Continuous Phase Modulation (CPM)

EE571
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Reading Material

- Proakis section 3.3
- Haykin sections 6.3 and 6.5

CPM

- CPM is a modulation scheme with memory
- In memory less modulations, such as PSK and FSK, the modulated signal switch from one value to the other independent from the previous value.
- This abrupt switching results in relatively large spectral side lobes outside the main spectral band of the signal.

Continuous Phase Frequency Shift Keying (CPFSK)

- CPFSK is a special case of CPM.
- The conventional FSK signal is generated by shifting the carrier by an amount $m\Delta f$, where $1 \leq m \leq M$.
- The switching from one frequency to another will result in large spectral side lobes.
- The solution is to modulate a single carrier whose frequency is changed continuously.
- This is called CPFSK

CPFSK

- To represent a CPFSK signal, we begin with a PAM signal

$$d(t) = \sum_n I_n g(t - nT)$$

- Where $\{I_n\}$ denoted the sequence of amplitudes obtained by mapping k-bit blocks of binary digits from the information sequence $\{a_n\}$ into the amplitude levels ±1, ±3, …, ±(M-1)
- *g(t)* is a rectangular pulse of amplitude 1/2T and duration T seconds.
- The signal d(t) is used to frequency modulate the carrier.

CPFSK

- The equivalent lowpass waveform *v(t)* is expressed as:

$$v(t) = \sqrt{\frac{2E}{T}} e^{j\left[4\pi T f_d \int_{-\infty}^{t} d(\tau) d\tau + \phi_0\right]}$$

- Where f_d is the peak frequency deviation and ϕ_0 is the initial phase of the carrier.
- The carrier modulated signal can be expressed as:

$$s(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + \phi(t; I) + \phi_0\right]$$

- Where $\phi(t; I) = 4\pi T f_d \int_{-\infty}^{t} d(\tau) d\tau$

$$= 4\pi T f_d \int_{-\infty}^{t} \left[\sum_n I_n g(t - nT)\right] d\tau$$

Note that d(t) contains discontinuities,
the integral of d(t) is continuous

CPFSK

Thus, over the interval *nT ≤ t ≤ (n+1)T*

$$\phi(t;I) = 2\pi f_d T \sum_{k=-\infty}^{n-1} I_k + 2\pi f_d T q(t - nT) I_n$$
$$= \theta_n + 2\pi h I_n q(t - nT)$$

where

$$h = 2 f_d T$$ → Modulation Index

$$\theta_n = \pi h \sum_{k=-\infty}^{n-1} I_k$$ → Represents the accumulation (Memory) of all symbols up to time *(n-1)T*

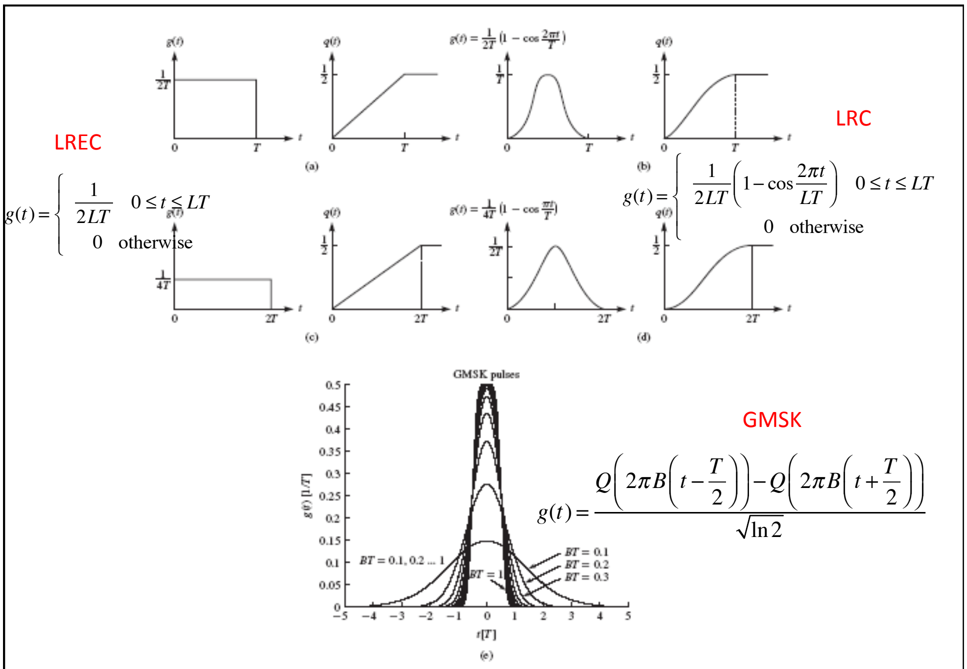
$$q(t) = \begin{cases} 0 & t < 0 \\ \frac{t}{2T} & 0 \le t \le T \\ \frac{1}{2} & t > T \end{cases}$$

Continuous Phase Modulation (CPM)

In general, the carrier phase is:

$$\phi(t;I) = 2\pi \sum_{k=-\infty}^{n} I_k h_k q(t - nT), \quad nT \le t \le (n+1)T$$

Where $\{I_k\}$ is the sequence of M-ary information
$\{h_k\}$ is a sequence of modulation indices
q(t) is some normalized waveform shape.



LREC

$$g(t)=\begin{cases}\frac{1}{2LT} & 0\le t\le LT\\ 0 & \text{otherwise}\end{cases}$$

LRC

$$g(t)=\begin{cases}\frac{1}{2LT}\left(1-\cos\frac{2\pi t}{LT}\right) & 0\le t\le LT\\ 0 & \text{otherwise}\end{cases}$$

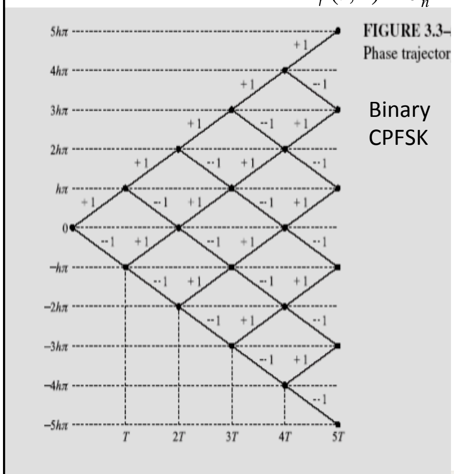
GMSK

$$g(t)=\frac{Q\left(2\pi B\left(t-\frac{T}{2}\right)\right)-Q\left(2\pi B\left(t+\frac{T}{2}\right)\right)}{\sqrt{\ln 2}}$$

FIGURE 3.3–4
Pulse shapes for full-response CPM (a, b) and partial-response CPM (c, d), and GMSK (e).

Phase Trajectory

$$\phi(t;I)=\theta_n+2\pi h I_n q(t-nT)$$



Binary CPFSK

Quaternary CPFSK

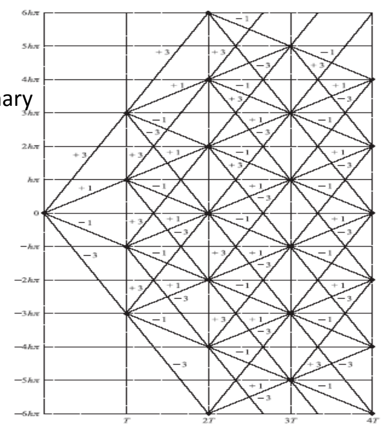
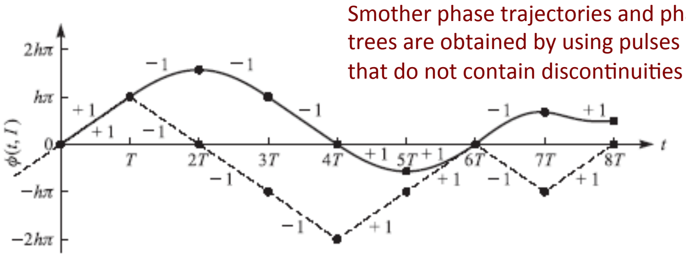


FIGURE 3.3–6
Phase trajectory for quaternary CPFSK.

Phase Trajectory



Smother phase trajectories and phase trees are obtained by using pulses that do not contain discontinuities

FIGURE 3.3–7
Phase trajectories for binary CPFSK (dashed) and binary, partial-response CPM based on raised cosine pulse of length $3T$ (solid). [*Source:* Sundberg (1986), © 1986 IEEE]

Minimum Shift Keying (MSK)

- MSK is a special case of Binary CPFSK in which h=1/2 and g(t) is a rectangular pulse of duration T
- The phase of the carrier in the interval *nT ≤ t ≤ (n+1)T*

$$\phi(t;I) = \frac{1}{2}\pi \sum_{k=-\infty}^{n-1} I_k + \pi I_n q(t-nT)$$
$$= \theta_n + \frac{1}{2}\pi I_n \left(\frac{t-nT}{T}\right)$$

- The modulated carrier is

$$s(t) = A\cos\left[2\pi f_c t + \theta_n + \frac{1}{2}\pi I_n \left(\frac{t-nT}{T}\right)\right]$$
$$= A\cos\left[2\pi\left(f_c + \frac{1}{4T} I_n\right)t - \frac{1}{2}n\pi I_n + \theta_n\right]$$

Minimum Shift Keying (MSK)

Since $I_n=\pm1$, MSK can be expressed as a sinusoid having one of two possible frequencies:

$$f_1 = f_c - \frac{1}{4T}$$

$$f_2 = f_c + \frac{1}{4T}$$

Note that $\Delta f=f_2-f_1=1/2T$, which is the minimum frequency separation required to ensure orthogonality of FSK signals

And the modulated carrier signal is:

$$s(t) = A\cos\left[2\pi f_i t + \theta_n + \frac{1}{2}\pi(-1)^{i-1}\right]$$

Signal Space Diagram of MSK

- Using trigonometric identities

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

- Where

$$s_I(t) = \pm\sqrt{\frac{2E_b}{T_b}}\cos\left(\frac{\pi}{2T_b}t\right), \qquad -T_b \le t \le T_b$$

$$s_Q(t) = \pm\sqrt{\frac{2E_b}{T_b}}\sin\left(\frac{\pi}{2T_b}t\right), \qquad 0 \le t \le 2T_b$$

MSK Power Spectral Density

- The symbol shaping function of the in-phase is

$$g(t)=\begin{cases}\sqrt{\frac{2E_b}{T_b}}\cos\left(\frac{\pi t}{2T_b}\right), & -T_b \le t \le T_b \\ 0, & \text{otherwise}\end{cases}$$

- The energy spectral density of this symbol shaping function is:

$$\psi_g(t)=\frac{32E_bT_b}{\pi^2}\left[\frac{\cos(2\pi T_b f)}{16T_b^2 f^2-1}\right]^2$$

- Hence, the power spectral density of the in-phase is $\psi_g(f)/2T_b$

MSK Power Spectral Density

- The symbol shaping function of the quadrature is

$$g(t)=\begin{cases}\sqrt{\frac{2E_b}{T_b}}\sin\left(\frac{\pi t}{2T_b}\right), & 0 \le t \le 2T_b \\ 0, & \text{otherwise}\end{cases}$$

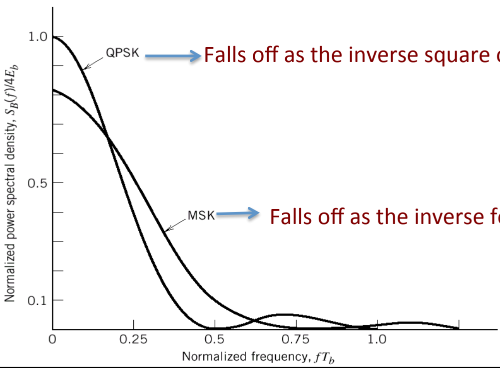
- Similar to the in-phase, the energy spectral density of this symbol shaping function is:

$$\psi_g(t)=\frac{32E_bT_b}{\pi^2}\left[\frac{\cos(2\pi T_b f)}{16T_b^2 f^2-1}\right]^2$$

MSK Power Spectral Density

Since the in-phase and quadrature are independent, the Baseband power spectral density is:

$$S_B(f) = 2\left[\frac{\Psi_g(f)}{2T_b}\right] = \frac{32E_b}{\pi^2}\left[\frac{\cos(2\pi T_b f)}{16T_b^2 f^2 - 1}\right]^2$$

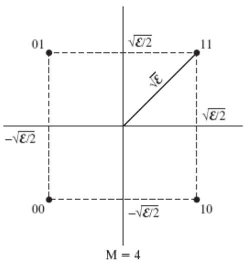


QPSK → Falls off as the inverse square of frequency

$$S_B(f) = 2E\,\text{sinc}^2(Tf) = 4E_b\,\text{sinc}^2(2T_b f)$$

MSK → Falls off as the inverse fourth power of frequency → lower side lobes

Recall QPSK



Conventional QPSK

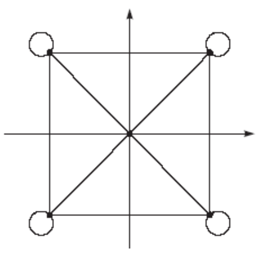


FIGURE 3.3–13
Possible phase transitions in QPSK signaling.

180° phase changes cause abrupt changes in the signal, resulting in large spectral side lobes

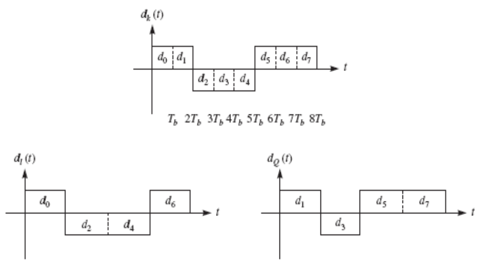


FIGURE 3.3–12
The in-phase and quadrature components for QPSK.

Offset QPSK (OQPSK)

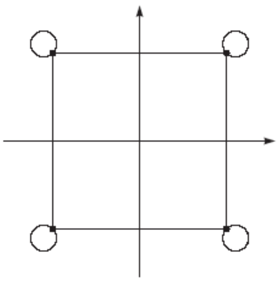


FIGURE 3.3–15
Phase transition diagram for OQPSK signaling.

The in-phase and quadrature components are misaligned by T_b

This prevents phase transitions of 180°
The maximum phase shifts are ±90°

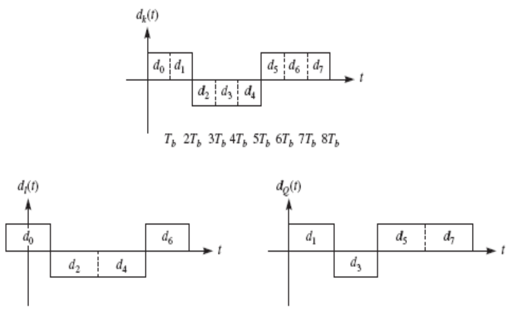


FIGURE 3.3–14
The in-phase and quadrature components for OQPSK signaling.

OQPSK

- The OQPSK signal can be written as

$$s(t) = A\left[\left(\sum_{n=-\infty}^{\infty} I_{2n}g(t-2nT)\right)\cos 2\pi f_c t + \left(\sum_{n=-\infty}^{\infty} I_{2n+1}g(t-2nT-T)\right)\sin 2\pi f_c t\right]$$

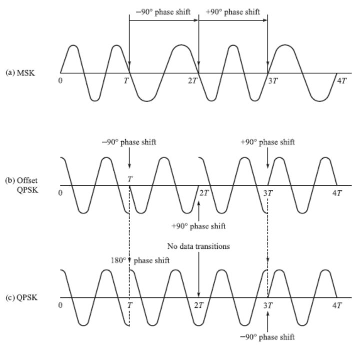


FIGURE 3.3–17
MSK, OQPSK, and QPSK signals.

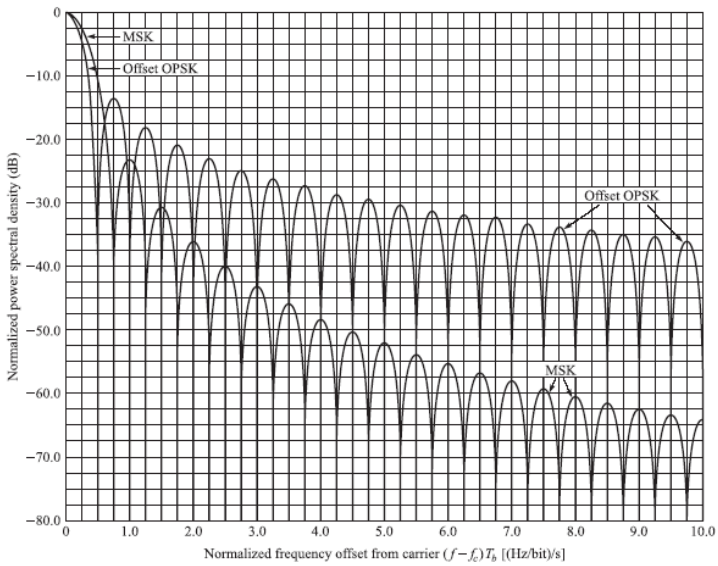


FIGURE 3.4–4
Power spectral density of MSK and OQPSK. [*Source:* Gronemeyer and McBride (1976); © IEEE.]