Problem 1: Linear Block Codes (12 points)

1. Consider the binary code C composed of the following four code words.

$$\begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \\ \end{array}$$

$$C=\{(11000), (10010), (01001), (11111)\}$$

a) What is the minimum distance of this code, *show your steps*? **(2 points)**

minimum distance is two = 2 by exhaustive check

$d(c_1, c_2) = d(11000, 10010) = 2$

$d(c_2,c_1)=2$, $d(c_1,c_3)=2$, $d(c_1,c_4)=3$

$d(c_2,c_3)=4$, $d(c_2,c_4)=3$, $d(c_3,c_4)=3$

b) Is the code linear? Prove your answer. **(2 points)**

No it is not linear sum of two codewords is not a valid code word $c_1+c_2 = 01010 \notin C$

- or it is not a linear code because the all zero codeword $\notin C$

c) Is the code perfect? Prove your answer. **(2 points)**

Since we have four code words $k=2$ $n=5$ $\Rightarrow$ $r=3$

$t=0$ because $d_{min}=2$ Hamming bound with equality ?

$\binom{n}{t} \overset{?}{=} 2^3$ $\binom{5}{0} \overset{?}{=} 8$ $1 \overset{?}{\neq} 8$ not perfect

2. Consider a systematic (8,4) block **code** whose parity-check **equations are:**

$$x_5 = u_1 \oplus u_2 \oplus u_4$$
$$x_6 = u_1 \oplus u_3 \oplus u_4$$
$$x_7 = u_1 \oplus u_2 \oplus u_3$$
$$x_8 = u_2 \oplus u_3 \oplus u_4$$

Where u_i **are** the message digits **and** x_i **are the** check digits:

(a) Find the generator **matrix for this** code. $C = [u_1 u_2 u_3 u_4 x_5 x_6 x_7 x_8]$ **(2 points)**

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(b) How many **errors** can the **code correct** **and/or** detect? **(2 points)**

d_{min} is the minimum weight for any linear combination

$d_{min} = 4$

Case I: detect up to 3 errors

Case II: correct 1 & detect two errors.

Case III: correct all single & some double errors.

(c) Is the **vector 1 0 1 0 1 0 1 0 a** valid code word? **(2 points)**

data is 1010 the code is linear

c is sum of first & third rows in G

10101001

the given code is not valid.

or by finding H & computing the syndrome

Problem 2: Cyclic Codes & Linear Codes (20 points)

Let C_1 be the binary cyclic code of length 15 generate by $g(x) = x^5 + x^4 + x^2 + 1$.

a) Compute the parity-check polynomial for C_1 and show that g(x) is a valid generator polynomial. **(3 points)**

$x^{15} + 1 = g(x)\, h(x)$

$h(x) = x^{10} + x^9 + x^8 + x^6 + x^5 + x^2 + 1$

g(x) is a valid generator polynomial because it is a factor of $x^{15}+1$

$$
\begin{array}{r}
x^{10} + x^9 + x^8 + x^6 + x^5 \\
x^5 + x^4 + x^2 + 1 \;\big)\; x^{15} + 1 \\
x^{15} + x^{14} + x^{12} + x^{10} \\
\hline
x^{14} + x^{12} + x^{10} + 1 \\
x^{14} + x^{13} + x^{11} + x^9 \\
\hline
x^{13} + x^{12} + x^{11} + x^{10} + x^9 + 1 \\
x^{13} + x^{12} + x^{10} + x^8 \\
\hline
x^{11} + x^9 + x^8 + 1 \\
x^{11} + x^{10} + x^8 + x^6
\end{array}
$$

$$
\begin{array}{r}
x^{10} + x^9 + x^6 + 1 \\
x^{10} + x^9 + x^7 + x^5 \\
\hline
x^7 + x^6 + x^5 + 1 \\
x^7 + x^6 + x^4 + x^2 \\
\hline
x^5 + x^4 + x^2 + 1 \\
x^5 + x^4 + x^2 + 1 \\
\hline
0
\end{array}
$$

b) Determine the number of check bits for the given code, r, and compute the number of valid code words of C_1. **(1 point)**

# of check bits r = degree $[g(x)] = 5$

# of valid codewords $= 2^k = 2^{10} = 1024$

c) Compute the code polynomial in C_1 and the associated code word for the following message polynomials using the polynomial multiplication encoding technique **(2 points)**

a. x^2

b. $x^7 + x^3 + x$

a) $c(x) = u(x)g(x)$

$= x^2(x^5 + x^4 + x^2 + 1) = x^7 + x^6 + x^4 + x^2$

$\equiv$ 00101011 00 00 00 0

b) $c(x) = (x^7 + x^3 + x)(x^5 + x^4 + x^2 + 1)$

$= x^{12} + x^{11} + x^9 + x^7$
$\quad + x^8 + x^7 + x^5 + x^3$
$\quad + x^6 + x^5 + x^3 + x$

$x^{12} + x^{11} + x^9 + x^8 + x^6 + x$

$\equiv$ 01 00 00 10 11 01 10 0

d) Compute the code polynomial in C_1 and the associated code word for the following message polynomial using the systematic encoding technique $x^7 + x^3 + x$. **(3 points)**

$b(x) = \text{Rem}\left[X^{n-k} u(x) / g(x)\right]$

$= \text{Rem}\left[X^5(X^7 + x^3 + x) / (x^5 + x^4 + x^2 + 1)\right] = x^4 + x^2 + 1$

$c(x) = b(x) + X^{n-k} u(x)$

$= 1 + x^2 + x^4 + x^6 + x^8 + x^{12}$

$\equiv$ 10 10 10 10 10 00 10 00

$$
\begin{array}{r}
x^7 + x^6 + x^5 + 1 \\
x^5 + x^4 + x^2 + 1 \;\big)\; x^{12} + x^8 + x^6 \\
x^{12} + x^{11} + x^9 + x^7 \\
\hline
x^{11} + x^9 + x^8 + x^7 + x^6 \\
x^{11} + x^{10} + x^8 + x^6 \\
\hline
x^{10} + x^9 + x^7 \\
x^{10} + x^9 + x^7 + x^5 \\
\hline
x^5 \\
x^5 + x^4 + x^2 + 1 \\
\hline
x^4 + x^2 + 1
\end{array}
$$

f) Compute the syndrome for the following received polynomial. $x^{14}+x^{10}+x^5+x^3$. (2 points)

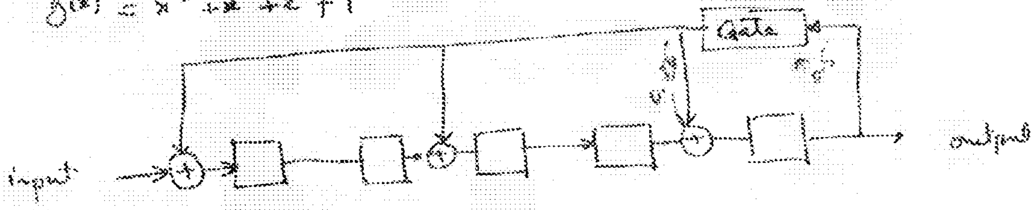
$S(x) = \text{rem}\left[r(x)/g(x) \right] = \text{rem}\left[\dfrac{x^{14}+x^{10}+x^5+x^3}{x^5+x^4+x^3+1} \right]$

yes syndrome = 0

valid code word.

g) Design a syndrome computation circuit for C_1. (2 points)

$g(x) = x^5 + x^4 + x^3 + 1$



h) Calculate the error pattern coverage of C_1. (2 point)

$$\frac{\text{number of invalid code words}}{\#\text{ of all code words}} = 1 - 2^{-r} = 1 - 2^{-5} = 0.9688$$

2 Consider an (8,4)block code whose generator matrix is given by:

$$\bar{\bar{G}} = \begin{bmatrix} 1 & 0 & 0 & 0 & p_{11} & p_{12} & p_{13} & p_{14} \\ 0 & 1 & 0 & 0 & p_{21} & p_{22} & p_{23} & p_{24} \\ 0 & 0 & 1 & 0 & p_{31} & p_{32} & p_{33} & p_{34} \\ 0 & 0 & 0 & 1 & p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix}$$

where the parity matrix $\bar{\bar{P}}$ is unknown. Given that four codewords are known as following: $\bar{x}_1 = [10001110], \bar{x}_2 = [01001011], \bar{x}_3 = [00100111]$,& $\bar{x}_4 = [00011101]$.

Find at least four more codewords. (3 points)

the parity matrix

$$\bar{\bar{P}} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

x_1 info 1000 first row = C_1

x_2 inf 0100 second row = C_2

x_3 inf 0010 third row = C_3

x_4 inf. 0001 fourth row = C_4

Since every row in G_n is a valid code word