IQ Space Frequency Time Codes for MIMO-OFDM Systems

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Abstract: **In this paper, we study concatenated coding for MIMO-OFDM systems. The proposed concatenated system achieves full spatial and frequency diversity at much lower complexity in terms of number of states without any bandwidth expansion. In general, coding for MIMO-OFDM systems is known as space frequency time (SFT) coding. This paper focuses on applying a powerful class of trellis codes known as IQ-TCM. We illustrate the benefits of IQ-TCM in concatenated SFT codes. The results of this study show the performance improvements of IQ-TCM over conventional TCM at the same number of states and spectral efficiency. The reason for that is the larger effective length of IQ-TCM. Also, the simulation results emphasize the importance of an appropriate interleaver design since there is a great loss in performance and diversity if the block interleaver is not designed carefully.**

Keywords: MIMO systems, Space Frequency Time Codes, Concatenated TCM-STBC, IQ-TCM

I. INTRODUCTION

Space frequency time (SFT) coding for OFDM applies spatial coding across multiple antennas, frequency coding across OFDM subcarriers and temporal coding across successive OFDM symbols. The first space frequency coding study was done by [1] where they adapted Tarokh's space time codes [2] to OFDM with multiple transmit antennas. However, these codes are designed for quasi-static fading channels. Thus, they are not optimized for OFDM channels and couldn't benefit from the available frequency diversity. In [3], it was shown that the maximum achievable diversity for a MIMO-OFDM system is $M_T L M_R$, where L is the frequency selective channel length and M_T and M_R are the number of transmit and receive antennas, respectively. In order to achieve this diversity, the minimum effective length of the SFT code should be equal to $M_T L$, which requires a large number of states for practical cases. Furthermore, since the OFDM channel in the frequency domain is highly correlated and slowly varying, interleaving across frequency tones is a vital requirement that allows the code to exploit the available frequency diversity.

To achieve full spatial-frequency diversity, trellis code design needs large number of states. In order to simplify the design and reduce the complexity of the code, [4] proposed to concatenate trellis coded modulation (TCM) with space time block coding (STBC). The spatial diversity is guaranteed by STBC while the frequency diversity is achieved by TCM. This separation allows for a less complex, lower number of states, TCM design. However, further reduction in number of states is possible and this is the main motivation of this work.

Our main contribution is proposing a SFT code that achieves full spatial and frequency diversity at much lower number of states compared to previous designed. The proposed code concatenates a powerful class of trellis codes, known as inphase-quadrature trellis coded modulation (IQ-TCM) [5], with STBC. IQ-TCM provides larger effective lengths compared to conventional TCM. At the same rate and number of states, it could at most doubles the minimum effective length. In addition, we examine the effect of interleaving on the frequency diversity of the concatenated SFT code.

II. MIMO-OFDM CHANNEL MODEL

A MIMO frequency selective channel (FSC) consists of $M_T M_R$ FSC. Assume that each FSC between the n^{th} transmit antenna and the m^{th} receive antennas be of length L, and denote it by $\mathbf{h}_{mn} = [h_0 \;\; h_1 \;\; \cdots \;\; h_{L-1}]^T$. OFDM transforms MIMO-FSC into N_c parallel MIMO flat fading channels, where N_c is the number of subcarriers. The OFDM channel in the frequency domain between the n^{th} transmit antenna and the m^{th} receive antenna is:

$$\mathbf{h}_{mn}^{f} = \mathbf{F}\mathbf{h}_{mn} \tag{1}$$

where $\mathbf{F}$ is a partition of the FFT matrix and it is defined as:

$$\mathbf{F}_{k,l} = \frac{1}{\sqrt{N_c}} \exp\left[-i\frac{2\pi}{N_c}(k-1)(l-1)\right]; \quad k = 0,1,\dots,N_c - 1 \quad l = 0,1,\dots,L-1 \tag{2}$$

Assume that $\mathbf{h}_{mn} \sim \mathcal{N}_c(\mathbf{0}, \mathbf{C}_{\mathbf{h}_{mn}})$, then the covariance matrix of $\mathbf{h}_{mn}^{f}$ will be:

$$\mathbf{C}_{\mathbf{h}_{mn}^{f}} = \mathbf{F}\mathbf{C}_{\mathbf{h}_{mn}}\mathbf{F}^{H} \tag{3}$$

Thus the OFDM channel in the frequency domain is highly correlated even when the paths of FSC are independent, i.e $\mathbf{C}_{\mathbf{h}}=\mathbf{I}$. The fade rate is slower at low number of paths and it is faster at higher number of paths.

III. IQ-SPACE FREQUENCY TIME CODES

An important advantage of the concatenated TCM-STBC is the design separation between temporal and spatial diversity. The spatial diversity is guaranteed by STBC which allows the designer to focus on TCM design to get more frequency diversity. There are number of trellis code designs that increase the minimum effective length (l_{min}) over fast fading channels. The minimum effect length is known as the diversity of the code. An interesting design that increases the minimum effective length of TCM is to code the inphase and quadrature components separately by two parallel TCM encoders [5]. This code is called IQ-TCM. It shows superior performance improvements over conventional TCM and it is easily implemented from off the shelf codes.

The minimum effective length of TCM is upper bounded by [5]:

$$l_{\min} \le \lfloor v / k \rfloor + 1 \tag{4}$$

where v is the number of memory elements in the encoder and k is the number of inputs. Thus, at the same number of states, reducing k increases l_{min}. When k is reduced by a half, l_{min} at most doubles and this is the reason behind the diversity increase of IQ-TCM.

In this section, we describe 2 bps/Hz efficiency system. The IQ-TCM encoder consists of two half rate 4-AM trellis codes. The trellis diagram and signal set of an 8-state code is shown in Figure 1. Each input is trellis coded and mapped to a 4-AM signal set. Then the output is combined to form a 16-QAM signal. At 8-states, the IQ-16QAM trellis code provides a frequency diversity of order four while it is only two for a conventional 8PSK TCM.

IQ-SFT encoder and decoder are shown in Figures 2 and 3, respectively. IQ-TCM output two codewords each is an OFDM symbol of length N_c. After interleaving, STBC encodes the two OFDM symbols using Alamouti code [6] at each subcarrier.

The receiver is equipped with one receive antenna as shown in Figure 3. Extension to more than one antenna is straight forward. After FFT, the received signals over two periods in the frequency domain are:

$$\begin{bmatrix} \mathbf{Y}^{t_1} & \mathbf{Y}^{t_2} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 & -\mathbf{s}_2^* \\ \mathbf{s}_2 & \mathbf{s}_1^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}^{t_1} & \boldsymbol{\eta}^{t_2} \end{bmatrix} \tag{5}$$

where $\mathbf{Y}^{t_1} = \begin{bmatrix} y_1^{t_1} & y_2^{t_1} & \cdots & y_{N_c}^{t_1} \end{bmatrix}^T$ is the OFDM received symbol at time t_1. Similarly, $\boldsymbol{\eta}^{t_1}$ is the complex AWGN vector of all subcarriers of zero mean and variance $N_0/2$ per dimension. Furthermore, the OFDM channel matrix in the frequency domain between transmit antennas n and receive antenna m is:

$$\mathbf{H}_{mn} = \begin{bmatrix} h_{mn,1} & 0 & \cdots & 0 \\ 0 & h_{mn,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{mn,N_c} \end{bmatrix} \tag{6}$$

where $h_{mn,l}$ is a complex Gaussian channel coefficient for the l^{th} subcarrier of zero mean and variance 0.5 per dimension. The $diag\left(\mathbf{H}_{mn}\right)$ is as defined in (1).

Since the OFDM system transforms FSC into N_c parallel flat fading channel as apparent in (6), the STBC combiner operates on each subcarrier separately. The STBC received signal at the i^{th} subcarrier is:

$$\begin{bmatrix} y_i^{t_1} & y_i^{t_2} \end{bmatrix} = \begin{bmatrix} h_{11,i} & h_{12,i} \end{bmatrix} \begin{bmatrix} s_{1,i} & -s_{2,i}^* \\ s_{2,i} & s_{1,i}^* \end{bmatrix} + \begin{bmatrix} \eta_i^{t_1} & \eta_i^{t_2} \end{bmatrix} \tag{7}$$

it is rearranged into:

$$\begin{bmatrix} y_i^{t_1} \\ y_i^{t_2 *} \end{bmatrix} = \begin{bmatrix} h_{11,i} & h_{12,i} \\ h_{12,i}^* & h_{12,i}^* \end{bmatrix} \cdot \begin{bmatrix} s_{1,i} \\ s_{2,i} \end{bmatrix} + \begin{bmatrix} \eta_i^{t_1} \\ \eta_i^{t_2 *} \end{bmatrix} \tag{8}$$

After that, STBC combiner is used to estimate the IQ coded symbols. The uncoupled estimates are then deinterleaved and passed to the IQ-trellis decoders.

IV. SIMULATION RESULTS

The channel is a MIMO-FSC of length L with equal power paths and each path experience an independent Rayleigh fading. We assume that the channel is constant over two OFDM symbols. This section compares the performance of the IQ-SFT code with other codes at 2 bps/Hz efficiency. The IQ concatenated code uses the IQ-16QAM TCM code as described in [5]. Its performance is compared to a conventional 8PSK TCM code designed for rapid fading channels [7] and to Tarokh's STTC [2]. Table 1 calculates the complexity of the SFT codes needed to achieve full spatial-frequency diversity ($M_T L M_R$). Tarokh's STTC needs to have at least $l_{min}=M_T L$ to achieve full diversity. On the other hand, the concatenated TCM-STBC provides spatial diversity by STBC and frequency diversity by TCM. This explains the huge reduction in complexity of the concatenated codes compared to Tarokh's code. In addition, the IQ codes further reduce the number of states of conventional TCM by factor of a square root. If the number of states for the IQ-16QAM code is x, then it is x^2 for the 8PSK TCM code.

The BER performance of the SFT codes over four taps FSC is shown in Figure 4. The codes have 8-states and the receiver has one antenna. The interleaver is a block interleaver of width four. At 8-states, the frequency diversity of the IQ code is four while it is two for 8PSK and Tarokh's QPSK codes. Thus, the IQ SFT code achieves full spatial and frequency diversity of order eight at just eight states. Tarokh's QPSK code only achieves a diversity of order 2 and its slope is parallel to the uncoded STBC.

For FSC of length two, the BER performance is shown in Figure 5. In this case, the maximum frequency diversity available is two and it is achieved by both concatenated schemes. Therefore, there are no additional diversity gains for the IQ-SFT code as apparent from the parallel slopes. However, the IQ-SFT code adds some coding gain (around 1.5dB at BER=10^{-4}).

V. INTERLEAVING EFFECT

In this section, we examine the effect of a block interleaver design on the performance of SFT codes. At four rays FSC and over 2×1 MIMO-OFDM channels, the performance of the 8-states IQ-16QAM SFT code is shown in Figure 6. It achieves full spatial-frequency diversity. However, the result shows that the diversity obtained is highly dependent on the interleaver width (W). The best performance is at W=4 and 8 while there is a diversity reduction at other values.

It has been shown in [8] that the optimal interleave in a coded OFDM system should separate any consecutive codeword symbols by a multiple of N_c/L in order to maximize the coding gain, which is impossible to build [Wan04]. However, a block interleaver with a width of L and a depth of N_c/L separates successive symbols by N_c/L or $N_c - N_c/L - 1$. Therefore, a block interleaver with a width of L is nearly optimal. However, after examining the interleaver design for a few cases, we find that a width of L is not always the best choice. This happens if the FSC has low number of rays. Specifically, at L=2 paths, the simulation result doesn't support the claim that the best block interleaver width should be two, as can be interpreted from Figure 7. The coded system got more gain and diversity at width of four and eight. However, in other cases, such as L= 4 and 8 in Figures 6 and 8, respectively, the best performance occurs at W=L in addition to other width values such as 2L.

VI. CONCLUSION

In this paper, we consider a reduced complexity bandwidth efficient design of space frequency time codes for MIMO-OFDM systems. Our approach is to concatenate IQ-trellis codes with space time block codes. The concatenated system separates frequency coding from space time coding and it has much lower complexity than a joint design. The IQ-TCM codes provide larger effective lengths at low number of states. In addition, we examined the effect of interleaving on the performance of SFT codes and on the achieved diversity. The main results of this study show the performance improvements of IQ-TCM over conventional TCM at the same number of states and spectral efficiency. The reason for that is the larger effective length of IQ-TCM. Also, the simulation results emphasize the importance of an appropriate interleaver design since there is a great loss in performance and diversity if the block interleaver is not designed carefully.

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Table 1: Complexity of SFT codes at 2bps/Hz and M_T=2 transmit antennas

FCS Length	Minimum number of states to achieve full diversity ($M_T L M_R$)		
L	Tarokh QPSK	8PSK	IQ-16QAM
2	64	4	2
3	1024	16	4
4	16384	64	8
5	262144	256	16
6	4194304	1024	32
7	67108864	4096	64

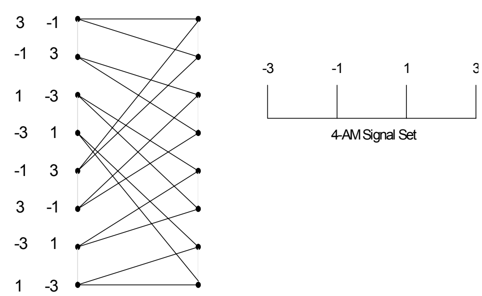

Figure 1: 8-states 4AM-TCM

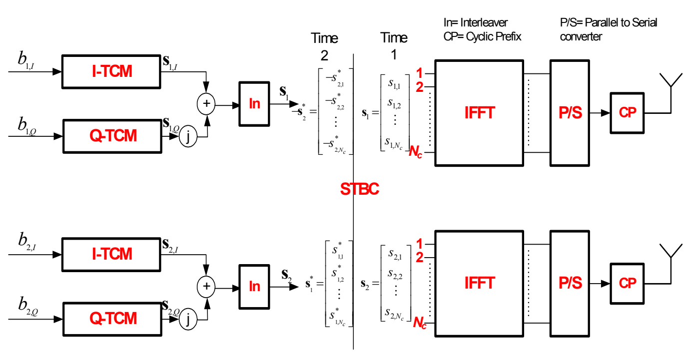

Figure 2: Block Diagram of IQ-SFT Encoder

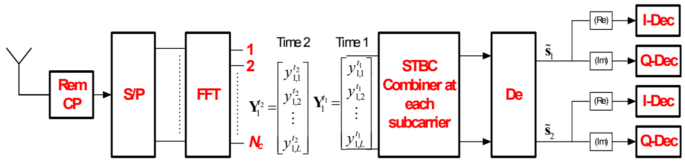

Figure 3: Block Diagram of IQ-SFT Decoder at one receive antenna

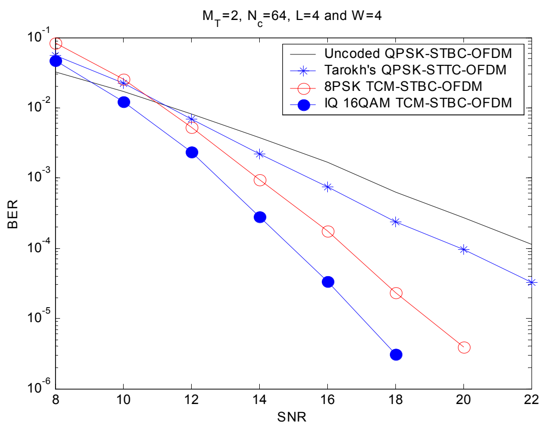

Figure 4: Performance comparison of 8-states SFT codes over four taps frequency selective channels

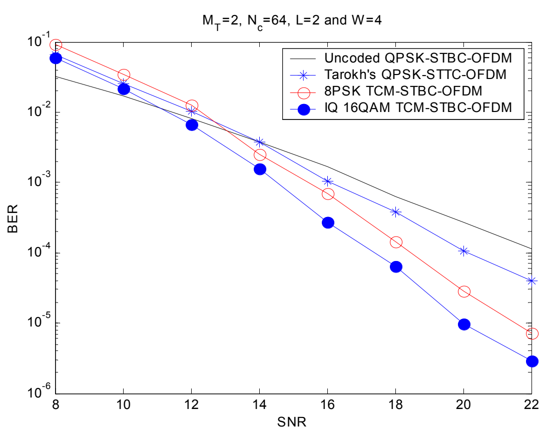

Figure 5: Performance comparison of 8-states SFT codes over two taps frequency selective channels

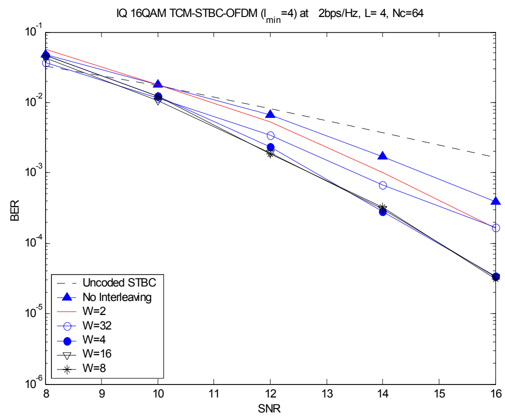

Figure 6: Interleaving effect on IQ-SFT code at 2x1 MIMO channels and at four rays FSC

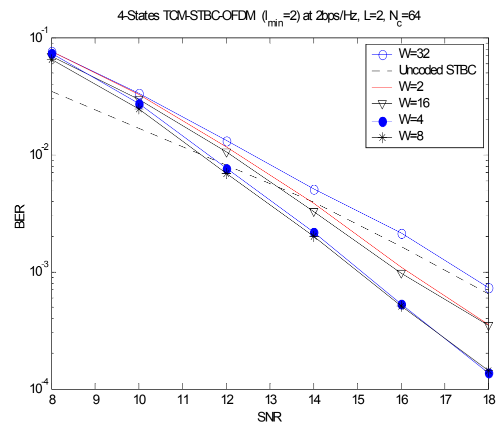

Figure 7: Interleaving effect of 4-states TCM-STBC-OFDM at a two rays frequency selective channel

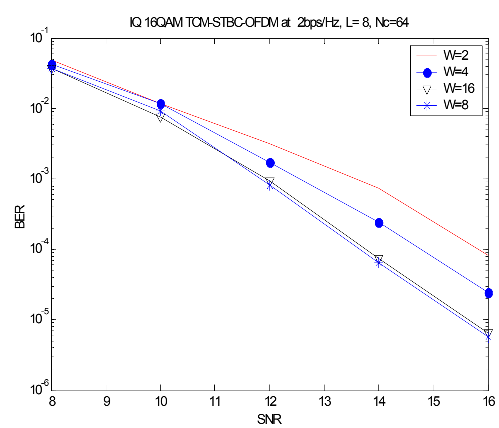

Figure 8: Effect of interleaving on the diversity and gain of the IQ-TCM-STBC-OFDM system at 2x1 MIMO channels and at eight rays FSC