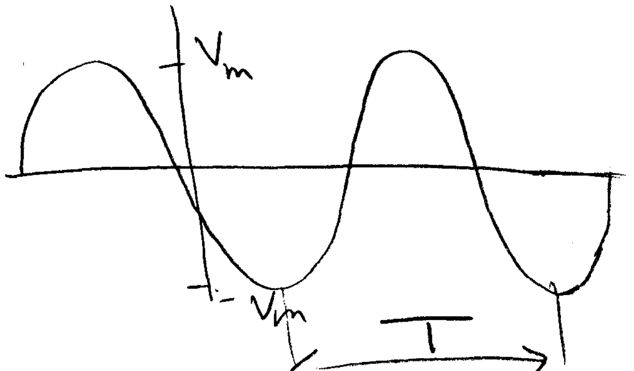
Introduction to 3-phase Circuits
Sections 11.1 — 11.3

Background

- The Sinusoidal Source
  - sinusoidal voltage source
  - sinusoidal current source.



$$v(t) = \underbrace{V_m}_{\text{Amplitude}} \cos(\underbrace{\omega}_{\text{frequency}} t + \underbrace{\phi}_{\text{phase angle}})$$

frequency $\omega = 2\pi f = \frac{2\pi}{T}$

phase angle "The value of $v(0) = V_m \cos(\phi)$"

$T \equiv$ Period (seconds)

$f = \frac{1}{T}$ (Hz)

- rms $\equiv$ Root Mean Square

$$V_{rms} = \frac{V_m}{\sqrt{2}} \leftarrow \text{Amplitude}$$

- The phasor
  - Euler's identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta \implies \begin{aligned} \cos\theta &= R\{e^{j\theta}\} \\ \sin\theta &= I\{e^{j\theta}\} \end{aligned}$$

  - For a sinusoidal voltage

$$\begin{aligned} v(t) &= V_m \cos(\omega t + \phi) \\ &= V_m R\{e^{j(\omega t+\phi)}\} \\ &= V_m R\{e^{j\omega t} e^{j\phi}\} \end{aligned}$$

$$v(t) = R\{\underbrace{V_m e^{j\phi}}_{\text{complex Number "phasor representation"}} e^{j\omega t}\}$$

- Phasor transform

Phasor transform

Polar Form $\longrightarrow \vec{V} = V_m e^{j\phi} = \mathbf{P}\{V_m \cos(\omega t + \phi)\}$

Rectangular Form $\longrightarrow \vec{V} = V_m \cos\phi + j V_m \sin\phi$

the phasor transform transfers the sinusoidal function from the time domain to the complex-number domain.

- Angle Notation

$$\vec{V} = V_m e^{j\phi} = V_m \angle \phi^{\circ}$$

change from Radian to degrees.

- Polar Form Vs. Rectangular Form

Let $\vec{V} = x + jy$

$\Rightarrow \quad V_m = \sqrt{x^2 + y^2}$

and $\quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$

- Multiplication and Division

Let $\vec{V_1} = V_1 e^{j\phi_1} = V_1 \angle \phi_1$

$\vec{V_2} = V_2 e^{j\phi_2} = V_2 \angle \phi_2$

$\therefore \quad \vec{V_1} \cdot \vec{V_2} = V_1 V_2 \angle \phi_1 + \phi_2$

$$\frac{\vec{V_1}}{\vec{V_2}} = \frac{V_1}{V_2} \angle \phi_1 - \phi_2$$

- Addition and Subtraction

$\vec{V} = \vec{V_1} + \vec{V_2}$

$= V_1 \angle \phi_1 + V_2 \angle \phi_2$

① Change to Rectangular "Complex" Numbers $\Longrightarrow$ $= V_1 \cos\phi_1 + j V_1 \sin\phi_1 + V_2 \cos\phi_2 + j V_2 \sin\phi_2$

② Add complex Number $\Longrightarrow$ $= \underbrace{[V_1 \cos\phi_1 + V_2 \cos\phi_2]}_{\text{Real Part}} + j \underbrace{\{V_1 \sin\phi_1 + V_2 \sin\phi_2\}}_{\text{Imaginary Part}}$

$\vec{V} = V \angle \phi$

11.1 Balanced Three-Phase Voltages

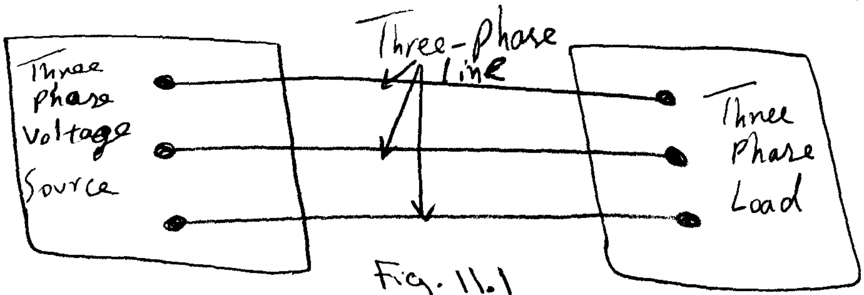
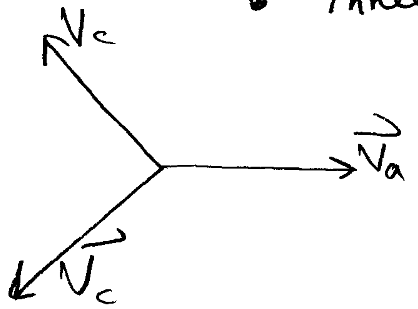


Fig. 11.1

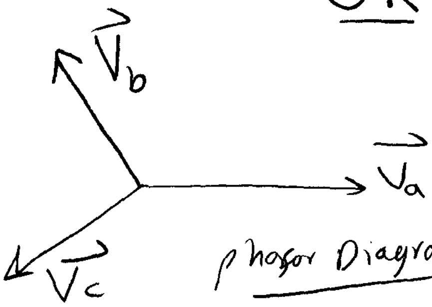
– A Blanced Three-Phase Voltages Consist of

- Three sinusoidal voltages ~~that~~.



$$\left.\begin{aligned}\vec{V}_a &= V_m\angle 0^\circ\\ \vec{V}_b &= V_m\angle -120^\circ\\ \vec{V}_c &= V_m\angle +120\end{aligned}\right\}\text{Positive Phase Sequence}$$

OR



phasor Diagram

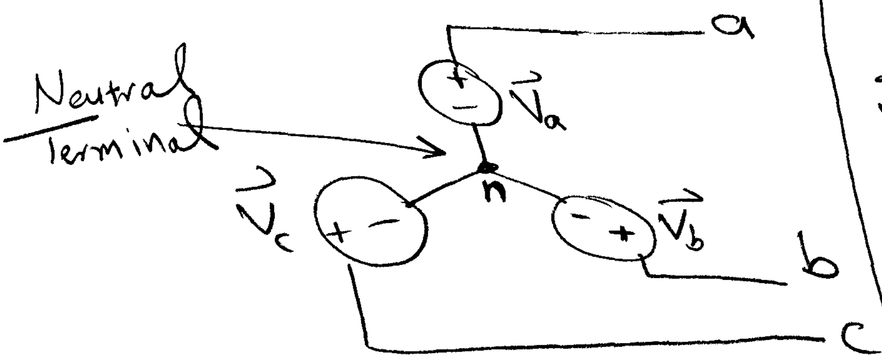
$$\left.\begin{aligned}\vec{V}_a &= V_m\angle 0^\circ\\ \vec{V}_b &= V_m\angle +120^\circ\\ \vec{V}_c &= V_m\angle -120^\circ\end{aligned}\right\}\text{Negative Phase Sequence}$$

– Important Notes:

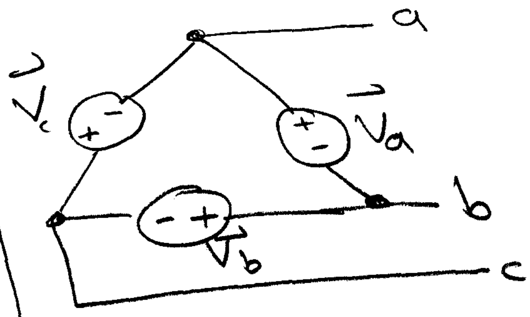
- The sum of balanced-three phase voltages is zero $\Rightarrow \vec{V}_a + \vec{V}_b + \vec{V}_c = 0 \longrightarrow$ Verify from Phasor Diagram
- If we know the phase sequence and one voltage in the set, we know the entire set.

11.2 Three-Phase Voltage Sources

Y- Connected Source

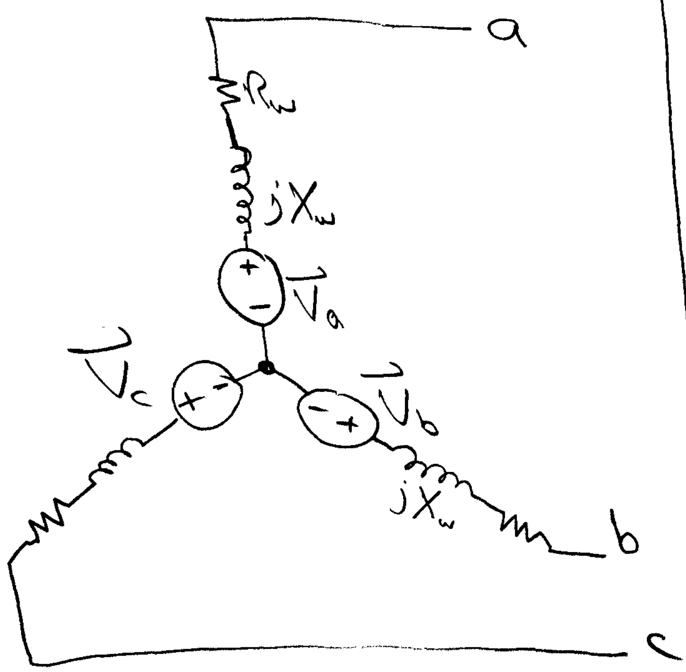


Δ- Connected Source

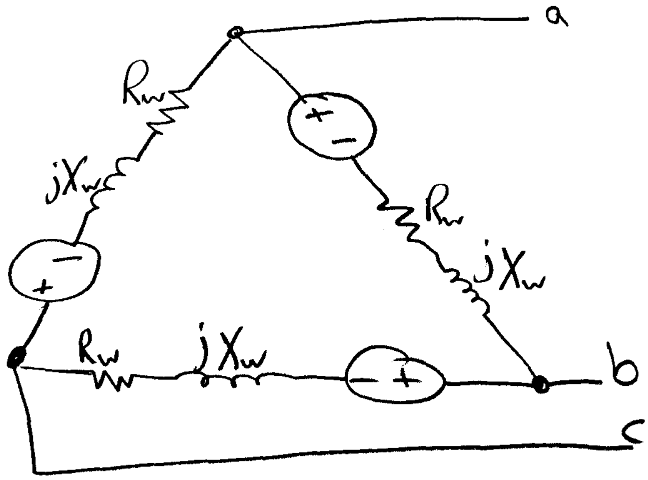


- Three-phase source model with winding impedance

Y- Connected



Δ- Connected



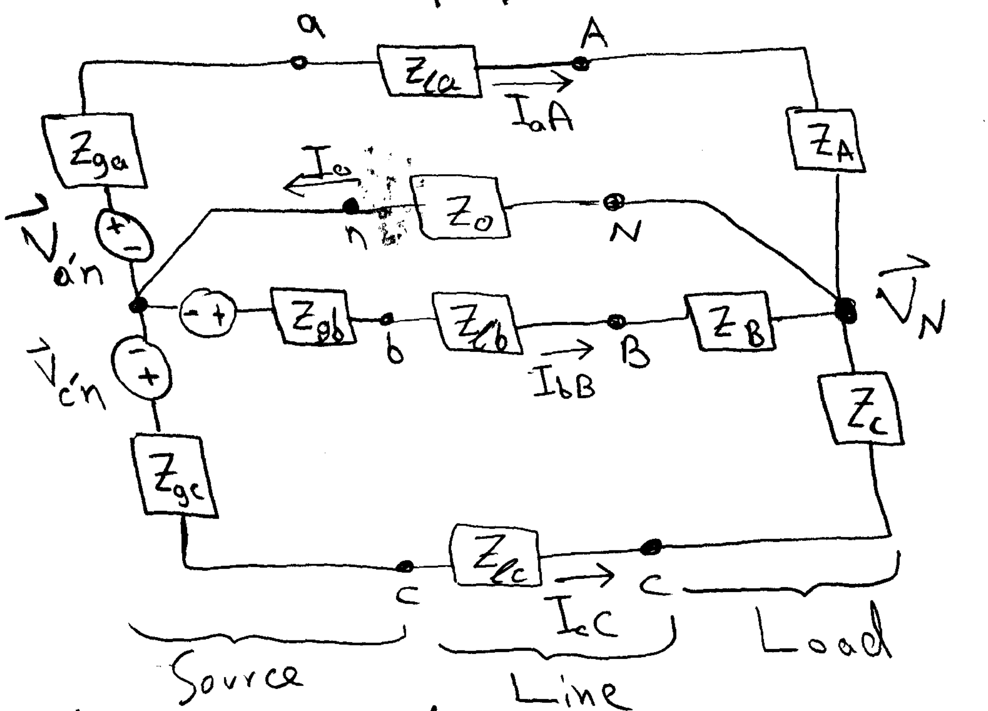
Since three-phase sources and loads can be either Y or Δ connected, there are four possible configurations:

Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

11.3 Analysis of Y-Y Circuit

– We begin by analyzing the Y-Y circuit, the remaining three arrangements can be reduced to a Y-Y equivalent circuit.

– A three-phase Y-Y system.



The node voltage equation at node N is:

$$\frac{\vec{V}_N}{Z} + \frac{\vec{V}_N - \vec{V}_{a'n}}{Z + Z + Z} + \frac{\vec{V}_N - \vec{V}_{b'n}}{Z_0 + Z_{1L} + Z_{a1}} + \frac{\vec{V}_N - \vec{V}_{c'n}}{Z + Z + Z} = 0$$

- For a balanced three-phase circuit, we have the following conditions:

1- The voltages sources form a set of balanced three-phase Voltages.

2- The impedance of each phase of the voltage source is the same $\Rightarrow Z_{ga} = Z_{gb} = Z_{gc}$

3- The impedance of each line conductor is the same.

$$\Rightarrow Z_{la} = Z_{lb} = Z_{lc}$$

4- The impedance of each phase of the load is the same

$$\Rightarrow Z_A = Z_B = Z_C$$

Thus, the three phase Y-Y system is:

$$\vec{V}_N \left(\frac{1}{Z_o} + \frac{3}{Z_\phi}\right) = \frac{\vec{V}_{a'n} + \vec{V}_{b'n} + \vec{V}_{c'n}}{Z_\phi}$$

where,

Single phase Impedance $\rightarrow Z_\phi = Z_A + Z_{la} + Z_{ga} = Z_B + Z_{lb} + Z_{gb} = Z_C + Z_{lc} + Z_{gc}$

Since $\vec{V}_{a'n} + \vec{V}_{b'n} + \vec{V}_{c'n} = 0$

$$\Rightarrow \quad \vec{V}_N = 0$$

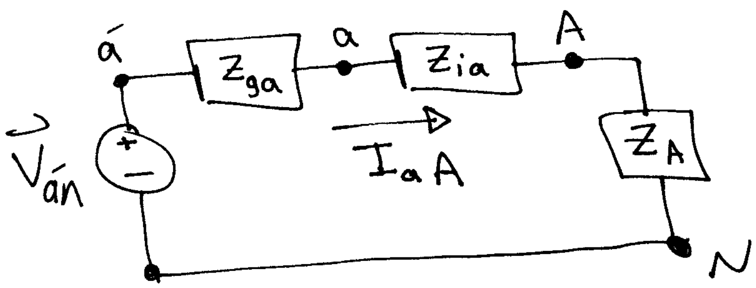
when the circuit is Balances,
the three line currents are

$$\vec{I}_{aA} = \frac{\vec{V}_{a'n}}{Z_\phi}$$

$$\vec{I}_{bA} = \frac{\vec{V}_{b'n}}{Z_\phi}$$

$$\vec{I}_{cc} = \frac{\vec{V}_{c'n}}{Z_\phi}$$

– Single-Phase Equivalent Circuit

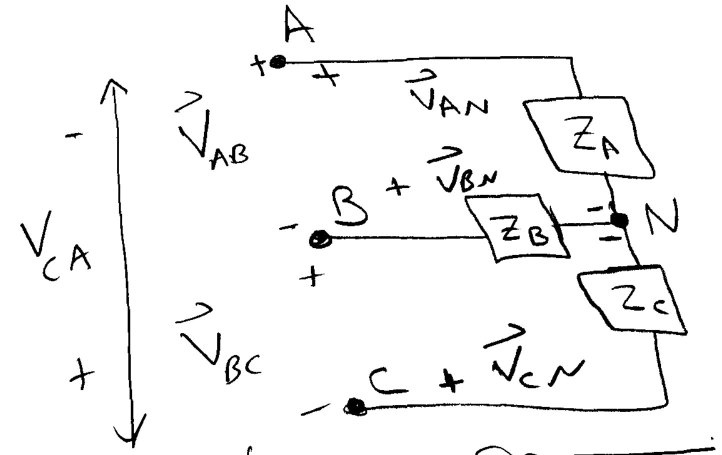


– If we analyse A single-phase, we can get the values A other phases

– Note that the current in the neutral conductor is

$$I_o = I_{aA} + I_{bB} + I_{cc}$$

$$= 0 \Rightarrow \text{for a balanced three-phase circuit.}$$



Line-to-Line voltages

Line-to-Neutral voltages

Q- What is the relation between Line-to-Line and Line-to-Neutral voltages?

- Assume a positive phase sequence, Let the line-to-neutral voltages be:

$$\left.\begin{aligned} \vec{V}_{AN} &= V_\phi \angle 0 \\ \vec{V}_{BN} &= V_\phi \angle -120 \\ \vec{V}_{CN} &= V_\phi \angle 120 \end{aligned}\right\}$$

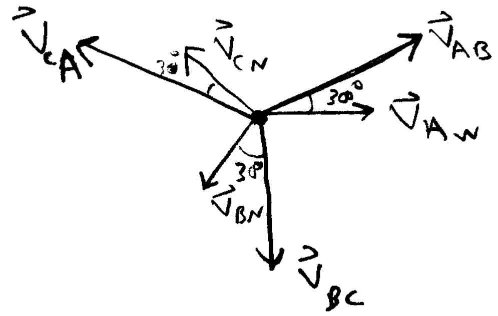
V_ϕ is the magnitude of the line-to-neutral voltage.

- Using Kirchhoff's voltage Law, the line-to-line voltages are:

$$\vec{V}_{AB} = \vec{V}_{AN} - \vec{V}_{BN} = V_\phi \angle 0 - V_\phi \angle -120^\circ$$
$$= \sqrt{3}\, V_\phi \angle 30^\circ$$
$$\vec{V}_{BC} = \vec{V}_{BN} - \vec{V}_{CN} = \sqrt{3}\, V_\phi \angle -90^\circ$$
$$\vec{V}_{CA} = \vec{V}_{CN} - \vec{V}_{AN} = \sqrt{3}\, V_\phi \angle 150^\circ$$

- Thus, Line-To-Line voltages
    - Perform a balanced three-phase voltages
    - the magnitude is $\sqrt{3}$ by the line-to-neutral.
    - the phase is 30° plus the line-to-neutral phase.

- phasor Diagrams



- Terminology

Line voltage: refers to the voltage across any pair of lines

(V_ϕ) Phase voltage: refers to the voltage across a single phase

Line current: refers to the current in a single line

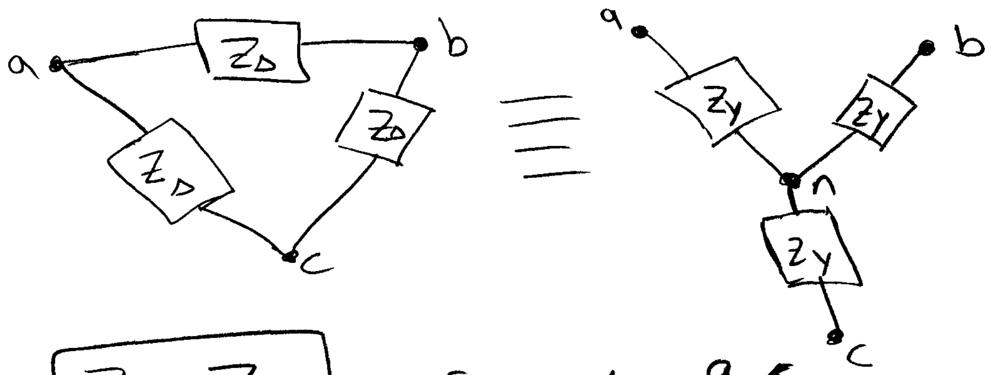
(I_ϕ) Phase current - refers to the current in a single phase

"ϕ letter used to denote phase"

- Example 11.1 in class assignment.

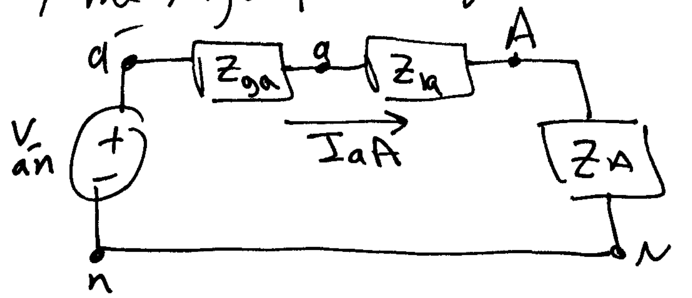
11.4 Analysis of the Y-Δ Circuit

- Δ connected Load can be transformed to Y connected Load.



$$\boxed{Z_Y = \frac{Z_\Delta}{3}}$$ → See section 9.6

- Then, the single phase equivalent circuit is

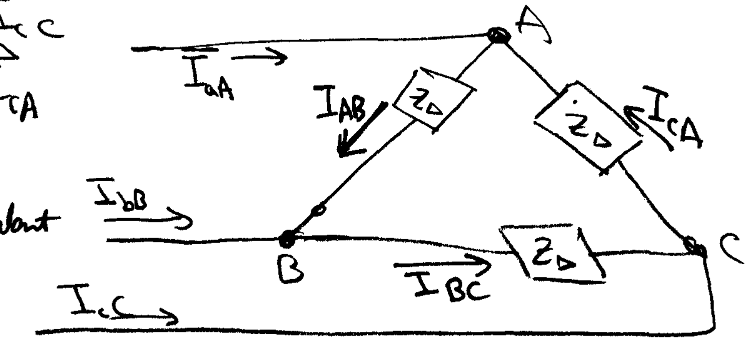


* Note that the phase voltages are the same as the line voltages

- What is the Relation between phase currents and Line currents?

Line Currents: $\vec{I}_{aA}$, $\vec{I}_{bB}$, and $\vec{I}_{cC}$

Phase Currents: $\vec{I}_{AB}$, $\vec{I}_{BC}$, and $\vec{I}_{CA}$



- First Find the line currents From the Single Phase equivalent Circuit as shown above.

- There is a Releation between Line and Phase currents.

Let the phase Currents be:

$$\vec{I}_{AB} = I_\phi \angle 0^\circ , \quad \vec{I}_{BC} = I_\phi \angle -120^\circ \quad \text{and} \quad \vec{I}_{CA} = I_\phi \angle 120^\circ$$

then the line currents will be:

$$\vec{I}_{aA} = \vec{I}_{AB} - \vec{I}_{CA} = I_\phi \angle 0 - I_\phi \angle 120^\circ$$

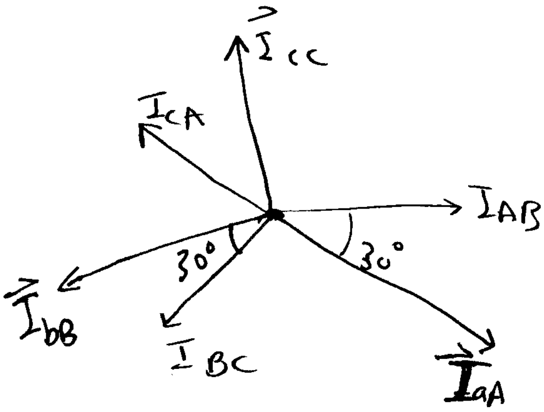
$$= \sqrt{3}\, I_\phi \angle -30^\circ$$

line current lags phase current by 30°

Similarly,

$$\vec{I}_{bB} = \sqrt{3}\, I_\phi \angle -150^\circ$$

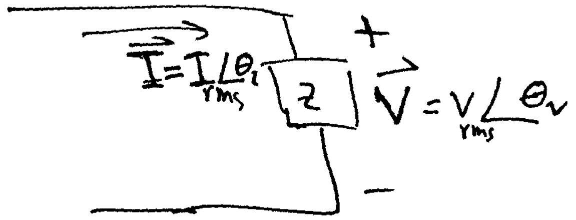
$$\vec{I}_{cC} = \sqrt{3}\, I_\phi \angle 90^\circ$$



See Example 11.2

11.5 Power Calculations in Balanced Three-Phase Circuits.

Background



The complex Power is

$$S = P + j\,Q$$

complex Power (S), average Power (P), Reactive Power (Q)

$|S| \equiv$ volt-amps

$P \equiv$ watts

$Q =$ var

$$S = \vec{V}\,\vec{I}^{*} \leftarrow \text{conjugate}$$

$$= V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$|S| = \sqrt{P^2 + Q^2}$$

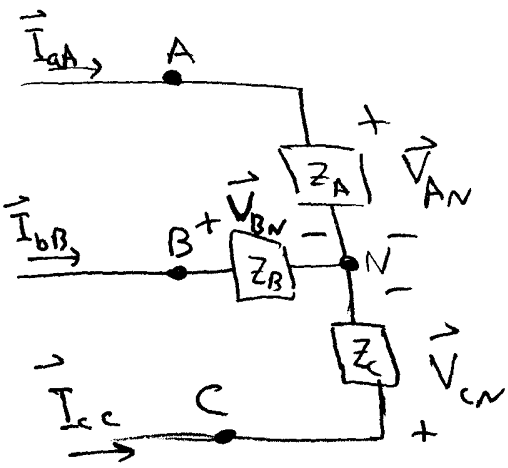
Average Power $\Rightarrow$ $P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$

Reactive Power $\Rightarrow$ $Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

Power factor $\Rightarrow$ $pf = \cos(\theta_v - \theta_i)$

Reactive Factor $\Rightarrow$ $rf = \sin(\theta_v - \theta_i)$

- Average Power in a Balanced Y load



The Power in the a-phase,

$$P_A = |V_{AN}|\,|I_{aA}| \cos(\theta_{vA} - \theta_{iA})$$

similarly, at other phases

$$P_B = |V_{BN}|\,|I_{bB}| \cos(\theta_{vB} - \theta_{iB})$$

$$P_C = |V_{CN}|\,|I_{cC}| \cos(\theta_{vC} - \theta_{iC})$$

in a Blanced three-phase system;

$$V_\phi = |V_{AN}| = |V_{BN}| = |V_{CN}|$$

$$I_\phi = |I_{aA}| = |I_{bB}| = |I_{cC}|$$

and

$$\theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}$$

Per phase Power $\Rightarrow$ ∴

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi$$

∴ The total power is

$$\boxed{P_T = 3P_\phi = 3\underbrace{V_\phi}_{\text{Phas voltage}}\,\underbrace{I_\phi}_{\text{phase current}} \cos\theta_\phi}$$

In Terms of Line currents and Voltages

Average Real Power $\longrightarrow$

$$\boxed{\begin{aligned} P_T &= 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\theta_\phi \\ &= \sqrt{3}\, V_L I_L \cos\theta_\phi \end{aligned}}$$

Reactive Power $\Rightarrow$ $Q_\phi = V_\phi I_\phi \sin\theta_\phi$

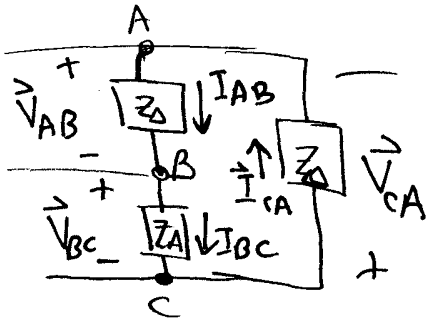
Total reactive Power $\Rightarrow$ $Q_T = 3\,Q_\phi = \sqrt{3}\,V_L I_L \sin\theta_\phi$

complex Power $\Rightarrow$ $S_\phi = P_\phi + j\,Q_\phi = \vec{V}_\phi \vec{I}^{\,*}_\phi$

Total complex Power $\Rightarrow$ $S_T = 3\,S_\phi = \sqrt{3}\,V_L I_L \angle\theta_\phi^\circ$

For Balanced Δ load:

For a balanced Δ load;



Average Power per phase

$$P_\phi = V_\phi I_\phi \cos\theta_\phi$$

Total power

$$P_T = 3\,V_\phi I_\phi \cos\theta_\phi$$
$$= 3\,V_L \left(\frac{I_L}{\sqrt{3}}\right)\cos\theta_\phi$$
$$= 3\,V_L I_L \cos\theta_\phi$$

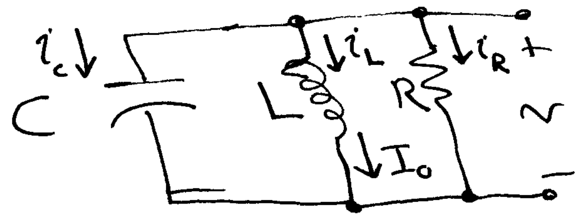
Total Reactive Power $Q_T = 3\,Q_\phi = 3\,V_\phi I_\phi \sin\theta_\phi$

Complex Power Per phase: $S_\phi = P_\phi + j\,Q_\phi = \vec{V}_\phi I^*_\phi$

$$S_T = 3\,S_\phi = \sqrt{3}\,V_L I_L \angle\theta_\phi$$

See Examples 11.3, 11.4, 11.5.

8.1 Parallel RLC Circuit



"Second-Order" Circuits

$$\frac{v}{R} + \frac{1}{L}\int_0^t v\,d\tau + I_0 + C\frac{dv}{dt} = 0$$

I_0: constant "initial condition"

Differentiate once

$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} + C\frac{d^2v}{dt^2} = 0$$

Second Order Differential Equation $\Rightarrow$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$$

How can we solve this?

- It makes sence to assume that the solution will be exponential. Why?

Thus, Assume that

$$v = Ae^{st}$$; where A and s are unknown constants

$$\Rightarrow As^2e^{st} + \frac{As}{RC}e^{st} + \frac{Ae^{st}}{LC} = 0$$

or

$$Ae^{st}\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right) = 0$$

the roots are; $\boxed{s^2 + \frac{s}{RC} + \frac{1}{LC} = 0}$ $\Rightarrow$ Characteritic equation, The roots are:

$$S_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$S_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Thus we have two solutions:

$$v_1 = A_1 e^{s_1 t} \quad \text{and} \quad v_2 = A_2 e^{s_2 t}$$

there sum is also a solution:

$$v = v_1 + v_2 = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

∴ The Natural Response of parallel RLC circuit is

$$\boxed{v = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$$ ⟸ Natural Response of parallel RLC

Let $\alpha = \frac{1}{2RC}$ and $w_0 = \frac{1}{\sqrt{LC}}$

∴ The roots of the characteristic equation are:

$$S_1 = -\alpha + \sqrt{\alpha^2 - w_0^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - w_0^2}$$

$\alpha \Rightarrow$ Neper frequency "rad/s"

$w_0 \Rightarrow$ Resonant radian frequency. "rad/s"

Case 1 $\alpha^2 > w_0^2 \Rightarrow S_1$ and S_2 are Real $\Rightarrow$ overdamped

Case 2 $\alpha^2 < w_0^2 \Rightarrow S_1$ and S_2 are complex conjugates $\Rightarrow$ Underdamped

Case 3 $\alpha^2 = w_0^2 \Rightarrow S_1$ and S_2 are Real and equal. $\Rightarrow$ critically damped.

8.2 Forms of the Natural Response of a Parallel RLC Circuit

Case 1 Overdamped Voltage Response

the Natural Response is in this Form:

$$v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Q) How to Find the constants A_1 and A_2?

- We can Find them from the initial conditions, which are the values of $v(0^+)$ and $\frac{dv(0^+)}{dt}$
- These values are determined from the initial voltage on the capacitor V_0 and the initial current in the inductor I_0.

Thus,

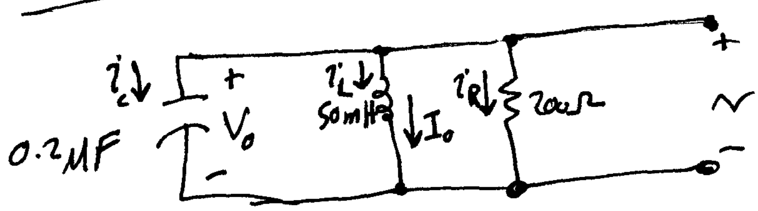
$$v(0^+) = A_1 + A_2 \quad ; \text{ } v(0^+) \text{ is the initial voltage on the capacitor } V_0$$

$$\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2 \quad ; \text{ where } \frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

and

$$i_C(0^+) = \frac{-V_0}{R} - I_0$$

Example 8.2:



For the above circuit, $v(0^+) = 12\text{V}$ and $i_L(0^+) = 30\text{ mA}$

a) Find the initial current in each branch of the circuit
b) Find the initial value of dv/dt
c) Find the expression for $v(t)$
d) sketch $v(t)$ in the interval $0 \le t \le 250\,\mu s$

Underdamped Voltage Response

* s_1 and s_2 are complex conjugates

Let $s_1 = -\alpha + j\sqrt{\omega_0^2 - \alpha^2}$

$= -\alpha + j\omega_d$; $\omega_d \equiv$ damped frequency $= \sqrt{\omega_0^2 - \alpha^2}$

$s_2 = -\alpha - j\omega_d$

* the underdamped voltage response is

$$v(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$$ ⟸ Voltage natural Response "Underdamped"

Proof

From Euler identity:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

Thus $v(t) = A_1 e^{(-\alpha + j\omega_d)t} + A_2 e^{-(\alpha + j\omega_d)t}$

$$= A_1 e^{-\alpha t} e^{j\omega_d t} + A_2 e^{-\alpha t} e^{-j\omega_d t}$$

$$= e^{-\alpha t}(A_1\cos\omega_d t + jA_1\sin\omega_d t + A_2\cos\omega_d t - jA_2\sin\omega_d t)$$

$$= e^{-\alpha t}\left[(A_1 + A_2)\cos\omega_d t + j(A_1 - A_2)\sin\omega_d t\right]$$

Let $B_1 = A_1 + A_2$ and $B_2 = j(A_1 - A_2)$

Since A_1 and A_2 are complex conjugates

$\Rightarrow B_1$ and B_2 are Real

Thus, $v(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$

* To Find B_1 and B_2

From the initial condition,

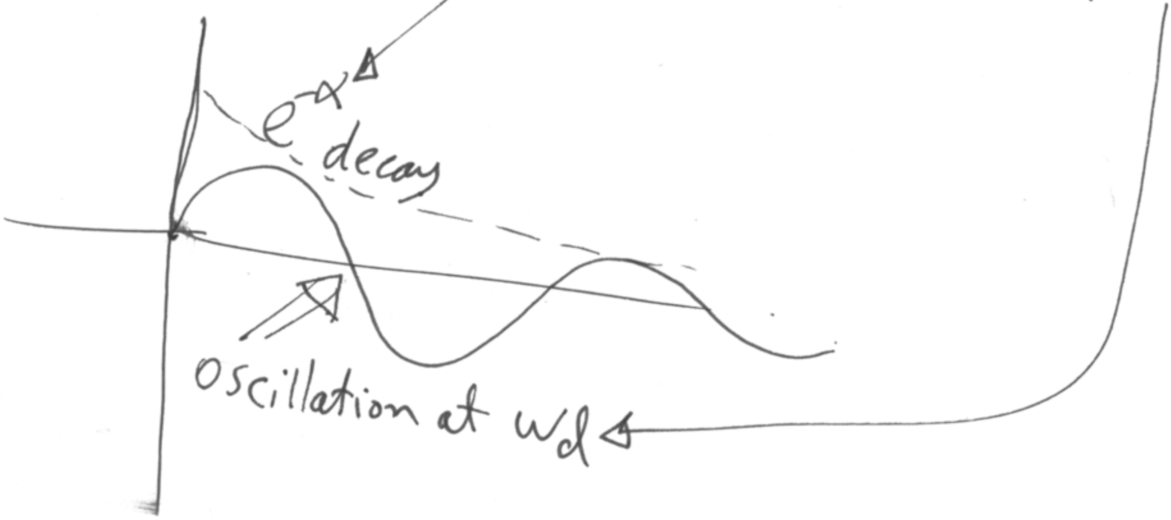
$$v(0^+) = V_0 = B_1$$

$$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = -\alpha B_1 + \omega_d B_2$$

* Closer look at the underdamped Responce.

$$v = B_1 e^{-\alpha t} \cos\omega_d t + B_2 e^{-\alpha t} \sin \underbrace{\omega_d}_{\text{damped Freq.}} t$$

damping factor.



The Critically Damped Voltage Response

when $w_0 = \alpha \Rightarrow s_1$ and s_2 are Real and equal

$$s_1 = s_2 = -\alpha = \frac{-1}{2RC}$$

$$\Rightarrow v = (A_1 + A_2)\, e^{-\alpha t} = A_0\, e^{-\alpha t}$$

However, this equation can't satisfy two independent Initial conditions.

Thus, the <u>correct</u> Response for <u>critically damped</u> Parallel RLC circuit is

$$\boxed{v(t) = D_1 t\, e^{-\alpha t} + D_2\, e^{-\alpha t}}$$

← This is from differential eqn Theory.

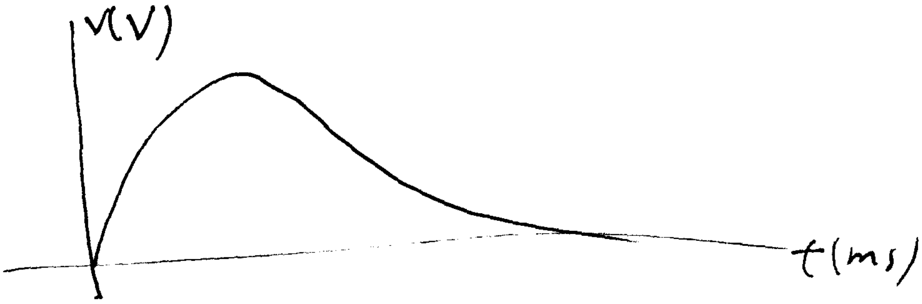
<u>To Find D_1 and D_2</u>, solve:

$$v(0^+) = V_0 = D_2$$

$$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = D_1 - \alpha D_2$$

<u>Example 8.5</u>

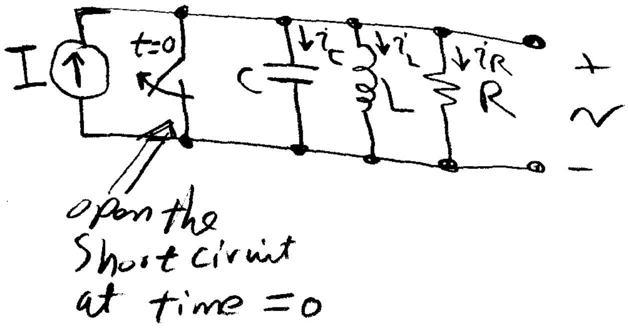
$$v(t) = 98{,}000\, t\, e^{-1000 t}\ \text{V}$$

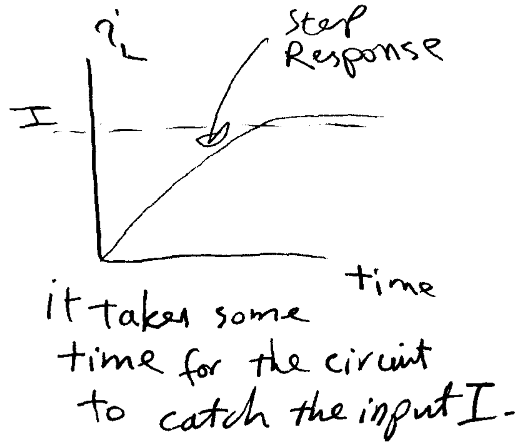


8.3 The Step Response of a Parallel RLC Circuit.

What is the step Response?

It is the response of an RLC circuit caused by the sudden input of a dc current.





* We focus on the current in the inductive branch [i_L].

* We assume that the initial energy is zero.

* The inductor current i_L :

From Kirchhoff's Current Law,

$$i_L + i_R + i_C = I$$

or

$$i_L + \frac{v}{R} + C\frac{dv}{dt} = I$$

Since $v = L\frac{di_L}{dt} \Rightarrow \frac{dv}{dt} = L\frac{d^2 i_L}{dt^2}$

$$\Rightarrow$$

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I}{LC}$$

- Similar to the natural Response

1. - Find the roots of the characteristic Equation

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0 \Rightarrow s = -\alpha \pm \sqrt{\alpha^2 - w_0^2}$$
$$\alpha = \frac{1}{2RC}, \quad w_0 = \frac{1}{\sqrt{LC}}$$

2. - Based on the roots $\{s_1, s_2\}$, we have three possible cases.

Case 1: $i_L = I + A_1 e^{s_1 t} + A_2 e^{s_2 t} \Rightarrow$ Overdamped

Case 2: $i_L = I + B_1 e^{-\alpha t} \cos w_d t + B_2 e^{-\alpha t} \sin w_d t \Rightarrow$ Underdamped

Case 3: $i_L = I + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t} \Rightarrow$ Critically Damped

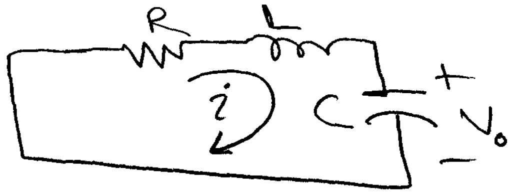
3. Find the constant from the initial Conditions at $i_L(0)$ and $\frac{di_L(0)}{dt}$.

* Solve Examples 8.6, 8.7, 8.8, 8.9, and 8.10

\+ Solve Assessment Problem 8.6

8.4 The Natural and Step Response of a series RLC Circuit.

Part 1: The Natural Response



— Summing the voltages inside the loop:

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,d\tau + V_0 = 0$$

— Differentiate and Rearrage Terms:

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

— The Characteristic equation for series RLC ckts

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\Rightarrow s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where, $\alpha = \frac{R}{2L}$ rad/s $\Rightarrow$ Neper freq.

$\omega_0 = \frac{1}{\sqrt{LC}}$ rad/s $\Rightarrow$ Resonant freq.

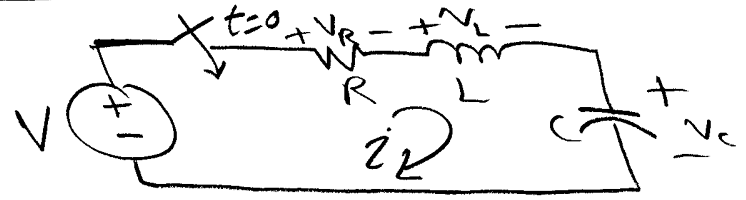
— Three Possible cases:

Overdamped: $i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

Underdamped: $i(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$

Critically damped: $i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

Part 2: Step Response of a Series RLC Circuit



— Applying Kirchhoff's voltage Law,

$$V = Ri + L\frac{di}{dt} + v_c$$

since $i = C\frac{dv_c}{dt} \Rightarrow \frac{di}{dt} = C\frac{d^2v_c}{dt^2}$

$$\Rightarrow \frac{d^2v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{v_c}{LC} = \frac{V}{LC}$$

— Three possible soultions:

Overdamped: $v_c = V_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

Underdamped: $v_c = V_f + B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$

Critically damped: $v_c = V_f + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

— Solve Example 8.11 and 8.12

— Solve Assessment Problem 8.7, 8.8.