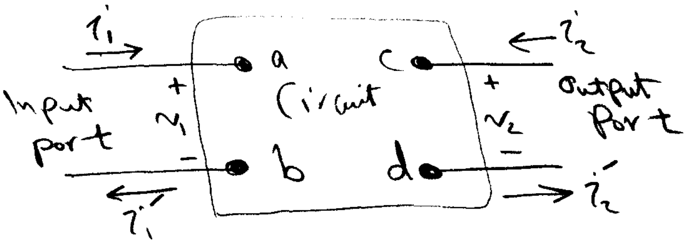
Ch. 18 Two-Port Circuits



The Z Parameters "The impedance Parameter"

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \ \Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \ \Omega$$

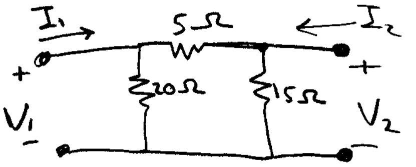
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \ \Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \ \Omega$$

Then,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Example



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{(20)(20)}{40} = 10\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Rightarrow V_2 = \frac{V_1(15)}{15+5} = 0.75 V_1$$

$$\Rightarrow Z_{21} = \left.\frac{0.75V_1}{\left(\frac{V_1}{10}\right)}\right|_{I_2=0} = 7.5\ \Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{(15)(25)}{40} = 9.375\ \Omega$$

For Z_{12}, when port 1 is open, I_1 is zero and the voltage is

$$V_1 = \frac{V_2}{5+20}(20) = 0.8 V_2$$

The Y parameters "The admittance"

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \ S$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \ S$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \ S$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \ S$$

Then,

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

The a Parameters "transmission"

$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}, \quad a_{12} = \left.\frac{-V_1}{I_2}\right|_{V_2=0} \Omega$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} S, \quad a_{22} = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$

Then,

$$V_1 = a_{11} V_2 - a_{12} I_2$$
$$I_1 = a_{21} V_2 - a_{22} I_2$$

The b parameters "Transmission"

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}, \quad b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0} \Omega$$

$$b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0} S, \quad b_{22} = \left.-\frac{I_2}{I_1}\right|_{V_1=0}$$

Then,

$$V_2 = b_{11} V_1 - b_{12} I_1$$
$$I_2 = b_{21} V_1 - b_{22} I_1$$

The h Parameters "hybrid"

$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}\ \Omega$, $h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$

$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$, $h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}\ S$

Then, $V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

The g Parameters "hybrid"

$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0}\ S$, $g_{12} = \frac{I_1}{I_2}\Big|_{V_1=0}$

$g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$, $g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0}\ \Omega$

See Example 18.2

- What is the releation between the two-port parameters?

You can convert between any Parameter to another.

See table 18.1 .

Solve ~~example~~ 18.3

Reciprocal Two-Port Circuit

The circuit is reciprocal ~~"symmetric"~~ if

$Z_{12} = Z_{21}$

$Y_{12} = Y_{21}$

$\Delta a = 1$

$\Delta b = 1$

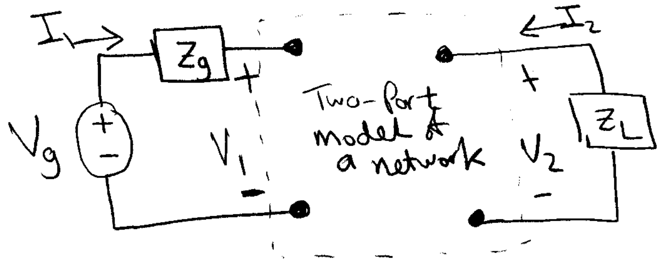
$h_{12} = -h_{21}$

$g_{12} = -g_{21}$

$\Delta a = a_{11}a_{22} - a_{12}a_{21}$

$\Delta b = b_{11}b_{22} - b_{12}b_{21}$

18.3 Analysis of the Terminated Two-Port Circuit



The analysis involves finding the following six characteristics.

1. $Z_{in} = \frac{V_1}{I_1} \equiv$ the input impedance
2. $I_2 \equiv$ the output current
3. $(V_{Th}, Z_{Th}) \equiv$ Thevenin Voltage and Impedance
4. $I_2 / I_1 \equiv$ current gain
5. $\frac{V_2}{V_1} \equiv$ voltage gain
6. $\frac{V_2}{V_g} \equiv$ voltage gain

The Z Parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad —(1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad —(2)$$

$$V_1 = V_g - I_1 Z_g \quad —(3)$$

$$V_2 = -I_2 Z_L \quad —(4)$$

(1) The input impedance

To Find the input impedance $Z_{in} = \frac{V_1}{I_1}$

$\Rightarrow$ From (4) and (2)

$$-I_2 Z_L = Z_{21} I_1 + Z_{22} I_2$$

$$\Rightarrow I_2 = \frac{-Z_{21} I_1}{Z_L + Z_{22}} \quad —(5)$$

Subtitute into (1),

$$V_1 = Z_{11} I_1 + Z_{12}\left(\frac{-Z_{21} I_1}{Z_L + Z_{22}}\right)$$

$$\frac{V_1}{I_1} = Z_{in} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_L + Z_{22}}$$

(2) The output current

From (3) and (1)

$$V_g - I_1 Z_g = Z_{11} I_1 + Z_{12} I_2$$

$$\therefore I_1 = \frac{V_g - Z_{12} I_2}{Z_{11} + Z_g} \quad —(6)$$

Subtitute into (5)

$$I_2 = \frac{-Z_{21}}{Z_L + Z_{22}}\left(\frac{V_g - Z_{12} I_2}{Z_{11} + Z_g}\right)$$

$$I_2 (Z_L + Z_{22})(Z_{11} + Z_g) = -Z_{21} V_g + Z_{21} Z_{12} I_2$$

$$\therefore I_2 = \frac{-Z_{21} V_g}{(Z_L + Z_{22})(Z_{11} + Z_g) - Z_{12} Z_{21}}$$

(3) Thevenin Voltage and Impedance

The Thevenin Voltage equals V_2 when $I_2 = 0$

From (2) $\Rightarrow$ $V_2 = Z_{21} I_1 = Z_{21} \frac{V_1}{Z_{11}}$

From (3) $V_1 = V_g - I_1 Z_g$

From (6) $I_1 = \frac{V_g - Z_{12} I_2}{Z_{11} + Z_g}$ ($I_2 \to 0$)

$$\therefore V_2 = \frac{Z_{21}}{Z_{11}}\left(V_g - \frac{V_g Z_g}{Z_{11} + Z_g}\right)$$

$$= \frac{Z_{21}}{Z_{11}}\left(\frac{Z_{11} V_g + Z_g V_g - V_g Z_g}{Z_{11} + Z_g}\right)$$

$V = Z_{21} \ldots V_g$

The Thevenin Impedance is defined as $\frac{V_2}{I_2}$ when V_g is shortend.

Thus, From ③ $V_1 = -I_1 Z_g$

and From ⑥ $I_1 = \frac{-Z_{12} I_2}{Z_{11} + Z_g}$

Subtitute into ②,

$$V_2 = Z_{21}\left(\frac{-Z_{12} I_2}{Z_{11}+Z_g}\right) + Z_{22} I_2$$

$$\frac{V_2}{I_2} = Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11} + Z_g}$$

④ The current gain

$$\frac{I_2}{I_1} = \frac{-Z_{21}}{Z_L + Z_{22}}$$

⑤ The voltage gain (V_2/V_1)

From ② and ④

$$V_2 = Z_{21} I_1 + Z_{22}\left(\frac{-V_2}{Z_L}\right) \quad \text{—— (7)}$$

From ① and ④

$$I_1 = \frac{V_1}{Z_{11}} + \frac{Z_{12} V_2}{Z_{11} Z_L} \quad \text{—— (8)}$$

Solve ⑦ and ⑧, we get

$$\frac{V_2}{V_1} = \frac{Z_{21} Z_L}{Z_{11} Z_L + Z_{11} Z_{22} - Z_{12} Z_{21}}$$

$$= \frac{Z_{21} Z_L}{Z_{11} Z_L + \Delta Z}$$

⑥ Voltage gain (V_2/V_g)

Combine ① and ③

$$I_1 = \frac{Z_{12} V_2}{Z_L (Z_{11} + Z_g)} + \frac{V_g}{Z_{11} + Z_g} \quad \text{—— (9)}$$

Combine ⑨ and ④ and ②

$$V_2 = \frac{Z_{21} Z_{12} V_2}{Z_L (Z_{11} + Z_g)} + \frac{Z_{21} V_g}{Z_{11} + Z_g} - \frac{Z_{22}}{Z_L} V_2 .$$

$$\Rightarrow \frac{V_2}{V_g} = \frac{Z_{21} Z_L}{(Z_{11} + Z_g)(Z_{22} + Z_L) - Z_{12} Z_{21}}$$

A summary of all equation for the two-port parameters is shown in Table 18.2