$, the decision statistics for the first stage is found.

$$\tilde{\mathbf{r}}_1 = \mathbf{\Theta}(\sigma_1)\mathbf{r} = \tilde{\mathbf{h}}_1\tilde{\mathbf{a}} + \tilde{\mathbf{v}} \quad (10)$$

where $\tilde{\mathbf{h}}_1$ is the $(N_R - N_T + L)\times L$ modified channel matrix, $\tilde{\mathbf{v}}$ is the $(N_R - N_T + L)\times 1$ noise vector, and $\tilde{\mathbf{a}}_1$ is the $L\times 1$ shortened transmitted symbol vector, $\tilde{\mathbf{a}} = ([s[1], s[2], \ldots, s[L])^T$. Note that due to nulling the receive diversity gain is reduced to $(N_R - N_T + L)$. Using the same technique, the received signal components and modified channel matrices for the subsequent stages are found. The maximum likelihood spatial sequence estimator (MLSE) can be written as

$$\hat{\mathbf{a}} = \arg\min_{\tilde{\mathbf{a}}\in M^L} \sum_{n=1}^{N_T} \left(\tilde{\mathbf{r}}_n - \tilde{\mathbf{h}}_n\tilde{\mathbf{a}}\right)^H \left(\tilde{\mathbf{r}}_n - \tilde{\mathbf{h}}_n\tilde{\mathbf{a}}\right) \quad (11)$$

where $(\bullet)^H$ denotes the conjugate transpose of the original quantity. The problem in (11) can be solved most efficiently with the Viterbi Algorithm (VA) [8].

Unlike in interference cancellation based algorithms like SIC and PIC, Spatial Sequence Estimation (SSE) can achieve variable receive diversity level that is equal for all the layers. In SIC, receive diversity increases for successive layers. However, the performance is limited by the performance of the first layer which has receive diversity of 1. The increased diversity level of successive layers is outdone by error propagation. Even though theoretically PIC has the potential to obtain full receive diversity, due to error propagation it never achieves that. The main advantage of the proposed algorithm is that it uses sequence estimation based detection scheme instead of ZF based detection. So the probability of error propagation through the layers is minimized. Another limitation of SIC or PIC is that the receiver must have at least the same number of transmit antennas. However, SSE can be applied even when the number of receive antenna is fewer than the number of transmit antenna. In Table 1 the allowable values of L are listed for different number of receive antennas.

The design parameter of the proposed algorithm is the number of antennas in the group, L, which sets the trade-off between complexity and performance. Also this parameter determines the receive diversity achieved by each layer. All the layers obtain equal receive diversity level of $N_R - N_T + L$. The two extreme cases are

a) full receive diversity of N_R. This corresponds to maximum likelihood detection with $L = N_T$.

b) receive diversity of 2. This is the minimum receive diversity achieved by SSE. Note that the receive diversity of 1 corresponds to Zero Forcing (ZF) based detection.

Cleary as more transmit antennas are grouped to from a state, i.e., L increases, the diversity advantage increases resulting performance improvement. However, the number of states in the trellis, M^{L-1}, and thus the complexity also increases exponentially. The SSE algorithm is summarized here:

1) A trellis is formed with M^{L-1} states, where the states are defined by (6),(7) and L is the number of antennas in each group.
2) The modified received signal and channel matrix for each stage of the algorithm is formed by using (10)
3) Viterbi Algorithm is applied to detect the most likely sequence according to (11).

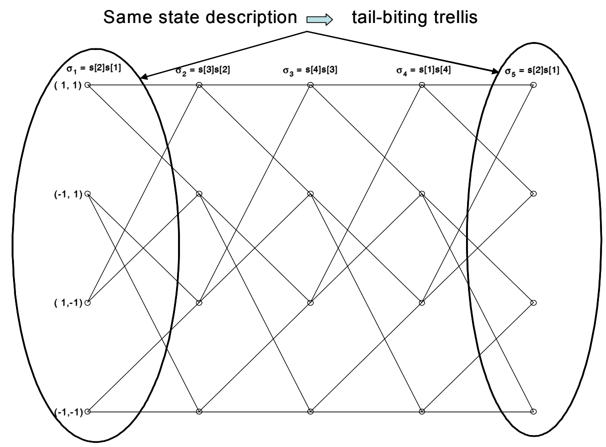

Figure 1: SSE trellis diagram for BPSK, $N_T = 4, L = 3$.

In Fig. 2, the performance of the proposed algorithm is compared against SIC and PIC based algorithms. The system parameters used in the simulation are $N_T = 4, N_R = 4$. The signaling format is QPSK. Perfect channel knowledge is assumed at the receiver. It is obvious that the SSE outperforms both SIC and PIC with one iteration. The performance of SIC without ordering is also presented as a reference. Also the performance of SSE for different group size, L, is compared. For $L = 2$ and 3, the receive diversity advantage for all the layers is 2 and 3, respectively. As L increases, the performance improves at the cost of complexity.

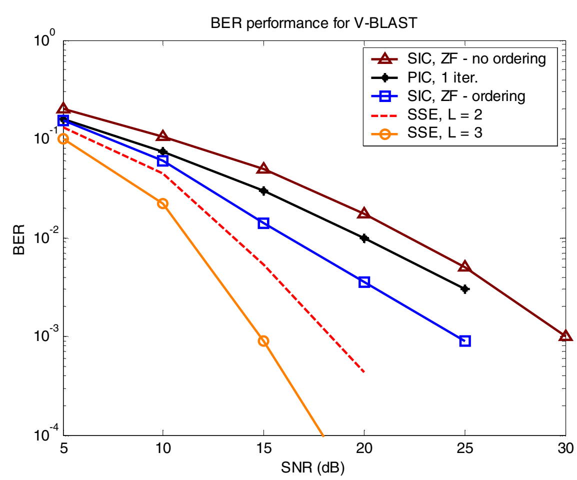

Figure 2: Performance comparison for different V-BLAST algorithms with QPSK, $N_T = N_R = 4$.

Table 1: SSE design parameter values.

System constraint	Allowable value of L
$N_T \le N_R$	$1 < L \le N_T$
$N_T > N_R$	$N_T - N_R < L \le N_T$

IV. REDUCED STATE SEQUENCE ESTIMATION (RSSE)

The computational complexity of MLSE is directly related to the number of states of the underlying trellis diagram, which is given by M^{L-1}. Hence the complexity of MLSE is prohibitively high for higher order modulation or large number of antennas in the group, L. To reduce the complexity to manageable limit, suboptimal sequence detection algorithms have to be considered. Among the suboptimal schemes, reduced state sequence estimation (RSSE) [10],[11] is considered to be one of the most promising candidates because of its regular structure and good performance. With RSSE, a finer tradeoff between performance and computational complexity is possible. For each element of the state space, i, $1 \le i \le \mu$, the signal constellation is partitioned into $N[i]$ subsets using Ungerboeck set partitioning and trellis state σ_1 is defined by the concatenation of the corresponding subset numbers $q[i]$, $1 \le i \le \mu$

$$\sigma_1 \triangleq (q[\mu], \ldots, q[2], q[1]) \quad (12)$$

In order to create a well-defined trellis, $q[\mu] \ge \ldots \ge q[2] \ge q[1]$ has to be valid [10]. The number of states of the RSSE trellis is given by

$$n_s = \prod_{i=1}^{\mu} N[i] \quad (13)$$

In the special case $N[1] = N[2] = \ldots = N[\mu] = M$, MLSE results.

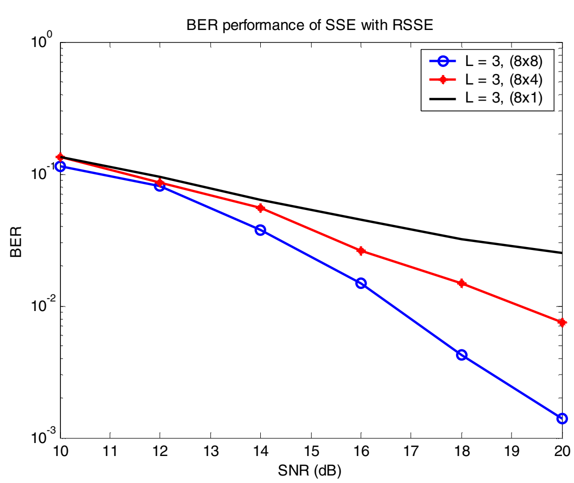

Figure 3: BER performance of SSE with RSSE, 8PSK, $N_T = N_R = 6$.

The performance of SSE with RSSE is shown in Fig. 3. Here we considered 8PSK with $N_T = N_R = 6$. The number of antennas in each group is $L = 3$. As we use $8 \times 8 (= 64)$ configuration, the performance is the best, however this also results in 64 states. By using 8×4 configuration, the number of states is reduced by half with a penalty of 2dB at 1% BER. The trade off with RSSE is obvious and must be set depending on the application requirement.

V. ITERATIVE REDUCED STATE SPATIAL SEQUENCE ESTIMATION

In trellis based detection, the error probability at the trellis edges become significant if the trellis is not initialized and terminated using known symbols [12]. The performance of SSE suffers form initialization/termination problem since it does not have known transmitted symbols at the trellis edges. However, exploiting the tail-biting feature, the parameters of the trellis can be updated iteratively resulting in improved performance. The performance of iterative SSE is shown in Fig. 4. By using two iterations, an 8×1 configuration performs within 1dB of an 8×8 configuration. The complexity is further reduced to only 4 states with very little performance degradation.

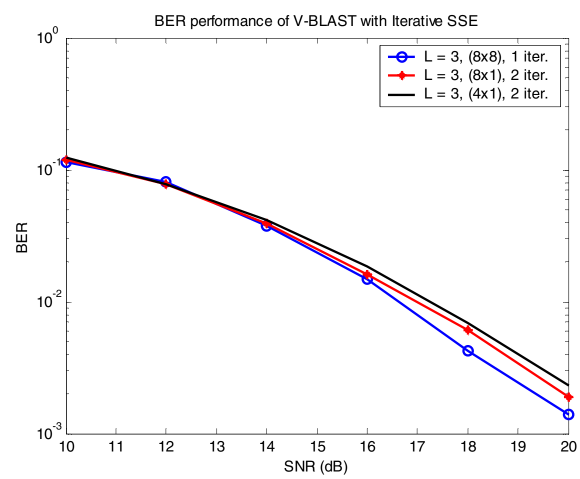

Figure 4: BER performance of iterative SSE with 8PSK, $N_T = N_R = 6$.

VI. CONCLUSION

In this paper, a novel sequence estimation based detection algorithm for V-BLAST is presented. The diversity advantage of SIC and PIC is limited by the error propagation. By using spatial sequence estimation, symbol detection is improved. This also improves the diversity gain achieved by different layers. To reduce the number of states in the trellis, reduced state sequence algorithm is also considered. For further improvement, iterative decoding is used to exploit the tail-biting nature of the trellis.

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