Chapter 4
Baseband Pulse Transmission

EE417
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Content

4.1 Introduction

- In this chapter we study the transmission of digital data (of whatever origin) over a ***baseband channel.***
- Data transmission over a band-pass channel using modulation is covered in Chapter 6.
- Baseband transmission of digital data requires the use of a low-pass channel with a bandwidth large enough to accommodate the essential frequency content of the data stream.
- Typically, however, the channel is ***dispersive*** in that its frequency response deviates from that of an ideal low-pass filter.

4.1 Error Sources in Baseband Transmission

***Intersymbol Interference* (ISI)**

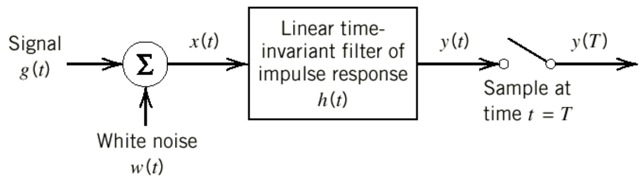
- The result of data transmission over a ***dispersive*** channel is that each received pulse is affected somewhat by adjacent pulses, thereby giving rise to a common form of interference called ***intersymbol interference*** (ISI).
- Intersymbol interference is a major source of bit errors in the reconstructed data stream at the receiver output.
- To correct for it, control has to be exercised over the pulse shape in the overall system.
- Thus much of the material covered in this chapter is devoted to ***pulse shaping*** in one form or another.

4.1 Error Sources in Baseband Transmission

- Another source of bit errors in a baseband data transmission system is the ***channel noise.***
- Naturally, noise and ISI arise in the system simultaneously. However, to understand how they affect the performance of the system, we first consider them separately; later on in the chapter, we study their combined effects.

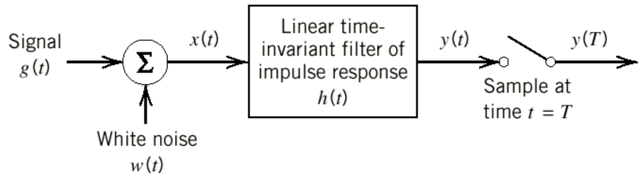
4.2 Matched Filter

- A fundamental result in communication theory deals with the ***detection*** of a pulse signal of known waveform that is immersed in additive white noise.
- The device for the optimum detection of such a pulse involves the use of a linear-time-invariant filter known as a ***matched filter.***
- It it called a matched filter because its impulse response is matched to the pulse signal.



4.2 Matched Filter

- The filter input *x(t)* consists of a pulse signal g*(t)* corrupted by additive channel noise *w(t),* as shown by
  x(t) = g*(t)* + *w(t),* $0 \le t \le T$ *,* where T is an arbitrary observation interval
- The pulse signal *g(t)* may represent a binary symbol 1 or 0 in a digital communication system.
- The *w(t)* is the sample function of a white noise process of zero mean and power spectral density $N_0/2$.
- It is assumed that the receiver has knowledge of the waveform of the pulse signal *g(t).* The source of uncertainty lies in the noise *w(t).*



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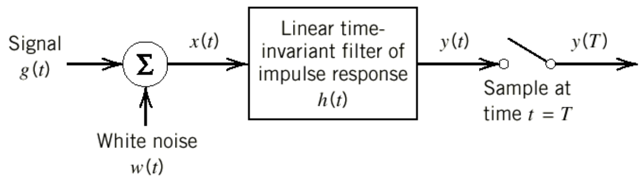
4.2 Matched Filter

- The function of the receiver is to detect the pulse signal *g(t)* in an optimum manner, given the received signal *x(t).*
- To satisfy this requirement, we have to optimize the design of the filter so as to minimize the effects of noise at the filter output in some statistical sense, and thereby enhance the detection of the pulse signal *g(t).*
- Since the filter is linear, the resulting output *y(t)* may be expressed as:

$$y(t) = g_o(t) + n(t)$$

- where $g_0(t)$ and *n(t)* are produced by the signal and noise components of the input *x(t),* respectively.



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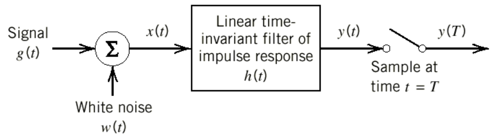
4.2 Matched Filter



- A simple way of describing the requirement that the output signal component $g_0(t)$ be considerably greater than the output noise component *n(t)* is to have the filter make the instantaneous power in the output signal $g_0(t)$, measured at time t = T, as large as possible compared with the average power of the output noise *n(t).*
- This is equivalent to maximizing the *peak pulse signal-to-noise ratio,* defined as

$$\eta = \frac{|g_o(T)|^2}{E[n^2(t)]}$$

- where $|g_o(T)|^2$ is the instantaneous power in the output signal, *E* is the statistical expectation operator, and $E[n^2(t)]$ is a measure of the average output noise power.
- **The requirement is to specify the impulse response *h(t)* of the filter such that the output signal-to-noise ratio is maximized**.

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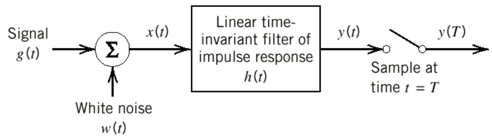
4.2 Matched Filter



Let *G(f)* denote the Fourier transform of the known signal *g(t),* and *H(f)* denote the frequency response of the filter. Then the Fourier transform of the output signal $g_0(t)$ is equal to *H(f)G(f),* and $g_o(t)$ is itself given by the inverse Fourier transform:

$$g_o(t) = \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi ft)\,df$$

Hence, when the filter output is sampled at time *t = T,* the signal power will be:

$$|g_o(T)|^2 = \left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)\,df\right|^2$$

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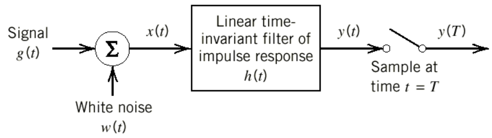
4.2 Matched Filter



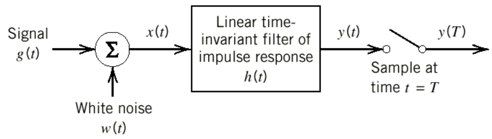
- Consider next the effect on the filter output due to the noise *w(t)* acting alone.
- The power spectral density $S_N(f)$ of the output noise *n(t)* is equal to the power spectral density of the input noise *w(t)* times the squared magnitude response $|H(f)|^2$
- Since *w(t)* is white with constant power spectral density $N_0/2$, it follows that:

$$S_N(f) = \frac{N_0}{2}|H(f)|^2$$

- The average power of the output noise *n(t)* is therefore

$$E[n^2(t)] = \int_{-\infty}^{\infty} S_N(f)\, df = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df$$

4.2 Matched Filter



- Thus, the peak pulse signal-to-noise ratio is:

$$\eta = \frac{\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)\, df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df}$$

- **For a given G(f), what is the frequency response H(f) of the filter that maximizes η ?**
- To find the solution to this optimization problem, we apply a mathematical result known as Schwarz's inequality to the numerator of the above Equation.

Schwarz's inequality

If we have two complex functions Φ_1(x) and Φ_2(x) in the real variable *x,* satisfying the conditions:

$$\int_{-\infty}^{\infty} |\phi_1(x)|^2\,dx < \infty \quad \text{AND} \quad \int_{-\infty}^{\infty} |\phi_2(x)|^2\,dx < \infty$$

Then

$$\left|\int_{-\infty}^{\infty} \phi_1(x)\phi_2(x)\,dx\right|^2 \leq \int_{-\infty}^{\infty} |\phi_1(x)|^2\,dx \int_{-\infty}^{\infty} |\phi_2(x)|^2\,dx$$

The equality in (4.9) holds if, and only if, we have

$$\phi_1(x) = k\phi_2^*(x)$$

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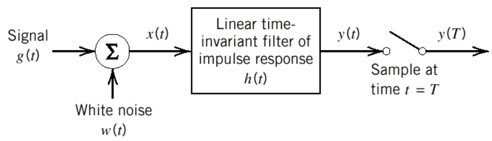
4.2 Matched Filter



- Therefore, applying Schwarz's inequality for $\phi_1(x) = H(f)$ and $\phi_2(x) = G(f)\exp(j\pi fT)$,

$$\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)\,df\right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2\,df \int_{-\infty}^{\infty} |G(f)|^2\,df$$

- Thus, the peak pulse signal-to-noise ratio is:

$$\eta \leq \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2\,df$$

- The right-hand side of this relation does not depend on the frequency response H(f) of the filter but only on the signal energy and the noise power spectral density.
- Consequently, the peak pulse signal-to-noise ratio *η* will be a maximum when H(f) is chosen so that the equality holds; that is,

$$H_{opt}(f) = kG^*(f)\exp(-j2\pi fT)$$

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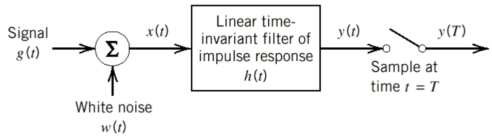
4.2 Matched Filter



In the time domain, the impulse response of the optimum filter is:

$$h_{opt}(t) = k \int_{-\infty}^{\infty} G^*(f) \exp[-j2\pi f(T-t)]\, df$$

Recall that for real signals g(t), the real part of the spectrum is even and the imaginary part is odd. Thus $G^*(f)=G(-f)$.

$$\begin{aligned} h_{opt}(t) &= k \int_{-\infty}^{\infty} G(-f) \exp[-j2\pi f(T-t)]\, df \\ &= k \int_{-\infty}^{\infty} G(f) \exp[j2\pi f(T-t)]\, df \\ &= kg(T-t) \end{aligned}$$

The impulse response of the optimum filter, except for the scaling factor *k,* is a time-reversed and delayed version of the input signal *g(t).* So, it is ***matched*** to the input signal.

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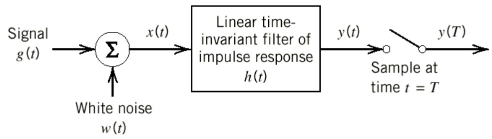
4.2 Matched Filter



- Thus, the peak pulse signal-to-noise ratio will be:

$$\eta_{max} = \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2\, df$$

- According to *Rayleigh's energy theorem, the integral of the squared magnitude spectrum of a pulse signal with respect to frequency is equal to the signal energy E*

$$E = \int_{-\infty}^{\infty} g^2(t)dt = \int_{-\infty}^{\infty} |G(f)|^2\, df$$

- Therefore $\eta_{max} = \dfrac{2E}{N_o}$

- *Thus, the peak pulse signal-to-noise ratio of a matched filter depends only on the ratio of the signal energy to the power spectral density of the white noise at the filter input*

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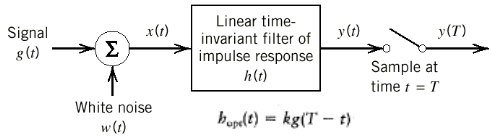
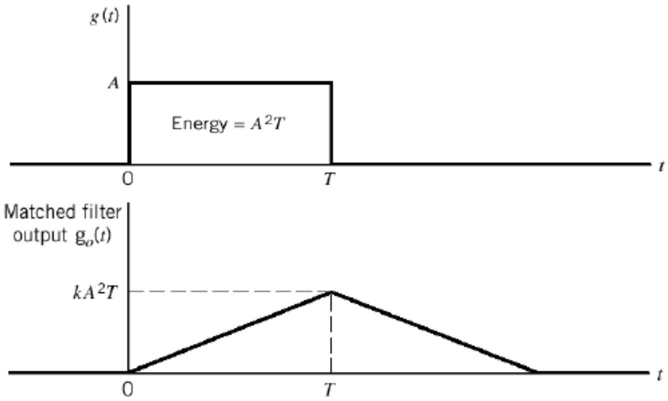
4.2 Matched Filter



$$h_{opt}(t) = kg(T - t)$$

$$H_{opt}(f) = kG^*(f)\exp(-j2\pi fT)$$

Example: Find the matched filter output $g_o(t)$ for the following signal:



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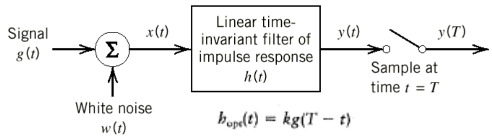
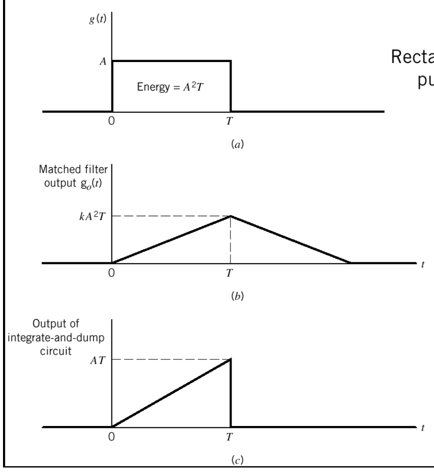
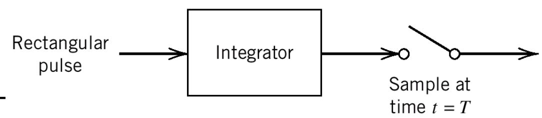
4.2 Matched Filter



$$h_{opt}(t) = kg(T - t)$$

For the special case of a rectangular pulse, the matched filter may be implemented using a circuit known as the *integrate-and-dump circuit*





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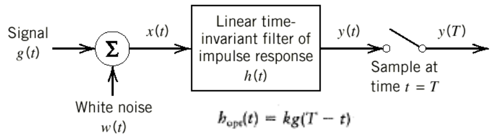
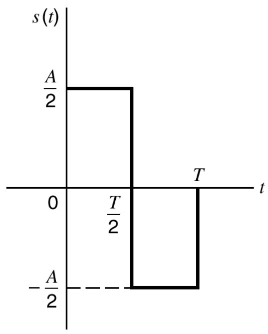
4.2 Matched Filter



$h_{opt}(t) = kg(T - t)$

$H_{opt}(f) = kG^*(f) \exp(-j2\pi fT)$

Example2: Consider the signal s(t)



a) Determine the impulse response of a filter matched to this signal and sketch it as a function of time.

b) Plot the matched filter output as a function of time.

c) What is the peak value of the output?

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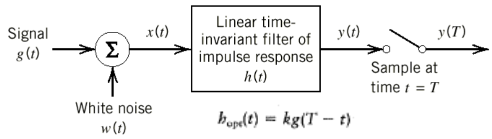
4.2 Matched Filter



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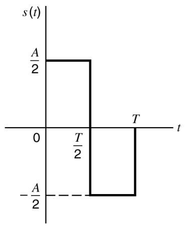
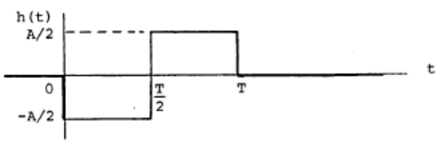
Example2: Consider the signal s(t)



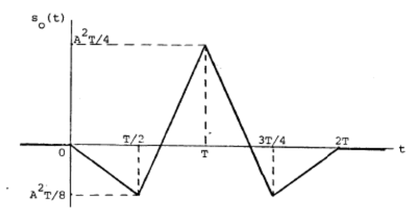
a) Determine the impulse response of a filter matched to this signal and sketch it as a function of time.



b) Plot the matched filter output as a function of time.

c) What is the peak value of the output?

Peak value = $A^2T/4$



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4.3 Error Rate Due to Noise

- Consider a binary PCM system based on *polar non-return-to-zero (NRZ) signaling.*
- In this form of signaling, symbols 1 and 0 are represented by positive and negative rectangular pulses of equal amplitude and equal duration.
- The channel noise is modeled as *additive white Gaussian noise w(t)* of zero mean and power spectral density $N_0/2$.
- In the signaling interval $0 \le t \le T_b$, the received signal is thus written as follows:

$$x(t) = \begin{cases} +A + w(t), & \text{symbol 1 was sent} \\ -A + w(t), & \text{symbol 0 was sent} \end{cases}$$

- where T_b is the ***bit duration***, and A is the ***transmitted pulse amplitude***.

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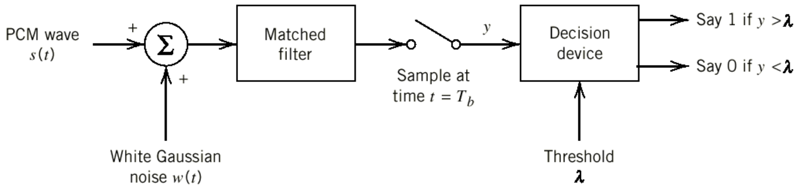
4.3 Error Rate Due to Noise

The structure of the receiver used to perform this decision-making process is:



There are two possible kinds of error to be considered:

- Symbol 1 is chosen when a 0 was actually transmitted; we refer to this error as an *error of the first kind.*
- Symbol 0 is chosen when a 1 was actually transmitted; we refer to this error as an *error of the second kind.*

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4.3 Error Rate Due to Noise

- To determine the average probability of error, we consider these two situations separately.
- Suppose that symbol 0 was sent. Then

  $x(t) = -A + w(t), \qquad 0 \le t \le T_b$
- The matched filter output, sampled at time $t = T_b$, is:

$$y = \int_0^{T_b} x(t)\,dt$$
$$= -A + \frac{1}{T_b}\int_0^{T_b} w(t)\,dt$$

- which represents the sample value of a random variable Y

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4.3 Error Rate Due to Noise

- Since the noise *w(t)* is white and Gaussian, we may characterize the random variable Y as follows:
  - The random variable Y is Gaussian distributed with a mean of —*A.*
  - The variance of the random variable Y is

$$\sigma_Y^2 = E[(Y + A)^2]$$
$$= \frac{1}{T_b^2} E\left[\int_0^{T_b}\int_0^{T_b} w(t)w(u)\,dt\,du\right]$$
$$= \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} E[w(t)w(u)]\,dt\,du$$
$$= \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} R_W(t, u)\,dt\,du$$

- where $R_w(t, u)$ is the autocorrelation function of the white noise *w(t).*
- Since *w(t)* is white with a power spectral density $N_0/2$, we have

$$R_W(t, u) = \frac{N_0}{2}\,\delta(t - u)$$ Where δ(t-μ) is a time-shifted delta function

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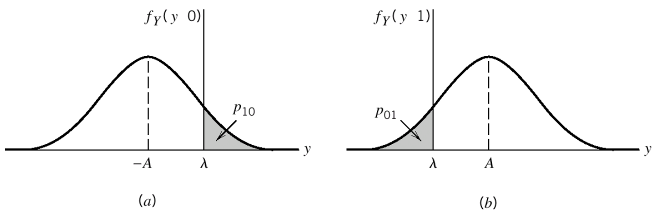
4.3 Error Rate Due to Noise

- Therefore, the variance of Y is:

$$\sigma_Y^2 = \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} \frac{N_0}{2}\,\delta(t-u)\,dt\,du$$
$$= \frac{N_0}{2T_b}$$

- The conditional probability density function of the random variable Y, given that symbol 0 was sent, is:

$$f_Y(y\,|\,0) = \frac{1}{\sqrt{\pi N_0/T_b}}\exp\left(-\frac{(y+A)^2}{N_0/T_b}\right)$$



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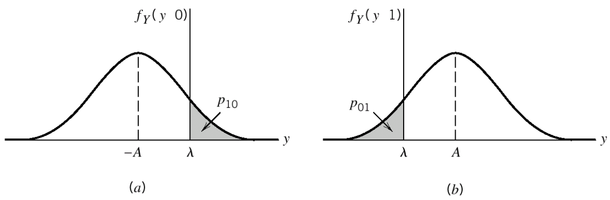
4.3 Error Rate Due to Noise

- Let p_{10} denote the *conditional probability of error, given that symbol 0 was sent.*

$$p_{10} = P(y > \lambda\,|\,\text{symbol 0 was sent})$$
$$= \int_\lambda^\infty f_Y(y\,|\,0)\,dy$$
$$= \frac{1}{\sqrt{\pi N_0/T_b}}\int_\lambda^\infty \exp\left(-\frac{(y+A)^2}{N_0/T_b}\right)dy$$



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Complementary Error Function

- *complementary error function* is defined as:

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-z^2)\, dz$$

- which is closely related to the Gaussian distribution.
- For large positive values of *u,* we have the following *upper bound* on the complementary error function:

$$\text{erfc}(u) < \frac{\exp(-u^2)}{\sqrt{\pi}u}$$

- Relation to Q-Function:

$$Q(v) = \frac{1}{\sqrt{2\pi}} \int_v^\infty \exp\left(-\frac{x^2}{2}\right) dx$$

$$Q(v) = \frac{1}{2}\,\text{erfc}\left(\frac{v}{\sqrt{2}}\right)$$

$$\text{erfc}(u) = 2Q(\sqrt{2}u)$$

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Error Function

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-z^2)\, dz$$

TABLE A6.6 ***The error function***[a]

u	*erf(u)*	*u*	*erf(u)*
0.00	0.00000	1.10	0.88021
0.05	0.05637	1.15	0.89612
0.10	0.11246	1.20	0.91031
0.15	0.16800	1.25	0.92290
0.20	0.22270	1.30	0.93401
0.25	0.27633	1.35	0.94376
0.30	0.32863	1.40	0.95229
0.35	0.37938	1.45	0.95970
0.40	0.42839	1.50	0.96611
0.45	0.47548	1.55	0.97162
0.50	0.52050	1.60	0.97635
0.55	0.56332	1.65	0.98038
0.60	0.60386	1.70	0.98379
0.65	0.64203	1.75	0.98667
0.70	0.67780	1.80	0.98909
0.75	0.71116	1.85	0.99111
0.80	0.74210	1.90	0.99279
0.85	0.77067	1.95	0.99418
0.90	0.79691	2.00	0.99532
0.95	0.82089	2.50	0.99959
1.00	0.84270	3.00	0.99998
1.05	0.86244	3.30	0.999998

[a] The error function is tabulated extensively in several references; see for example, Abramowitz and Stegun (1965, pp. 297–316).

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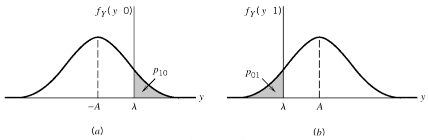
4.3 Error Rate Due to Noise



$$
\begin{aligned}
p_{10} &= P(y > \lambda \,|\, \text{symbol 0 was sent}) \\
&= \int_{\lambda}^{\infty} f_Y(y|0)\, dy \\
&= \frac{1}{\sqrt{\pi N_0/T_b}} \int_{\lambda}^{\infty} \exp\left(-\frac{(y+A)^2}{N_0/T_b}\right) dy
\end{aligned}
$$

- Define a new variable z as:

$$z = \frac{y + A}{\sqrt{N_0/T_b}}$$

- Thus

$$
\begin{aligned}
p_{10} &= \frac{1}{\sqrt{\pi}} \int_{(A+\lambda)/\sqrt{N_0/T_b}}^{\infty} \exp(-z^2)\, dz \\
&= \frac{1}{2} \operatorname{erfc}\left(\frac{A+\lambda}{\sqrt{N_0/T_b}}\right)
\end{aligned}
$$

- Similarly, *conditional probability of error, given that symbol 1 was sent is*.

$$
\begin{aligned}
p_{01} &= \frac{1}{\sqrt{\pi}} \int_{(A-\lambda)/\sqrt{N_0/T_b}}^{\infty} \exp(-z^2)\, dz \\
&= \frac{1}{2} \operatorname{erfc}\left(\frac{A-\lambda}{\sqrt{N_0/T_b}}\right)
\end{aligned}
$$

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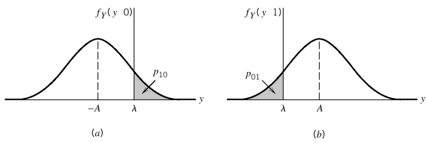
4.3 Error Rate Due to Noise



- Let p_a and p_1 denote the *a priori* probabilities of transmitting symbols 0 and 1, respectively.
- Hence, the *average probability of symbol error* P_e in the receiver is given by:

$$
\begin{aligned}
P_e &= p_0 p_{10} + p_1 p_{01} \\
&= \frac{p_0}{2} \operatorname{erfc}\left(\frac{A+\lambda}{\sqrt{N_0/T_b}}\right) + \frac{p_1}{2} \operatorname{erfc}\left(\frac{A-\lambda}{\sqrt{N_0/T_b}}\right)
\end{aligned}
$$

- What is the optimal value of the threshold λ that minimizes the error probability P_e ?
- We need to derive P_e and equate it to zero.
- For this optimization we use *Leibniz's rule*

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Leibniz's Rule

$$\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} \exp(-z^2)\, dz$$

- Consider the integral $\int_{a(u)}^{b(u)} f(z, u)\, dz$
- *Leibniz's rule* states that the derivative of this integral with respect to *u* is

$$\frac{d}{du} \int_{a(u)}^{b(u)} f(z, u)\, dz = f(b(u), u)\, \frac{db(u)}{du} - f(a(u), u)\, \frac{d(a(u))}{du} + \int_{a(u)}^{b(u)} \frac{\partial f(z, u)}{\partial u}\, dz$$

- For the problem at hand, we note from the definition of the complementary error function that: $f(z, u) = \frac{2}{\sqrt{\pi}} \exp(-z^2)$

$$a(u) = u$$
$$b(u) = \infty$$

- The application of Leibniz's rule to the complementary error function thus yields

$$\frac{d}{du} \operatorname{erfc}(u) = -\frac{1}{\sqrt{\pi}} \exp(-u^2)$$

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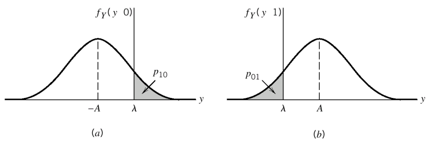
4.3 Error Rate Due to Noise



- Hence, differentiating P_e with respect to λ by making use of the Leibniz's rule, then setting the result equal to zero and simplifying terms, we obtain the optimum threshold as:

$$\lambda_{\text{opt}} = \frac{N_0}{4AT_b} \log\left(\frac{p_0}{p_1}\right)$$

- For the special case when symbols 1 and 0 are equiprobable, we have

$$p_1 = p_0 = \frac{1}{2}$$

- And that leads to $\lambda_{\text{opt}} = 0$

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4.3 Error Rate Due to Noise

- This result is intuitively satisfying as it states that, for the transmission of **equiprobable** binary symbols, we should choose the threshold at the midpoint between the pulse heights —*A* and *+A* representing the two symbols 0 and 1.
- Note that for this special case we also have $p_{01} = p_{10}$
- A channel for which the conditional probabilities of error p_{01} and p_{10} are equal is said to be *binary symmetric.*

4.3 Error Rate Due to Noise

- Correspondingly, for **equiprobable binary polar NRZ PCM**, the average probability of symbol error

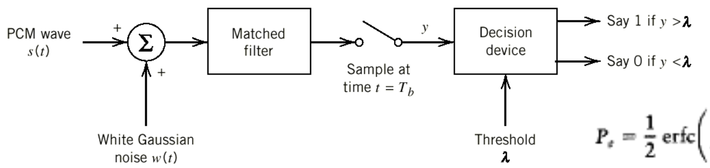
$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\frac{A}{\sqrt{N_0/T_b}}\right)$$

- Since *transmitted signal energy per bit* is defined as $E_b = A^2 T_b$
- Accordingly, we may finally formulate the average probability of symbol error to be:

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

- which shows that *the average probability of symbol error in a binary symmetric channel depends solely on E_b / N_0, the ratio of the transmitted signal energy per bit to the noise spectral density.*

4.3 Error Rate Due to Noise

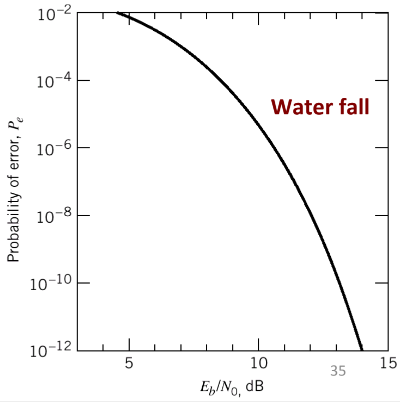


$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

- Using the upper bound on the complementary error function, we may correspondingly bound the average probability of symbol error for the PCM receiver

$$P_e < \frac{\exp(-E_b/N_0)}{2\sqrt{\pi E_b/N_0}}$$

- The PCM receiver therefore exhibits an ***exponential*** improvement in the average probability of symbol error with increase in E_b/N_0



4.3 Error Rate: Example

Q) A binary PCM system using polar NRZ signaling operates just above the error threshold with an average probability of error equal to 10^{-5}. Suppose that the signaling rate is doubled. Find the new value of the average probability of error. You may use Table A6.6 to evaluate the complementary error function.

A) For a binary PCM system, with NRZ signaling, the average probability of error is

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_o}}\right)$$

The signal energy per bit is $E_b=A^2T_b$, where A is the pulse amplitude and T_b is the bit duration.

If the signaling rate is doubled, the bit duration T_b is reduced by half.

➔ E_b is reduced by half

4.3 Error Rate: Example (cont)

- Let $u=\sqrt{\frac{E_b}{N_o}}$, then for $P_e=10^{-6}=\frac{1}{2}erfc(u)$, we get u=3.3
- Now when the signaling rate is doubled, the new value of P_e is:

$$P_e=\frac{1}{2}erfc\left(\frac{u}{\sqrt{2}}\right)$$
$$=\frac{1}{2}erfc(2.33)=10^{-3}$$

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4.4 Intersymbol Interference (ISI)

- The next source of bit errors in a baseband-pulse transmission system that we wish to study is intersymbol interference (ISI), which arises when the communication channel is *dispersive.*

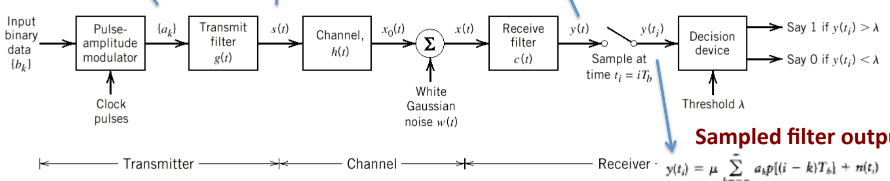
Transmitted Signal

Receive filter output

$$a_k=\begin{cases}+1 & \text{if symbol } b_k \text{ is } 1\\ -1 & \text{if symbol } b_k \text{ is } 0\end{cases}$$

$$s(t)=\sum_k a_k g(t-kT_b)$$

$$y(t)=\mu\sum_k a_k p(t-kT_b)+n(t)$$



Sampled filter output

$$y(t_i)=\mu\sum_{k=-\infty}^{\infty}a_k p[(i-k)T_b]+n(t_i)$$
$$=\mu a_i+\mu\sum_{\substack{k=-\infty\\k\neq i}}^{\infty}a_k p[(i-k)T_b]+n(t_i)$$

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4.4 Intersymbol Interference

- The receive filter output is written as

$$y(t) = \mu \sum_k a_k p(t - kT_b) + n(t)$$

- where μ is a scaling factor and the pulse *p(t)* is to be defined.
- The scaled pulse *μp(t)* is obtained by a double convolution involving the impulse response g*(t)* of the transmit filter, the impulse response *h(t)* of the channel, and the impulse response *c(t)* of the receive filter, as shown by:

$$\mu p(t) = g(t) \star h(t) \star c(t)$$

- In the frequency domain, we get:

$$\mu P(f) = G(f)H(f)C(f)$$

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4.4 Intersymbol Interference

- The receive filter output *y(t)* is sampled at time $t_i = iT_b$ (with *i* taking on integer values), yielding:

$$\begin{aligned} y(t_i) &= \mu \sum_{k=-\infty}^{\infty} a_k p[(i-k)T_b] + n(t_i) \\ &= \underbrace{\mu a_i}_{\text{Desired Signal}} + \underbrace{\boxed{\mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p[(i-k)T_b]}}_{\text{intersymbol interference}} + \underbrace{n(t_i)}_{\text{White Gaussian Noise}} \end{aligned}$$

- How to deal with ISI ?
  - Eliminate ISI by an appropriate pulse design
  - Cancel ISI by equalizers

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4.5 Nyquist's Criteria for Distortionless Baseband Binary Transmission

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k p[(i-k)T_b] + n(t_i)$$

$$= \mu a_i + \boxed{\mu \sum_{\substack{k=-\infty \\ k\neq i}}^{\infty} a_k p[(i-k)T_b]} + n(t_i)$$

Keep a_i at the sampling time t_i

Eliminate the ISI at sampling time t_i

- Thus, we need to design the pulse $p(t)$ such as:

$$p(iT_b - kT_b) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases}$$

Where p(0)=1

- Therefore, the receiver output $y(t_i)$ will be:

$$y(t_i) = \mu a_i$$

4.5 Nyquist's Criteria in the Frequency Domain

- Recall that sampling in the time domain produces periodicity in the frequency domain.
- In particular, we may write

$$P_\delta(f) = R_b \sum_{n=-\infty}^{\infty} P(f - nR_b)$$

Where $R_b=1/T_b$ is the bit rate in bits per second

- $P_\delta(f)$ is the Fourier transform of an infinite periodic sequence of delta functions of period T_b , whose individual areas are weighted by the respective sample values of *p(t).*

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} [p(mT_b)\ \delta(t - mT_b)] \exp(-j2\pi ft)\ dt$$

$$P_\delta(f) = \int_{-\infty}^{\infty} p(0)\ \delta(t) \exp(-j2\pi ft)\ dt$$
$$= p(0)$$

4.5 Nyquist's Criteria in the Frequency Domain

- Thus, Nyquist's Criteria in the Frequency Domain is:

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b$$

- In the time domain, the pulse should satisfy:

$$p(iT_b - kT_b) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases}$$

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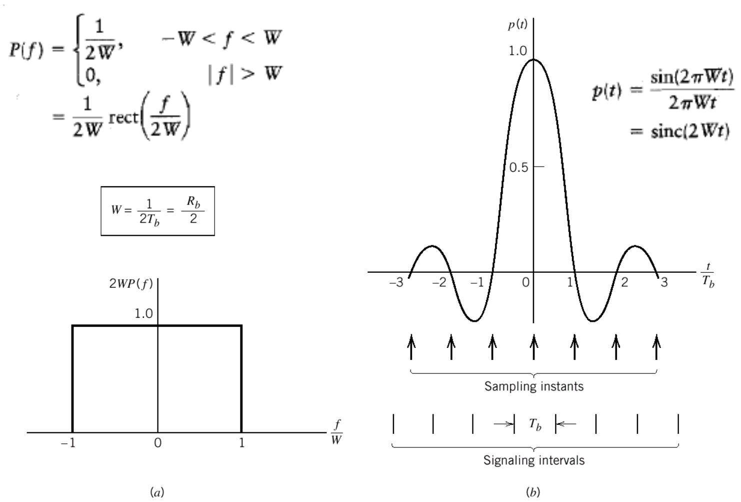
4.5 Ideal Nyquist Channel



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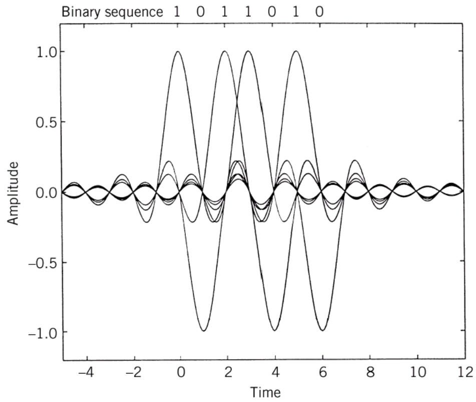
4.5 Ideal Nyquist Channel

- A series of sinc pulses corresponding to the sequence 1011010.



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4.5 Practical difficulties of the Ideal Nyquist Channel

1. It requires that the magnitude characteristic of P(f) be flat from –*W to W,* and zero elsewhere. This is physically unrealizable because of the abrupt transitions at the band edges ± *W.*
2. The function *p(t)* decreases as 1/t f*or* large *t,* resulting in a slow rate of decay. This is also caused by the discontinuity of P(/) at ± W. Accordingly, there is practically no margin of error in sampling times in the receiver.

- Solution ➔ Raised Cosine Spectrum

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4.5 Raised Cosine Spectrum

- We may overcome the practical difficulties encountered with the ideal Nyquist channel by extending the bandwidth from the minimum value W = $R_b/2$ to an adjustable value between W and 2W.
- We now specify the overall frequency response P(*f*) to satisfy a condition more elaborate than that for the ideal Nyquist channel; specifically, we retain three terms and restrict the frequency band of interest to [—W , W], as shown by:

$$P(f)+P(f-2W)+P(f+2W)=\frac{1}{2W}, \quad -W \le f \le W$$

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4.5 Raised Cosine Spectrum

- A particular form of *P(f)* that embodies many desirable features is provided by a *raised cosine spectrum.* This frequency response consists of a *flat* portion and a *rolloff* portion that has a sinusoidal form, as follows:

$$P(f)=\begin{cases}\dfrac{1}{2W}, & 0 \le |f| < f_1 \\ \dfrac{1}{4W}\left\{1-\sin\left[\dfrac{\pi(|f|-W)}{2W-2f_1}\right]\right\}, & f_1 \le |f| < 2W-f_1 \\ 0, & |f| \ge 2W-f_1\end{cases}$$

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4.5 Raised Cosine Spectrum

- The frequency parameter f_1 and bandwidth *W are* related by

$$\alpha = 1 - \frac{f_1}{W}$$

- The parameter α is called the *rolloff factor;* it indicates the *excess bandwidth* over the ideal solution, *W.* Specifically, the transmission bandwidth B_T is defined by:

$$\begin{aligned} B_T &= 2W - f_1 \\ &= W(1+\alpha) \end{aligned}$$

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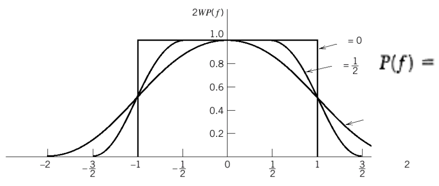
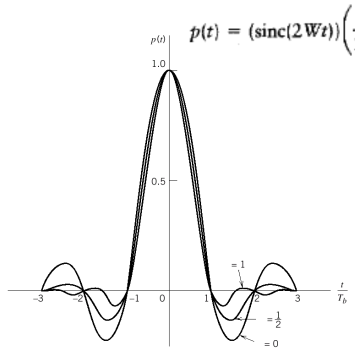
4.5 Raised Cosine Spectrum



(a)

$$P(f) = \begin{cases} \dfrac{1}{2W}, & 0 \le |f| < f_1 \\ \dfrac{1}{4W}\left\{1 - \sin\left[\dfrac{\pi(|f| - W)}{2W - 2f_1}\right]\right\}, & f_1 \le |f| < 2W - f_1 \\ 0, & |f| \ge 2W - f_1 \end{cases}$$

$$p(t) = (\mathrm{sinc}(2Wt))\left(\frac{\cos(2\pi\alpha Wt)}{1 - 16\alpha^2W^2t^2}\right)$$



(b)

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4.5 Raised Cosine Spectrum

$$p(t) = (\text{sinc}(2Wt))\left(\frac{\cos(2\pi\alpha Wt)}{1 - 16\alpha^2 W^2 t^2}\right)$$

- The time response *p(t)* consists of the product of two factors:
  - The factor sinc(2Wt) characterizing the ideal Nyquist channel
  - A second factor that decreases as $l/\backslash t\backslash^2$ for large $\backslash t \backslash$.
- The first factor ensures zero crossings of *p(t)* at the desired sampling instants of time *t = iT* with *i* an integer (positive and negative).
- The second factor reduces the tails of the pulse considerably below that obtained from the ideal Nyquist channel, so that the transmission of binary waves using such pulses is relatively insensitive to sampling time errors.

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4.5 Raised Cosine: Example

Q) A computer puts out binary data at the rate of *56* kb/s. The computer output is transmitted using a baseband binary PAM system that is designed to have a raised-cosine spectrum. Determine the transmission bandwidth required for each of the following rolloff factors: α = 0.25, 1.0.

A) The transmission bandwidth of a raised cosine spectrum is

$B_T = W(1+\alpha)$, where W is the Nyquist channel bandwidth $W = \frac{R_b}{2}$

Then for the above case, W=28 kHz.

For **α=0.25,** BT=35 kHz.

For **α=1,** BT=56 kHz.

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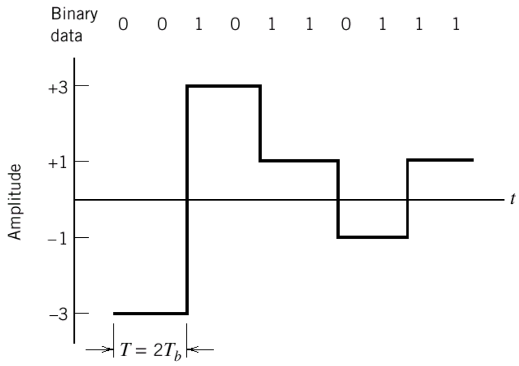
4.7 Baseband M-ary PAM Transmission

Output of a quaternary system



(*a*)

Representation of the 4 possible dibits, based on Gray encoding.

Dibit	Amplitude
00	−3
01	−1
11	+1
10	+3

(*b*)

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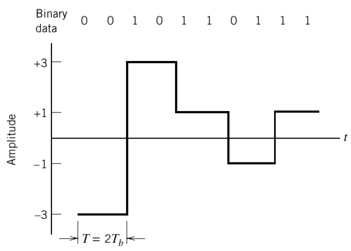
4.7 Baseband M-ary PAM Transmission



(*a*)

Dibit	Amplitude
00	−3
01	−1
11	+1
10	+3

(*b*)

Consider then an M-ary PAM system with a signal alphabet that contains M equally likely and statistically independent symbols, with the symbol duration denoted by T seconds.
We refer to 1/T as the *signaling rate* of the system, which is expressed in *symbols per second,* or *bauds*

$$T = T_b \log_2 M$$

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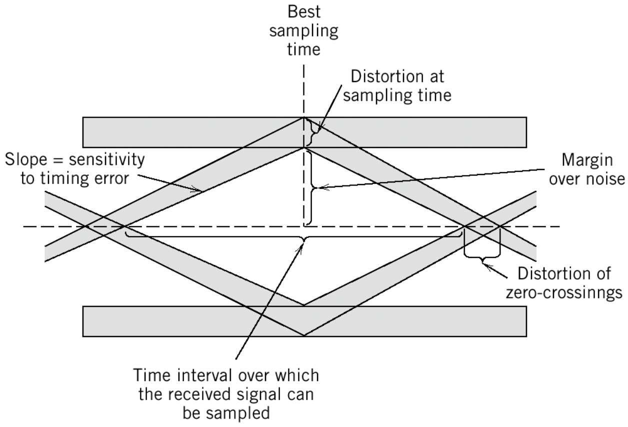
4.7 Baseband M-ary PAM Transmission

- Therefore, in a given channel bandwidth, we find that by using an M-ary PAM system, we are able to transmit information at a rate that is $\log_2 M$ faster than the corresponding binary PAM system.
- However, to realize the same average probability of symbol error, an M-ary PAM system requires more transmitted power.
- Specifically, we find that for M much larger than 2 and an average probability of symbol error small compared to 1, the transmitted power must be increased by the factor $\mathbf{M^2/\log_2 M}$, compared to a binary PAM system.

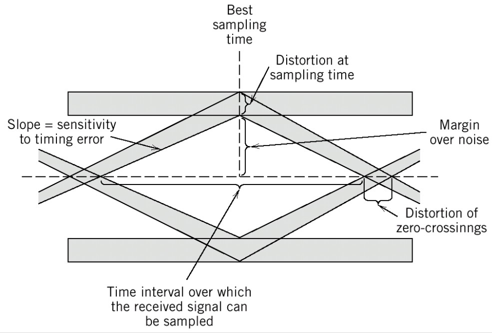
4.11 Eye Pattern

- *eye pattern,* which is defined as the synchronized superposition of all possible realizations of the signal of interest (e.g., received signal, receiver output) viewed within a particular signaling interval.



4.11 Eye Pattern

- The width of the eye opening defines the *time interval over which the received signal can be sampled without error from intersymbol interference;* it is apparent that the preferred time for sampling is the instant of time at which the eye is open the widest.
- The *sensitivity of the system to timing errors* is determined by the rate of closure of the eye as the sampling time is varied.
- The height of the eye opening, at a specified sampling time, defines the *noise margin* of the system.



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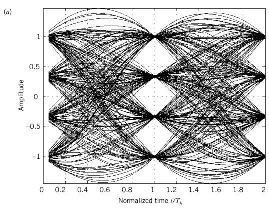
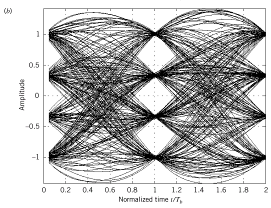
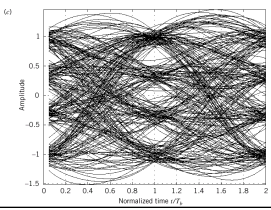
4.11 Eye Pattern: Effect of channel noise



(a) Eye diagram for noiseless quaternary system.



(b) Eye diagram for quaternary system with SNR = 20 dB.



(c) Eye diagram for quaternary system with SNR = 10 dB.

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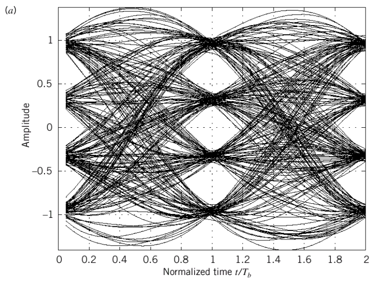
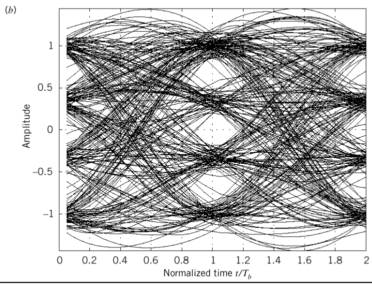
4.11 Eye Pattern: Effect of Bandwidth Limitation



(a) Eye diagram for noiseless band-limited quaternary system: cutoff frequency f_o = 0.975 Hz.



(b) Eye diagram for noiseless band-limited quaternary system: cutoff frequency f_o = 0.5 Hz.

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