

# Comparative Study of MIMO-OFDM Uplink Scheduling Criteria

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**Abstract:** Uplink scheduling for MIMO-OFDM users is considered in this paper. Each user spatially multiplexes his data over multiple transmit antennas using V-BLAST scheme. This spatial multiplexing (SM) scheme provides high data rates while multi-user diversity obtained from scheduling improves the performance of the uplink system. The greedy scheduler selects one user at a time based on a criterion that minimizes aggregate OFDM symbol error rate. The main results of this study show that the scheduler that maximizes the optimal MIMO capacity doesn't work well for a V-BLAST system. Instead, a scheduler that maximizes the V-BLAST capacity is derived specifically from the V-BLAST detection algorithm. Furthermore, we propose and compare suboptimal schedulers that are based on the received MIMO-OFDM channels before processing. The result shows that selecting the user with maximum minimum MIMO-OFDM channel singular value performs very close to the optimal selection criteria with less processing.

**Keywords:** MIMO-OFDM, Uplink Scheduling, V-BLAST-OFDM, Multiuser Diversity.

## I. INTRODUCTION

Most of the studies on MIMO-OFDM techniques were focused on optimizing the physical layer. However, in a multiuser environment, optimizing the physical layer for each user doesn't necessarily optimize the system performance nor does it take advantage of the statistical independence of the fading channels among the users in a typical mobile environment. Furthermore, different users have different needs in terms of data rates, power limits and Quality of Service (QoS). These requirements make scheduling an important technique for optimizing the performance of a communication system and utilizing the system resources efficiently. Scheduling transmission to the best user leads to a form of selection diversity known as multiuser diversity [1].

In single-input single-output (SISO) systems, where each mobile and the base station have one antenna, it was shown that selecting the user that has the maximum signal to noise ratio (MaxSNR) maximizes the total information capacity of the uplink system [2]. This scheduler is known as MaxSNR scheduling. Over MIMO channels, most of the studies are based on theoretical information capacity [3-5] and on the downlink, which is the broadcast channel from the base station to the mobile unit. It has been shown in [6] that space time block coding (STBC) and scheduling aren't a good match. In fact, scheduling to a user with a single antenna can outperform scheduling using STBC. The reason is that STBC averages the fades while the scheduler tends to benefit from high peaks in the fading channel. In addition, the multiuser diversity obtained from scheduling is much higher than the spatial diversity of STBC, so STBC diversity doesn't add much benefit. On the other hand, spatial multiplexing (SM) schemes are more synergistic with scheduling. This is

because they provide high data rates while the scheduler provides multiuser selection diversity.

In a MIMO system, scheduling can be done to a single user or multiple users. Scheduling to multiple users, i.e. allowing more than one user to transmit or receive at the same time, is shown to be optimal in terms of maximizing system capacity and throughput. In [3], downlink scheduling to multiple users improved the average throughput compared to single user scheduling. Furthermore, the optimal uplink MIMO scheduling based on an information theoretical approach was considered in [7]. They showed that the scheduler should allocate all the power to at most  $M_R$  users, where  $M_R$  is the number of receive antennas at the base station. Also, they found that the optimal power resource allocation is water-filling in space and time. In [4], the authors found that multiuser scheduling reduces the average delay experienced by the users compared to single-user scheduling.

In this paper, we investigate scheduling for uplink MIMO-OFDM systems. These systems are becoming the physical layer standard for high data rate communication systems, such as IEEE 802.16 [10] and IEEE 802.11n [11]. We focus on single user scheduling where each user spatially multiplexes his system using V-BLAST [8]. Although it is not optimal, it is more practical and easily implemented. The search space for best transmission is much less than the multiuser case and a multiuser diversity of order  $K$ , where  $K$  is the number of users, can be achieved. The scheduler selects one user at a time based on a criterion that minimizes the aggregate error rate of the uplink MIMO system. Each user spatially multiplexes their data over the transmit antennas to provide high data rates while the multiuser diversity obtained from scheduling improves the performance of the uplink system. Our main contribution is proposing and comparing several scheduling algorithms for uplink MIMO-OFDM systems. The scheduling criteria are designed to minimize OFDM symbol error rates. In addition, V-BLAST capacity maximization scheduling criteria is proposed and the results show that the scheduler that maximizes optimal MIMO capacity doesn't work well for V-BLAST. The V-BLAST maximum capacity scheduler is derived specifically from its detection algorithm. Furthermore, we investigate the performance of suboptimal scheduling criteria that are based on the MIMO-OFDM channel directly.

## II. MIMO-OFDM CHANNEL MODEL

This section presents a MIMO-OFDM channel model in the frequency domain. The fast Fourier transform (FFT) matrix ( $T$ ) for  $N_c$  subcarriers is defined as:

$$\mathbf{T}_{k,j} = \frac{1}{\sqrt{N_c}} \exp\left[-i \frac{2\pi}{N_c} (k-1)(j-1)\right]; \quad (1)$$

$$k, j = 0, 1, \dots, N_c - 1$$

where  $\mathbf{T}_{k,j}$  is the  $(k,j)$  element of  $\mathbf{T}$ .

A MIMO frequency selective channel (FSC) consists of  $M_T \times M_R$  single-input single-output (SISO) FSCs. OFDM transforms the MIMO-FSC into  $N_c$  parallel MIMO flat fading channels. Let SISO-FSC between the  $n^{\text{th}}$  transmit antenna and the  $m^{\text{th}}$  receive antennas be of length  $L$  and denote it as  $\mathbf{h}_{mn} = [h_0 \ h_1 \ \dots \ h_{L-1}]^T$ . The OFDM channel in the frequency domain between the  $n^{\text{th}}$  transmit antenna and the  $m^{\text{th}}$  receive antenna is:

$$\mathbf{h}_{mn}^f = \mathbf{F} \mathbf{h}_{mn} \quad (2)$$

where  $\mathbf{F}$  is a partition of  $\mathbf{T}$  and it is defined as:

$$\mathbf{F}_{k,l} = \frac{1}{\sqrt{N_c}} \exp\left[-i \frac{2\pi}{N_c} (k-1)(l-1)\right]; \quad (3)$$

$$k = 0, 1, \dots, N_c - 1$$

$$l = 0, 1, \dots, L - 1$$

Assume that  $\mathbf{h} \sim \mathcal{N}_c(\mathbf{0}, \mathbf{C}_h)$ , then the covariance matrix of  $\mathbf{h}^f$  will be:

$$\mathbf{C}_{\mathbf{h}^f} = \mathbf{F} \mathbf{C}_h \mathbf{F}^H \quad (4)$$

The correlation coefficient between subcarriers  $i$  and  $j$  is:

$$\rho_{i,j} = \frac{\mathbf{f}_i \mathbf{C}_h \mathbf{f}_j^H}{\sqrt{\mathbf{f}_i \mathbf{C}_h \mathbf{f}_i^H \mathbf{f}_j \mathbf{C}_h \mathbf{f}_j^H}} \quad (5)$$

where  $\mathbf{f}_i, \mathbf{f}_j$  are the  $i^{\text{th}}$  and  $j^{\text{th}}$  row of  $\mathbf{F}$ , respectively.

### III. SYSTEM MODEL

We consider scheduling a single user at a time. The average SNR and channel statistics are assumed to be the same for all users; they either at a similar distance or strict power control is applied. There are  $K$  users and each user transmits through  $M_T$  transmit antennas and the receiver has  $M_R$  receive antennas. The VBLAST-OFDM system spatially multiplexes  $M_T$  OFDM symbols, each has  $N_c$  subcarriers, over  $M_T$  transmit antennas. A block diagram of a single user VBLAST-OFDM system is shown in Figure 1. At the receiver and after OFDM demodulation, the received signal for user  $k$  and at the  $i^{\text{th}}$  subcarrier is:

$$\mathbf{y}_{k,i} = \mathbf{H}_{k,i} \mathbf{x}_{k,i} + \boldsymbol{\eta}_{k,i} \quad (6)$$

where  $\mathbf{y}_{k,i}$  is an  $M_R \times 1$  received vector,  $\mathbf{H}_{k,i}$  is an  $M_R \times M_T$  MIMO channel matrix for the  $k^{\text{th}}$  user at the  $i^{\text{th}}$  subcarrier,  $\mathbf{x}_{k,i}$  is an  $M_T \times 1$  transmitted symbols from user  $k$  at subcarrier  $i$ , and  $\boldsymbol{\eta}_{k,i}$  is an  $M_R \times 1$  i.i.d complex AWGN vector of zero mean and variance  $N_0/2$  per dimension.

A block diagram of the uplink system with scheduling is shown in Figure 2. The MIMO-OFDM channels are assumed to be independent frequency selective Rayleigh fading MIMO channels of length  $L$  taps, where each coefficient on each tap is an i.i.d complex Gaussian random variable with zero mean. The variance is scaled such that the total fade power is one.

### IV. SCHEDULING CRITERIA

This section describes the MIMO-OFDM scheduling criteria used in our study.

Assuming that an optimal MIMO encoder and decoder are available, the first criterion is to maximize the optimal MIMO capacity. This scheduler finds the minimum MIMO capacity among the OFDM subcarriers for each user. After that, the minimum MIMO capacities for all users are compared and the maximum one is selected. In other words, this scheduler, called *MaxMinMIMOCapc*, selects a user  $k$  whose MIMO-OFDM channel satisfies the following criterion:

$$\max_{k=1, \dots, K} \left\{ \min_{i=1, \dots, N_c} \left\{ \log_2 \left( \det \left( \mathbf{I}_{M_R} + \frac{SNR}{M_T} \mathbf{H}_{k,i} \mathbf{H}_{k,i}^H \right) \right) \right\} \right\} \quad (7)$$

where  $\mathbf{I}_{M_R}$  is the identity matrix and  $\mathbf{A}^H$  is the conjugate-transpose (Hermitian) of  $\mathbf{A}$ . Also,  $K$  is the number of users and  $N_c$  is the number of subcarriers.

For V-BLAST users, selecting the user that has the maximum SNR, as in [2], is not optimal and scheduling based on maximization of MIMO channel capacity as in (7) is also not optimal for V-BLAST since it uses suboptimal detection algorithm.

Since V-BLAST is an open loop system and all layers have the same rate, an outage in capacity will occur if an outage happens in at least one layer. Therefore, the V-BLAST capacity is dominated by the weakest layer and it is given by [9]:

$$C_{VBLAST}^{ZF} = M_T \cdot \min_{m=1, 2, \dots, K, M_T} \left\{ \log_2 \left( 1 + \frac{SNR}{M_T \|\mathbf{W}_{ZF,m}\|^2} \right) \right\} \quad (8)$$

where  $\mathbf{W}_{ZF,m}$  is the ZF projection row for the  $m^{\text{th}}$  layer and  $M_T$  is the number of layers (transmit antennas).

V-BLAST detector performs a series of interference nulling and cancellation operations. At the  $n^{\text{th}}$  stage, the ZF nulling matrix is:

$$\mathbf{W}_{ZF} = (\mathcal{H}_n^H \mathcal{H}_n)^{-1} \mathcal{H}_n^H \quad (9)$$

where  $\mathcal{H}_n$  is the MIMO channel matrix after canceling the  $n-1$  detected layers.

The detected layer at this stage, assume it is the  $m^{\text{th}}$  layer, is the strongest layer which has:

$$\|\mathbf{W}_{ZF,m}\|^2 = \min \left( \text{diag} \left( \left[ \mathcal{H}_n^H \mathcal{H}_n \right]^{-1} \right) \right) \quad (10)$$

and its post-processing SNR is:

$$SNR_m^{ZF} = \frac{SNR}{M_T \|\mathbf{W}_{ZF,m}\|^2} \quad (11)$$

After detecting all layers, the capacity of V-BLAST is determined by the weakest layer as in (8). The norm of the ZF projection row of the weakest layer for user  $k$  is:

$$w_k = \max_{m=1, 2, \dots, K, M_T} \left\{ \|\mathbf{W}_{ZF,m}^k\|^2 \right\} \quad (12)$$

where  $\mathbf{W}_{ZF,m}^k$  is the ZF projection row for the  $i^{\text{th}}$  layer of user  $k$ .

Based on the above analysis, the second criterion for selecting the best user over MIMO-OFDM channels is to first calculate the VBLAST capacity, as defined in (8), at each subcarrier for each user. Then, the minimum value for each user is

found. After that, all users are compared and the maximum one is allowed to transmit. This scheduler is called *MaxMinVBLASTCapc* and it selects user  $k$  whose MIMO-OFDM channel satisfies the following criterion:

$$\max_{k=1,K} \left\{ \min_{i=1,K,N_c} \left\{ C_{VBLAST}^{ZF} (\mathbf{H}_{k,i}) \right\} \right\} \quad (13)$$

To find the above criterion, the scheduler needs to perform some processing on the received MIMO-OFDM channels. To reduce the computations needed, we examine suboptimal schedulers that are based on the received MIMO channels before V-BLAST processing. The first one chooses the user with the largest minimum MIMO-OFDM channel power. It is called *MaxMinSNR* and it selects user  $k$  whose MIMO-OFDM channel satisfies the following criterion:

$$\max_{k=1,K} \left\{ \min_{i=1,K,N_c} \left\{ \text{trace}(\mathbf{H}_{k,i} \mathbf{H}_{k,i}^H) \right\} \right\} \quad (14)$$

This scheduler mimics the optimal scheduler for single antenna systems.

The other scheduler considered in this study measures the eigenspread of the MIMO-OFDM correlation channel matrix at each subcarrier  $(\mathbf{H}_{k,i} \mathbf{H}_{k,i}^H)$ . The first step is to find the maximum eigenspread of the MIMO-OFDM channels among the OFDM subcarriers for each user. After that, the user with the minimum one will be selected. This scheduler is called *MinMaxES* and it selects user  $k$  such that:

$$\min_{k=1,K} \left\{ \max_{i=1,K,N_c} \left\{ s(\mathbf{H}_{k,i} \mathbf{H}_{k,i}^H) \right\} \right\} \quad (15)$$

The eigenspread is defined as  $s(\mathbf{H}_{k,i} \mathbf{H}_{k,i}^H) = \lambda_{\max} / \lambda_{\min}$  where  $\lambda_{\max}$  and  $\lambda_{\min}$  are the largest and smallest eigenvalues of  $\mathbf{H}_k \mathbf{H}_k^H$ . The eigenspread gives insight into the orthogonality of the channels. The smaller the value of  $s$ , the closer the matrix is to be orthogonal. The minimum value of  $s$  is one, and it occurs when the channel matrix is orthogonal.

The last criterion considered in this study is based on the singular values of the MIMO-OFDM channel. Let  $\rho_{\max}$  and  $\rho_{\min}$  be the largest and smallest singular values of  $\mathbf{H}_k$ , then we have the following relation:

$$\rho_{\min} = \frac{\rho_{\max}}{\sqrt{s}} \quad (16)$$

Thus, selecting the user that has the largest minimum singular value of  $\mathbf{H}_{k,i}$  takes into account both the channel power and the eigenspread of  $\mathbf{H}_{k,i}$ . This scheduler is called *MaxMinSV* and it finds first the minimum singularvalue of the MIMO-OFDM channel for each user across the OFDM symbol, and then the user with the maximum one is selected. In other words, it selects user  $k$  such that his MIMO-OFDM channel is:

$$\max_{k=1,K} \left\{ \min_{i=1,K,N_c} \left\{ \rho_{\min}(\mathbf{H}_{k,i}) \right\} \right\} \quad (17)$$

The performance of these algorithms are compared to round-robin (RR) scheduling which is a passive algorithm that cycles equally through all users irrespective of their channel status and it doesn't obtain any multiuser diversity.

## V. SIMULATION RESULTS

The simulation result of the uplink MIMO-OFDM scheduling is presented in this section. The aggregate OFDM symbol error rate (SER) performance of the uplink system with scheduling is shown in Figure 3. The number of user is ten and the spectral efficiency is 8bps/Hz. The simulation is done over 4×4 frequency selective MIMO channels of length four and with 64 OFDM subcarriers. The scheduling algorithm selects the best user among ten users (Greedy). Thus, the maximum available multi-user diversity is ten. The MaxSNR scheduler captures very little multiuser diversity and it gains around 1dB compared to the RR algorithm. On the other hand, the best scheduler is the one that maximizes V-BLAST capacity among the MIMO-OFDM channels (MaxMinVBLASTCapc). The gain is 12dB compared to RR and 8dB compared to MaxMinMIMOCapc. The MinMaxES and the MaxMinSV schedulers capture most of the multiuser diversity but MaxMinSV provides more gain since it takes into account the power of the MIMO-OFDM channel. They perform very close to MaxMinVBLASTCapc which has more diversity at high SNR (sharper slope). The results in this figure also show that using maximum MIMO capacity as the scheduling criterion doesn't perform very well for V-BLAST. The reason is the suboptimality of the V-BLAST detection algorithm.

## VI. CONCLUSION

This paper proposed and compared several uplink scheduling algorithms for MIMO-OFDM users based on V-BLAST scheme. The scheduler selects one user at a time and each user spatially multiplexes his data over the transmit antennas. V-BLAST-OFDM capacity maximizing scheduler that minimizes the aggregate OFDM symbol error rate of the uplink system is proposed in this paper. A suboptimal scheduler that performs very close to the optimal is to schedule to the user that has the maximum minimum singular value of the MIMO-OFDM channel. Furthermore, the results showed that scheduling based on maximum MIMO capacity is not optimal for V-BLAST. This is due to the suboptimality of the V-BLAST detection algorithm while the MIMO capacity criterion assumes optimal encoding and decoding.

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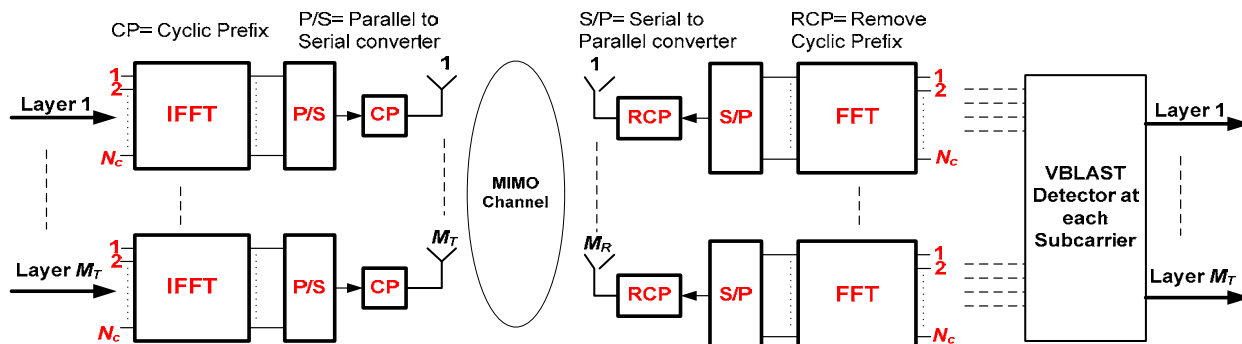


Figure 1: Block Diagram of VBLAST-OFDM Single User System

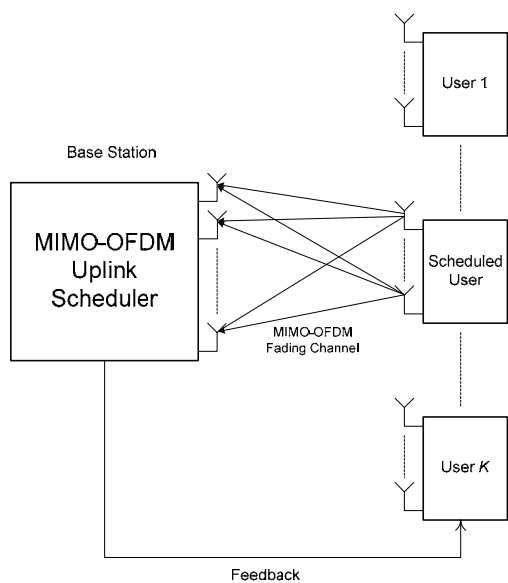


Figure 2: Block Diagram of Uplink MIMO Scheduling

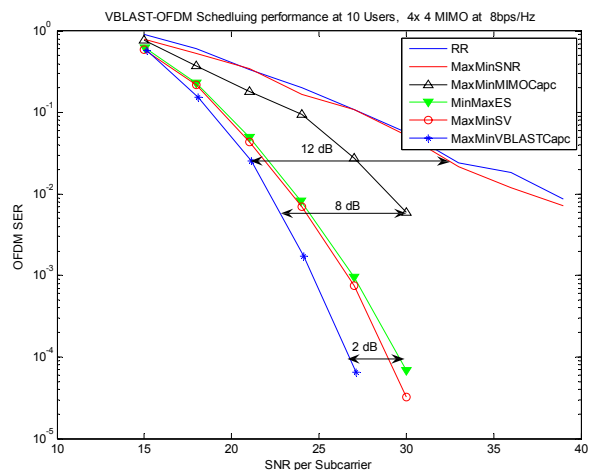


Figure 3: Aggregate OFDM SER of 4x4 QPSK MIMO-OFDM uplink scheduling at 64 subcarriers and over FSC of length four