

Outage Capacity Comparison of Multi-Layered STBC and V-BLAST Systems

Samir Al-Ghadhban, R. M. Buehrer and B. D. Woerner

Mobile and Portable Radio Research Group
Virginia Polytechnic Institute and State University
E-Mail: samir@vt.edu

Abstract: In this paper, we examine the capacity of high data rate open loop MIMO architectures. The focus of the study is to compare the information capacity of multi-layered space time block coded (MLSTBC) systems with V-BLAST and STBCs. MLSTBC combines transmit diversity and spatial multiplexing. The single user data are divided into layers of information and each layer is encoded with a STBC. The result of this study shows that for the same number of transmit-receive antennas, MLSTBC is more power efficient than V-BLAST, since it provides more diversity. Furthermore, at low SNRs and low outage probabilities, MLSTBC is more spectrally efficient. Thus, it is more suitable for low power high data rate wireless applications.

Keywords: Outage Capacity, MIMO, MLSTBC, V-BLAST

I. INTRODUCTION

Multiple-input multiple-output (MIMO) communication systems can offer high data rates through spatial multiplexing and they can improve the link quality through diversity. V-BLAST [1] is a spatial multiplexing scheme that transmits independent layers of information through a MIMO channel. However, it has poor energy performance and doesn't fully exploit the available diversity. On the other hand, space time block coding [2] provides full transmit and receive diversity but with a maximum code rate of one which is achieved at two transmit antennas.

Combining V-BLAST and STBC results in a layered architecture with transmit diversity in each layer. This can be called a multi-layered space time block code (MLSTBC) [3]. The idea of this scheme is to demultiplex a single user's data into parallel layers of information. Then, each layer is encoded by a STBC. Each code is called a group, because the total number of transmit antennas are divided into groups and each group is assigned a STBC. This architecture was first considered in [4] where they used space time trellis codes (STTC) as the component codes. In a multi-user environment, a multi-user STBC system with minimum mean-squared error (MMSE) detection was studied in [5]. In [3], different decoding algorithms for MLSTBC were compared over flat fading MIMO channels. One advantage of using STBC over STTC is that the orthogonal structure and the short code length can be exploited at the receiver to reduce the minimum required number of receive antennas [5]. For MLSTTC [4,6], the number of receive antennas should be at least equal to the total number of transmit antennas. However, for MLSTBC, it is equal to the number of layers.

In this paper, we evaluate the outage capacity of a MLSTBC system which uses a serial group interference nulling and cancellation (SGINC) [3] detection algorithm and we compare it with other open loop MIMO transmit techniques, such as V-BLAST, STBC and optimal MIMO,

with the same number of transmit and receive antennas. The main finding of this work shows that adding a STBC layer on V-BLAST improves the capacity of the MIMO system at low SNRs and at low outage probabilities.

II. MLSTBC SYSTEM MODEL

The MLSTBC transmitter consists of K parallel space time block encoders which are independent and synchronized (Figure 1). Each encoder transmits through N_G antennas and the receiver has M receive antennas. The total number of transmit antennas is $N = K \cdot N_G$. The MIMO channel is assumed to be an independent Rayleigh flat fading MIMO channel where each coefficient is a complex Gaussian random variable with mean zero and variance of 0.5 per dimension. The received matrix over T time slots, where T is the STBC length, is given by:

$$\mathbf{Y} = \mathbf{H}\mathbf{S} + \mathbf{V}$$

$$= [\mathbf{H}_1 \quad \mathbf{H}_2 \quad \dots \quad \mathbf{H}_K] \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \\ \vdots \\ \mathbf{S}_K \end{bmatrix} + \mathbf{V} \quad (1)$$

where \mathbf{H}_i is an $M \times N_G$ MIMO channel matrix of the i^{th} group and \mathbf{S}_i is the $N_G \times T$ STBC of the i^{th} group. Also, \mathbf{V} is the AWGN matrix over T time slots. Due to the short code length, the receiver can rearrange the received matrix to a vector [5]. After doing that, we get a discrete MIMO model similar to V-BLAST:

$$\mathbf{y} = \hat{\mathbf{H}}\mathbf{x} + \boldsymbol{\eta}$$

$$= [\hat{\mathbf{H}}_1 \quad \hat{\mathbf{H}}_2 \quad \dots \quad \hat{\mathbf{H}}_K] \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_K \end{bmatrix} + \boldsymbol{\eta} \quad (2)$$

where \mathbf{y} is the $M \cdot T \times 1$ received vector, $\hat{\mathbf{H}}_i$ is the $M \cdot T \times N_G$ orthogonal channels matrix for the i^{th} group, \mathbf{x}_i is the $N_G \times 1$ transmitted symbols from the i^{th} group, and $\boldsymbol{\eta}$ is the $M \cdot T \times 1$ AWGN vector.

III. CAPACITY FORMULAS

The instantaneous capacity of V-BLAST with K layers and with zero forcing interference nulling (ZF) and serial cancellation is given by [7]:

$$C_{VBLAST}^{ZF} = K \cdot \min_{i=1,2,\dots,K} \left\{ \log_2 \left(1 + \frac{\rho}{K \|\mathbf{W}_{ZF,i}\|^2} \right) \right\} \quad (3)$$

where $\mathbf{W}_{ZF,i}$ is the ZF projection vector of the i^{th} layer, ρ is the SNR per receive antenna, and $\|\cdot\|$ is the Frobenius norm.

Furthermore, the instantaneous capacity of an orthogonal STBC of rate r_c and N_G transmit antennas is [9]:

$$C_{STBC} = r_c \log_2 \left(1 + \frac{\rho}{N_G} \|\mathbf{H}\|_F^2 \right) \quad (4)$$

In a MLSTBC system, the SGINC algorithm detects each group separately after canceling previously detected groups and nulling interfering groups. Based on an ordering criterion, assume that the first detected group is the i^{th} group. Then, the algorithm calculates the orthonormal bases of the null space of \mathcal{H}_i , where:

$$\mathcal{H}_i = [\hat{\mathbf{H}}_1 \quad \cdots \quad \hat{\mathbf{H}}_{i-1} \quad \hat{\mathbf{H}}_{i+1} \quad \cdots \quad \hat{\mathbf{H}}_K] \quad (5)$$

Denote the orthonormal bases of \mathcal{H}_i by \mathcal{N}_i , the received signal for the i^{th} group after nulling is:

$$\tilde{\mathbf{y}}_i = \mathcal{N}_i^T \mathbf{y} = \tilde{\mathbf{H}}_i \mathbf{x}_i + \tilde{\mathbf{n}}_i \quad (6)$$

where $\tilde{\mathbf{H}}_i$ is the resultant channel matrix after nulling. After that, the contribution of the i^{th} group will be subtracted from (2) and the process will be repeated serially for each group. The ordering is based on the Frobenius norm of $\tilde{\mathbf{H}}_i$. The layer with maximum $\|\tilde{\mathbf{H}}_i\|_F^2$ is detected first.

Since MLSTBC is a single user system and the transmitter doesn't know the channel and all groups transmit at the same rate, an outage will occur if an outage happens in one layer "the weakest". Therefore, the instantaneous capacity of a K group STBC system is:

$$C_{MLSTBC}^{GNIC} = K \cdot \min_{i=1,2,\dots,K} \left\{ r_c \log_2 \left(1 + \frac{\rho}{K \cdot N_G} \left(\frac{\|\tilde{\mathbf{H}}_i\|_F^2}{T} \right) \right) \right\} \quad (7)$$

where T is the STBC length. The Frobenius norm of $\tilde{\mathbf{H}}_i$ is divided by T because the dimension of the channel matrix has been increased T times after rearranging the original channel matrix as indicated in (2).

IV. SIMULATION RESULTS

This section compares the capacities of the detection algorithms of MLSTBC, V-BLAST and STBC. In addition, the optimal MIMO capacity is included as a reference. For MLSTBC, each component code is a rank two Alamouti STBC [8]. The capacity of the different systems is estimated by generating random complex Gaussian channel realizations from which the instantaneous capacity is calculated and then the capacity probability distribution function (pdf) is approximated.

One main difference between MLSTC and V-BLAST at the same number of transmit-receive antennas is that the earlier has more spatial diversity than the later while the later has more layers. For example, with a 4×4 MIMO system, MLSTBC has two layers and each layer has a transmit diversity of two. At the receiver, the first detected layer has a receive diversity of three. This is because the detector needs

one antenna to null out one interfering layer and the rest provide diversity. On the other hand, V-BLAST has four layers and no transmit diversity. In addition, the first detected layer has no receive diversity because the algorithm needs three antennas to null out three interfering layers.

Figure 2 plots the capacity complementary cumulative distribution function (CCDF) of the considered MIMO schemes for 4×4 MIMO channels. The STBC is the orthogonal code of rate $3/4$ [2]. The results show that there is a crossover in capacity that is a function of SNR. At low SNRs and at low outage probabilities, the capacity of V-BLAST is lower than both STBC and MLSTBC. The reason is that the later schemes provide more diversity which benefits the capacity at low SNRs. On the other hand, at high SNRs, the capacity of V-BLAST improves significantly and it is higher than MLSTBC for outage probabilities greater than 3%.

The spectral efficiencies of the 4×4 MIMO schemes are shown in Figure 3. It shows the tradeoffs between the MLSTBC and V-BLAST systems. First, the spectral efficiency of V-BLAST varies a lot with the outage probability, unlike MLSTBC. For example, at 10 bps/Hz, V-BLAST needs around 9 dB to sustain this rate when going from 10% to 1% outage. On the other hand, MLSTBC needs only 2 dB. This is a result of lack of diversity of V-BLAST. Furthermore, the figure shows that the spectral efficiency rate of increase of V-BLAST is faster than MLSTBC. Its slope is parallel to the optimal MIMO at high SNRs since it is a full spectral multiplexing scheme. However, at low outage probabilities, MLSTBC is more spectral efficient than V-BLAST for a wide range of SNRs.

The outage probability as a function of SNR is shown in Figure 4 at 4 bps/Hz efficiency. The result reemphasizes the fact that MLSTBC has more diversity than V-BLAST. In the case of 4×4 MIMO channels, the first detected layer has a diversity order of 2×3 while it doesn't have any diversity in V-BLAST. Furthermore, MLSTBC is more power efficient at low and moderate SNRs than STBC. That is a result of having diminishing gains with higher diversity orders. Therefore, utilizing some of the antennas for spatial multiplexing doesn't harm the performance.

Figure 5 shows the effect of increasing total number of transmit antennas (N) on the spectral efficiency while the number of receive antennas is fixed at eight. The capacities of MLSTBC and V-BLAST first increase when adding more layers as expected but after a certain number of layers, a reduction in capacity occurs especially when $N = 2M$ in MLSTBC and when $N = M$ in V-BLAST. This is a result of receive diversity reduction caused by the nulling operation in the detection algorithms of both systems. In other words, the capacity could be maximized by selecting the best number of layers at a given SNR. As a heuristic rule inferred from the plots, if the intended region of operation is at high SNRs, set the number of layers (K) to $M-1$. On the other hand, if the region of operation is at low and moderate SNRs, set K to be equal to $M/2$.

V. CONCLUSION

The capacity of the SGINC detection algorithm of the MLSTBC system was studied and compared to other open loop transmit techniques at the same number of transmit-receive antennas. The simulation results show that MLSTBC is more spectrally efficient at low SNRs and at low outage probabilities than V-BLAST. Furthermore, since MLSTBC has more transmit-receive diversity, it is more power efficient. Therefore, it makes a good candidate for low power high data rate wireless applications. Moreover, the study shows that there is a capacity reduction in MLSTBC and V-BLAST after adding a certain number of layers. That is a result of the nulling operation involved in the detection algorithms.

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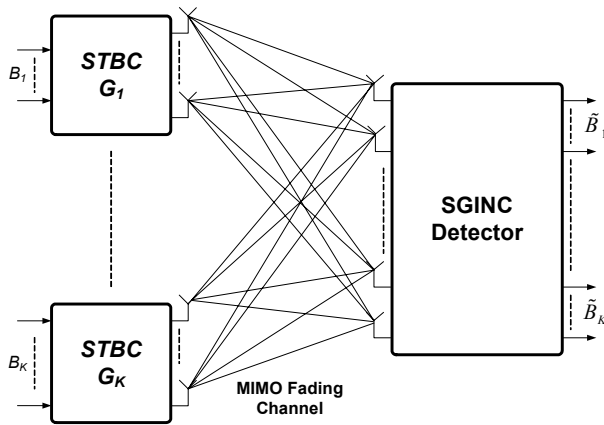


Figure 1: The MLSTBC block diagram

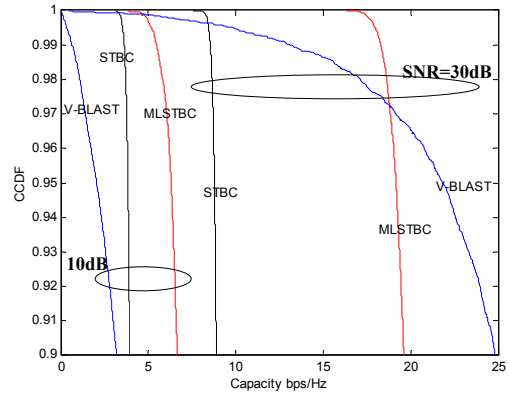


Figure 2: Capacity CCDFs of MLSTBC, V-BLAST and the STBC at 4x4 MIMO channels

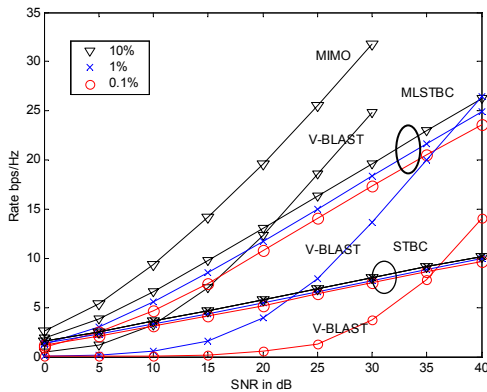


Figure 3: Spectral efficiency of MLSTBC, V-BLAST and STBC at 4x4 MIMO channels and at 10, 1, 0.1% outage probabilities.

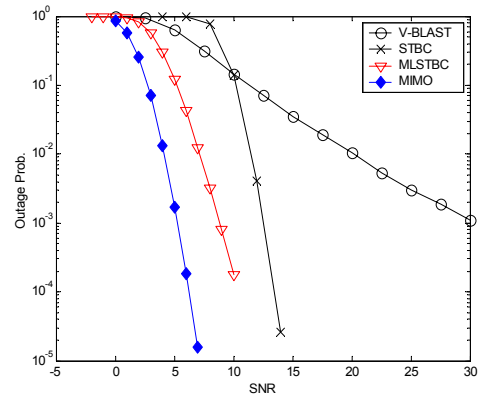


Figure 4: Outage probability of MLSTBC, V-BLAST and STBC at 4 bps/Hz and over 4x4 MIMO channels

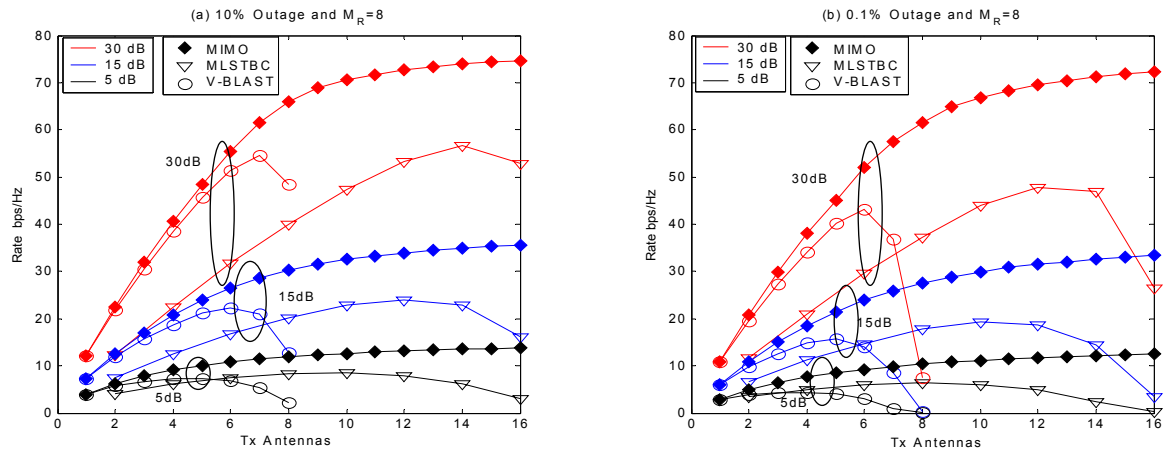


Figure 5: Spectral efficiency versus number of transmit antennas for MLSTBC, V-BLAST and the optimal MIMO at eight receive antennas ($M=8$).