The Transformer

Ch. 9.10 and 9.11

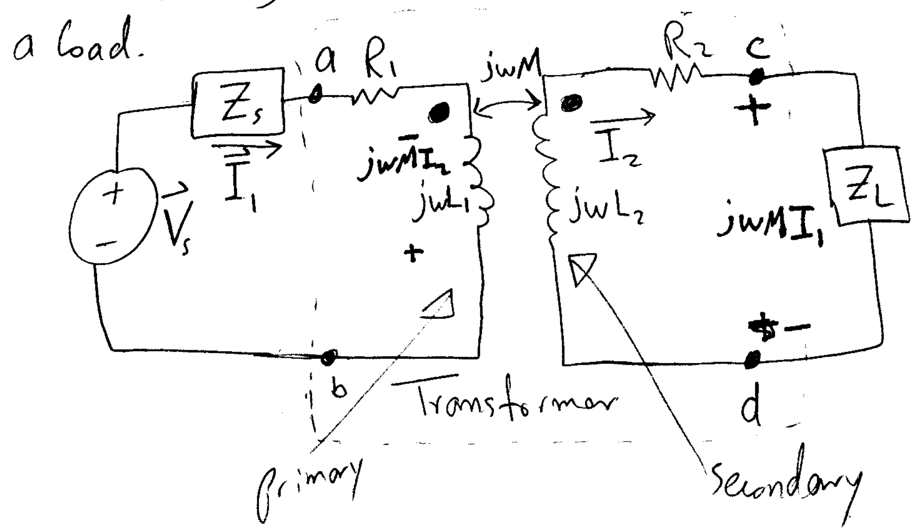
Two common Types

1- Linear transformer ⇒ Communication Circuits
Block dc signals and Match impedances.

2- Ideal transformer ⇒ model ferromagnetic transformers in power systems.

9.10 Linear Transformer

The following system use a transformer to connect a source to a load.



R_1 = the resistance of the primary winding.
R_2 = the resistance of the secondary winding
L_1 = self-inductance of the primary winding
L_2 = self-inductance of the secondary winding.
M = mutual inductance.

$$\vec{V}_s = \vec{I}_1 Z_s + \vec{I}_1 R_1 + \vec{I}_1 j\omega L_1 - j\omega M I_2$$

$$= (Z_s + R_1 + j\omega L_1)\vec{I}_1 - j\omega M I_2 \longrightarrow (1)$$

$$0 = -j\omega M I_1 + (R_2 + j\omega L_2 + Z_L)\vec{I}_2 \longrightarrow (2)$$

Let $Z_{11} = Z_s + R_1 + j\omega L_1$ and $Z_{22} = R_2 + j\omega L_2 + Z_L$

Solving the above two equations:

$$\vec{I}_1 = \frac{Z_{22}}{Z_{11}Z_{22}+w^2M^2}\vec{V}_s$$

and $$\vec{I}_2 = \frac{jwM}{Z_{11}Z_{22}+w^2M^2}\vec{V}_s = \frac{jwM}{Z_{22}}\vec{I}_1$$

— The impedence seen by the internal source voltage is

$$\frac{V_s}{I_1} = Z_{int} = \frac{Z_{11}Z_{22}+w^2M^2}{Z_{22}} = Z_{11} + \frac{w^2M^2}{Z_{22}}$$

Primary winding

Secondary winding

— The impedence at the terminals of the source

Reflected Impedance

$$Z_{ab} = Z_{int} - Z_s$$
$$= Z_{11} + \frac{w^2M^2}{Z_{22}} - Z_s = R_1 + jwL_1 + \overbrace{\frac{w^2M^2}{(R_2+jwL_2+Z_L)}}^{\text{Reflected Impedance}}$$

Load impedence only appears here.

— Reflected Impedance

The Third Term in Zab is called the reflected impedance.

$$Z_r = \frac{w^2M^2}{\underbrace{R_2+jwL_2+Z_L}_{Z_{22}}}$$ ⟹ it shows how the primary side sees the secondary side coil and load impedance.

Transformer ch.9

Let $Z_L = R_L + jX_L$

$$\Rightarrow Z_r = \frac{\omega^2 M^2}{R_2 + R_L + j(\omega L_2 + X_L)}$$

$$= \frac{\omega^2 M^2 \left[(R_2 + R_L) - j(\omega L_2 + X_L) \right]}{(R_2 + R_L)^2 + (\omega L_2 + X_L)^2}$$

$$= \frac{\omega^2 M^2}{|Z_{22}|^2} \left[(R_2 + R_L) - j(\omega L_2 + X_L) \right]$$

conjugate

$$= \frac{\omega^2 M^2}{|Z_{22}|^2} Z_{22}^*$$

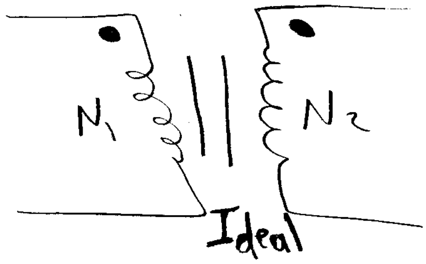
Scaler factor.

Solve Example 9.13

9.11 The Ideal Transformer

Assumptions:

1. The coefficient of coupling is unity ($k=1$)
2. The self-inductance of each coil is infinite ($L_1=L_2=\infty$)
3. The coil losses, ~~due to~~ are negligible.



The graphic symbol for an ideal transformer.

Note: The core is a magnetic Material ||

* Voltage relation for an ideal transformer

$$\boxed{\frac{\vec{V}_1}{N_1} = \frac{\vec{V}_2}{N_2}}$$

* Current Relationship for an ideal transformer

$$\boxed{\vec{I}_1 N_1 = \vec{I}_2 N_2}$$

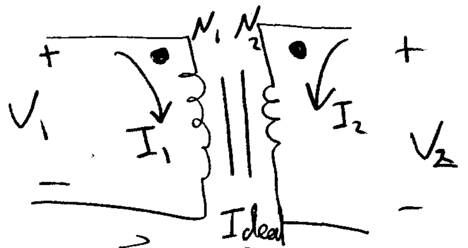
Dot convention for ideal transformers

- If the coil voltages $\vec{V}_1$ and $\vec{V}_2$ are both positive or negative at the dot-marked terminal, use a plus sign $\Rightarrow \left|\frac{V_1}{N_1}\right| = \left|\frac{V_2}{N_2}\right|$

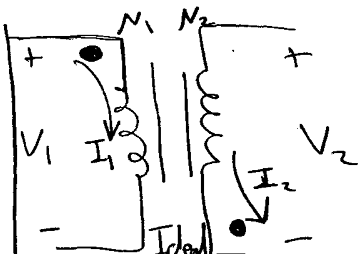
  orthemwise.
  $$\Rightarrow \left|\frac{V_1}{N_1}\right| = -\left|\frac{V_2}{N_2}\right|$$

- If the coil currents $\vec{I}_1$ and $\vec{I}_2$ are both directed into ~~the~~ or out ~~of~~ the dot marked terminal, use a minus sign $\Rightarrow |I_1N_1| = -|I_2N_2|$

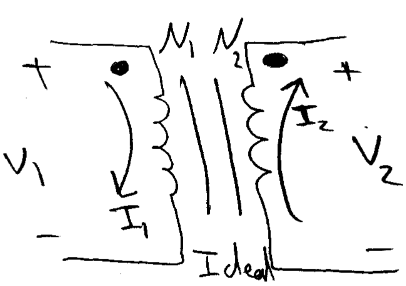
  otherwise $\Rightarrow |I_1N_1| = |I_2N_2|$



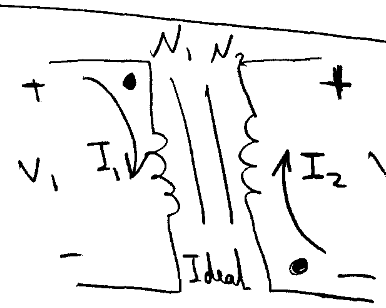
$$\frac{\vec{V}_1}{N_1} = \frac{\vec{V}_2}{N_2}$$
$$N_1\vec{I}_1 = -N_2\vec{I}_2$$



$$\frac{\vec{V}_1}{N_1} = -\frac{\vec{V}_2}{N_2}$$
$$N_1\vec{I}_1 = N_2\vec{I}_2$$



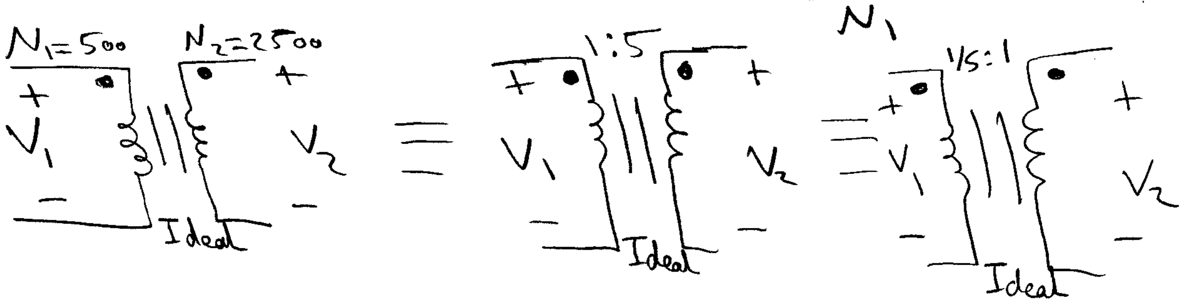
$$\frac{\vec{V}_1}{N_1} = \frac{\vec{V}_2}{N_2}$$



$$\frac{\vec{V}_1}{N_1} = -\frac{\vec{V}_2}{N_2}$$

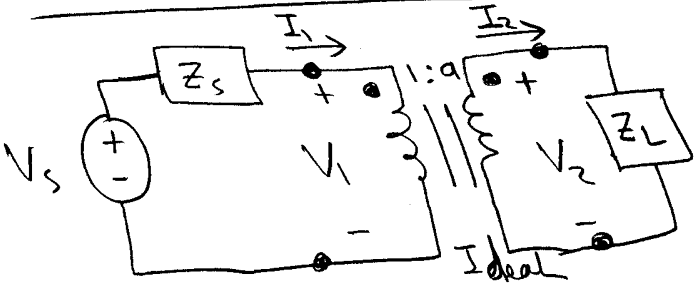
Turn Ratio

The turn ratio is defined as $a = \frac{N_2}{N_1}$



Solve Example 9.44 in Textbook

What is the Impedance seen by the source?



since $\vec{V}_1 = \frac{\vec{V}_2}{a}$

and $\vec{I}_1 = a\vec{I}_2$

$$\Rightarrow Z_{in} = \frac{V_1}{I_1} = \frac{1}{a^2}\frac{V_2}{I_2}$$

$$\Rightarrow \boxed{Z_{in} = \frac{1}{a^2} Z_L}$$

impedance seen by the source

load impedance

∘ Ideal transformers can be used for impedance matching either to lower or raise the impedance level.